Joint Relay Selection and Power Allocation for Two-way Relay Channels with Asymmetric Traffic Requirements

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Abstract

This paper studies relay selection and power allocation for amplify-and-forward (AF) based two-way relay networks (TWRN) with asymmetric traffic requirements (ATR). A joint relay selection and power allocation algorithm is proposed to decrease the outage probability of TWRN with ATR. In this algorithm, two sources exchange information with the help of the relay during two time slots. We first calculate the optimal power allocation parameters based on instantaneous channel state information (CSI), and then derive a tight lower bound of outage probability. Furthermore, we propose a simplified relay selection criterion, which can be easily calculated as harmonic mean of instantaneous channel gains, according to the outage probability expressions. Simulation results verified the theoretical analyses we presented. It is shown that the outage probability of our algorithm improves 3-4dB comparing with that of other existing algorithms, and the lower bound is tight comparing with actual value for the entire signal-to-noise ratio (SNR) region.

Key words: Two-way relay, outage probability, Amplify-and-Forward, asymmetric traffic requirements, joint relay selection and power allocation

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1. Introduction

Cooperative communications has been widely studied recently for its ability to enhance the throughput and reliability of wireless networks [1]. However, traditional one-way relay protocol causes an inherent spectral efficiency loss since all nodes operate under half-duplex mode. Two-way relaying is proposed to improve spectral efficiency by avoiding the 1/2 pre-log factor in capacity calculating [2]. Several network-coding schemes have been presented for two-way relay systems, such as amplify-and-forward (AF), decode-and-forward (DF), etc. Two-way AF relaying is more practically attractive than DF relaying due to the simple processing at the relay node. The received signal-to-noise ratio (SNR) expression of two-way AF relaying had been given in [3] and the achievable sum rate had been derived in [2].

Relay selection (RS) protocol is widely used to improve the performance of two-way relaying system. Most of the RS schemes select one best relay to forwarding the information by a certain criterion. Relay selection in two-way AF relaying over Rayleigh fading had been presented in [4]–[8]. A relay selection scheme was proposed for AF-based TWRN, where bounds on the sum rate, symbol error rate and outage probability were analyzed in high SNR range [4]. Closed-form expression of the outage probability was derived for analog network coding-based TWRN with single RS in [5]. The selection criterion in above two schemes was based on maximizing the minimum receiving signal-to-noise ratio. And some other criteria such as maximizing sum rate, minimizing outage probability or maximizing mutual information were proposed in [6]-[8].

Optimal power allocation (OPA) is often associated with relay selection to enhance system performance. A joint scheme combined relay selection and power allocation by maximizing the balanced SNR was firstly proposed in [9], in which the author proposed that the minimum error probability was obtained while the SNR of bidirectional channels was equal to each other. In [10], author proposed a power allocation algorithm which is capable of simultaneously minimizing the outage probability and maximizing the total mutual information under total constant power constraint for a single-relay two-way system. The algorithm was extended to multi-relay system in [11], where the relay with minimum outage probability was selected to assist information forwarding. The power allocation algorithms in [10]-[11] were based on instantaneous CSI under a total transmit power budget. However, these algorithms were limited to a two-way system with symmetric traffic requirements.

To gain more insight, we consider a more general setup including symmetric and asymmetric traffic flows and channels, which is more practical in realistic communications. To the best of our knowledge, there have been few papers on investigating outage probability performance of joint relay and optimum power allocation algorithm in this setup, which has motivated our work. The authors in [12] had firstly investigated the system outage performance of TWRN-AF with asymmetric traffic requirements. The authors in [13] combined relay selection with incremental relaying for TWRN-ATR in which two sources exchanged information through both relay and direct path. In [14], a golden sec-

tion method was used to obtain power allocation parameters of the two-way communications. In [15], the author examined the impact of traffic asymmetry on system outage probability, and proposed a power allocation algorithm on the basis of different channel gains and required rates of the bi-directional networks. However, though most of previous power allocation algorithms, which based on average channel gains, have relatively lower computation complexity but cannot provide a stable and efficient performance gain for different channel states.

In this paper, we propose an algorithm based on instantaneous CSI to allocate transmit power among two sources and the best relay. It is worthy noticed that in the outage probability expression, the widely used approximation $\frac{xy}{x+y} \approx \min(x, y)$, which is different than actual value by at most 3dB due to $\frac{1}{2} \le \frac{xy/(x+y)}{\min(x, y)} \le 1$, may cause up to K times deviation in the performance analysis of multi-relay systems [11]. Therefore, we propose a novel tight lower bound to decrease the gap between analytical results and actual results. The bound is more accurate than any existing bound for the entire SNR region. Furthermore, a simple relay selection criterion is proposed based on the outage probability expression of the whole system.

The rest of this paper is organized as follows. In Section II, we present the system model. In Section III, we propose a joint relay selection and power allocation algorithm for two-way system with ATR, and derive a tight lower bound of outage probability. Numerical simulations are presented in Section IV and Section V concludes the paper.

2. System model

As shown in **Fig. 1**, the system we considered is a time division duplex (TDD) bidirectional network, where two sources tend to exchange information via one of the K relay nodes over Rayleigh flat-fading channels. We use S_1, S_2 and R_i to denote the first source, the second source, and the *i*th relay, respectively. The sources select the best relay from all the candidate relays to assist information exchanging. All nodes are equipped with single antenna, and operate in a half-duplex mode. The signals are coded by using binary phase shift keying (BPSK) modulation at each source. We assume that there is no direct path between sources. However, the performance analysis can be easily extended to the case with direct path.

Considering channel reciprocity, the channel gains between S_1 and R_i, R_i and S_2 are denoted by h_i, f_i respectively, which are complex Gaussian random variables (RV) with zero mean and variances Ω_{h_i} and Ω_{f_i}. The additive white Gaussian noise (AWGN) at S_1, S_2 and R_i are assumed to be a complex Gaussian random variable with zero mean and unit variance. It is assumed that sources have full CSI knowledge to implement perfect

self-interference cancelation, relay selection and optimal power allocation in AF-based TWRN.

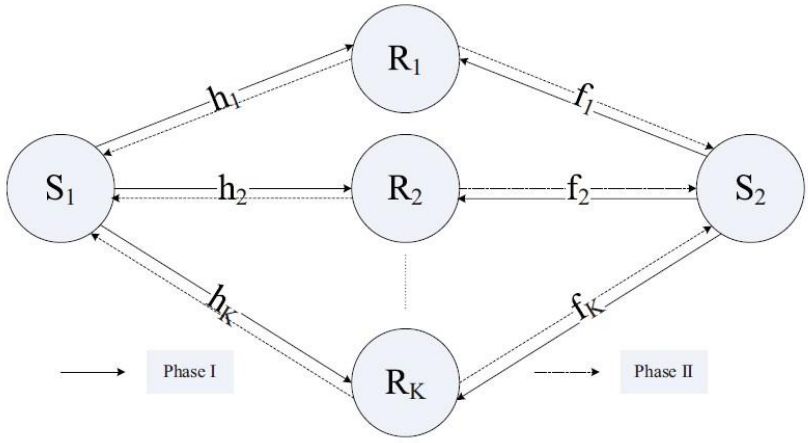

Fig. 1. System Model

The protocol shown in **Fig.1** takes two time slots to exchange information. At first time slot, S_1 and S_2 send signals to all candidate relay nodes. The received signal at *i*th relay is expressed as

$$x_i = \sqrt{E_1} h_i x_1 + \sqrt{E_2} f_i x_2 + n_i , \tag{1}$$

where x_1, x_2 are signals from S_1 and S_2 respectively, E_1 and E_2 are transmit powers at S_1 and S_2, n_i is Gaussian noise at relay *i*.

At second time slot, a relay with best performance amplifies x_i with an amplifying coefficient ρ and transmits it to S_1 and S_2. The amplifying coefficient ρ can be chosen as

$$\rho_i = \sqrt{\frac{E_r}{E_1 |h_i|^2 + E_2 |f_i|^2 + N_i}} , \tag{2}$$

where E_r denotes power consumption at the selected relay.
In the following research, it is approximated as

$$\rho_i = \sqrt{\frac{E_r}{E_1 |h_i|^2 + E_2 |f_i|^2}} , \tag{3}$$

which has been proved very close to the exact value for the entire SNR region [16]-[17].
The received signal at S_1 and S_2 can be expressed as

$$y_{i1} = \sqrt{E_1} \rho_i h_i^2 x_1 + \sqrt{E_2} \rho_i h_i f_i x_2 + \rho_i h_i n_i + n_1 , \tag{4}$$

$$y_{i2} = \sqrt{E_2}\rho_i f_i^2 x_2 + \sqrt{E_1}\rho_i h_i f_i x_1 + \rho_i f_i n_i + n_2, \tag{5}$$

where n_1, n_2 denote Gaussian noise at S_1 and S_2. We can subtract the self-interference from y_{i1} and y_{i2}, then obtain

$$\tilde{y}_{i1} = y_{i1} - \sqrt{E_1}\rho_i h_i^2 x_1 = \sqrt{E_2}\rho_i h_i f_i\ x_2 + \rho_i h_i n_i + n_1, \tag{6}$$

$$\tilde{y}_{i2} = y_{i2} - \sqrt{E_2}\rho_i f_i^2 x_2 = \sqrt{E_1}\rho_i h_i f_i\ x_1 + \rho_i f_i n_i + n_2. \tag{7}$$

The sources can obtain signals x_2 and x_1 by decoding the received signals $\tilde{y}_{i1}$ and $\tilde{y}_{i2}$.

The instantaneous SNR at S_1 and S_2 can be respectively expressed as

$$\gamma_{i1} = \frac{E_r E_2 |h_i|^2 |f_i|^2}{(E_r + E_1)|h_i|^2 + E_2 |f_i|^2}, \tag{8}$$

$$\gamma_{i2} = \frac{E_r E_1 |h_i|^2 |f_i|^2}{(E_r + E_2)|f_i|^2 + E_1 |h_i|^2}. \tag{9}$$

The corresponding mutual information at S_1 and S_2 is given as $I_{i1} = \frac{1}{2}\log_2(1+\gamma_{i1})$, $I_{i2} = \frac{1}{2}\log_2(1+\gamma_{i2})$. Pre-log factor 1/2 presents information exchange takes two time slots.

3. Outage probability analysis

In this section, outage probability performance of joint relay selection and optimum power allocation scheme with asymmetric traffic requirements is analyzed. Furthermore, a tight lower bound of outage probability of this joint scheme is derived.

Since the considered protocol is a multi-source system, either source's outage will cause outage of the whole system. With asymmetric rates requirements R_{th1}, R_{th2} $(R_{th1} \neq R_{th2})$ at the source S_1 and S_2, we can express the outage probability of AF-based TWRN-ATR protocol as

$$P_{outage}(R_i) = \Pr(I_{i1} < R_{th1}\ or\ I_{i2} < R_{th1}) = \Pr(\gamma_{i1} < \mathrm{r}_{th1}\, or\, \gamma_{i2} < \mathrm{r}_{th2}), \tag{10}$$

where $r_{th1} = 2^{2R_{th1}} - 1, r_{th2} = 2^{2R_{th2}} - 1$.

Total power constraint of the joint scheme is assumed as $E_{\max} = 3E$, as 3E ($E_1 = E_2 = E_r = E$) power consumption in traditional equal power allocation (EPA) system.

The optimum power allocation (OPA) problem is represented as

$$\min_{E_1,E_2,E_r,i}\left(P_{outage}(R_i)\right)\ \ subject\ to\ E_1+E_2+E_r=3E\,. \tag{11}$$

Following from (8)-(10) and opportunistic relay selection criterion $k=\arg\max_i\left(\min\left(\gamma_{i1},\gamma_{i2}\right)\right)$ [2], the optimization problem in (11) is equivalent to the following formulation [13].

$$\max_{E_1,E_2,E_r}\min\left(\gamma_{i1},\gamma_{i2}\right)\ subject\ to\ E_1+E_2+E_r=3E\,. \tag{12}$$

The max-min problem can be solved in the next theorem.

Theorem 1: When $E_1+E_2+E_r=3E$, the optimum power allocation that minimizes the outage probability of the AF-based TWRN protocol with ATR is given by

$$E_1=\frac{3r_{th2}\left|f_i\right|}{(r_{th1}+r_{th2})(\left|h_i\right|+\left|f_i\right|)}E\,, \tag{13}$$

$$E_2=\frac{3r_{th1}\left|h_i\right|}{(r_{th1}+r_{th2})(\left|h_i\right|+\left|f_i\right|)}E\,, \tag{14}$$

$$E_r=\frac{3(r_{th2}\left|h\right|+r_{th1}\left|f_i\right|)}{(r_{th1}+r_{th2})(\left|h_i\right|+\left|f_i\right|)}E\,. \tag{15}$$

Proof: See Appendix A.

It is obviously that the optimal scheme allocates more power to the weaker traffic flow, which has worse channel gain and higher rate requirement, for lower outage probability of the whole system. With the allocation parameters, the SNR in (8) (9) can be expressed as

$$\gamma_{i,1}=\frac{3r_{th1}E}{r_{th1}+r_{th2}}\frac{\left|h_i f_i\right|^2}{(\left|h_i\right|+\left|f_i\right|)^2}\,, \tag{16}$$

$$\gamma_{i,2}=\frac{3r_{th2}E}{r_{th1}+r_{th2}}\frac{\left|h_i f_i\right|^2}{(\left|h_i\right|+\left|f_i\right|)^2}\,. \tag{17}$$

Now we calculate the outage probability for $S_1-R_i-S_2$ sub-channel.

$$\begin{aligned}P_{outage}(R_i)&=\Pr(I_{i1}<R_{th1}\ or\ I_{i2}<R_{th1})\\&=\Pr(\gamma_{i1}<r_{th1}or\gamma_{i2}<r_{th2})\\&=\Pr(\frac{3r_{th1}E}{r_{th1}+r_{th2}}\frac{\left|h_i f_i\right|^2}{(\left|h_i\right|+\left|f_i\right|)^2}<r_{th1}or\frac{3r_{th2}E}{r_{th1}+r_{th2}}\frac{\left|h_i f_i\right|^2}{(\left|h_i\right|+\left|f_i\right|)^2}<r_{th2})\,.\end{aligned} \tag{18}$$

Outage probability of the whole system can be expressed as

$$P_{outage}(R)=\prod_{i=1}^{K}P_{outage}(R_i)\,. \tag{19}$$

Since the two mutual information I_{i1} and I_{i2} are highly correlated, it is very difficult to directly calculate the outage probability of (19) even in high SNR region. Thus we derive a tight lower bound of $P_{outage}(R)$ in the following theorem.

Theorem 2: The lower bound of outage probability with the help of K relays can be expressed as

$$\begin{aligned}P_{lower}(R)=\prod_{i=1}^{K}\Bigg(1-\sum_{n=0}^{N-1}\Bigg(&\frac{a_n^2\lambda_{h_i}}{a_n^2\lambda_{h_i}+\lambda_{f_i}}\exp((-(b_n+1)^2\lambda_{h_i}+\frac{(b_n+1)^2}{a_n^2}\lambda_{f_i})\Upsilon)\\&-\frac{b_n^2\lambda_{h_i}}{b_n^2\lambda_{h_i}+\lambda_{f_i}}\exp((-(b_n+1)^2\lambda_{h_i}+\frac{(b_n+1)^2}{b_n^2}\lambda_{f_i})\Upsilon)\\&+\frac{a_n^2\lambda_{f_i}}{\lambda_{h_i}+a_n^2\lambda_{f_i}}\exp((-\frac{(b_n+1)^2}{a_n^2}\lambda_{h_i}+(b_n+1)^2\lambda_{f_i})\Upsilon)\\&-\frac{b_n^2\lambda_{f_i}}{\lambda_{h_i}+b_n^2\lambda_{f_i}}\exp((-\frac{(b_n+1)^2}{b_n^2}\lambda_{h_i}+(b_n+1)^2\lambda_{f_i})\Upsilon)\Bigg)\\&-\frac{\lambda_{f_i}}{b_N^2\lambda_{h_i}+\lambda_{f_i}}\exp((-b_N^2\lambda_{h_i}+\lambda_{f_i})\Upsilon)-\frac{\lambda_{h_i}}{\lambda_{h_i}+b_N^2\lambda_{f_i}}\exp((-\lambda_{h_i}+b_N^2\lambda_{f_i})\Upsilon)\Bigg),\end{aligned}\tag{20}$$

where $a_n=1+(n+1)\Delta$, $b_n=1+n\Delta$, $\Upsilon=\frac{r_{th1}+r_{th2}}{3E}$.

Proof: See Appendix B

The values of Δ and N decide the accuracy of outage probability bound. When $\Delta<<1$, (36) can be approximated as $\frac{|h_i||f_i|}{|h_i|+|f_i|}\approx\frac{|h_i|}{(2+n\Delta)}$, thus $P_{lower}(R)$ becomes extremely tight.

The relay selection problem can be expressed as

$$k=\arg\min_i(P_{outage}(R_i)).\tag{21}$$

Noticing that $P_{outage}(R_i)$ is a monotonically function of equivalent SNR, the relay selection problem can be simplified as follows

$$k=\arg\max_i\left(\frac{|h_i||f_i|}{|h_i|+|f_i|}\right)(for\ i=1,2,...,K).\tag{22}$$

The above criterions can instead the criterion in [11] for less computation complexity. Therefore, the relay selection process can be described as follows: One of the sources,

which have full CSI knowledge, calculates harmonic mean of $|h_i|$ and $|f_i|$, selects the relay with the maximum harmonic mean, and then broadcasts the index to all relays.

4. Numerical results

In this section, Monte-Carlo simulations are performed to verify the accuracy of our analytical results and compare performance gain with other algorithm in AF-based TWRN with ATR.

In **Fig. 2**, we depict the system outage probability versus SNR under two different power allocation algorithms in single relay system with symmetric and asymmetric channels. We set (i) $\Omega_{h_i} = \Omega_{f_i} = 1$; (ii) $\Omega_{h_i} = 1$, $\Omega_{f_i} = 0.1$ and the required rates $R_{th1} = 0.5$, $R_{th2} = 1$ at S_1 and S_2 respectively. As shown in the figure, the proposed OPA algorithm outperforms the EPA counterpart in the entire SNR region. Especially, the performance gap gets widen as SNR increases till the gap reaches the maximum value of 5dB. Meanwhile, the proposed OPA algorithm can help maintain the system balance for asymmetric traffic requirements. Furthermore, using instantaneous CSI instead of average CSI to calculate power allocation parameters, the proposed algorithm significantly outperforms algorithm in [15] although system complexity is slightly increased.

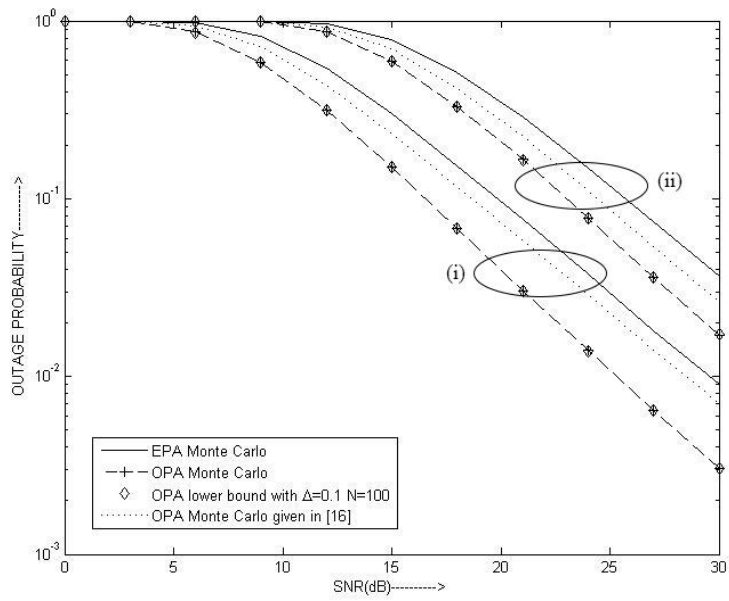

Fig. 2. outage probability in single relay system with (i) symmetric channels and (ii) asymmetric channels

Then we add relay selection to the above comparison. We set $\Omega_{h_i} = \Omega_{f_i}$ ($for\ i = 1, 2 ... K$) in symmetric scenario, and $\Omega_{h_i} = \{0.3, 0.8, 0.5, 0.6\}$, $\Omega_{f_i} = \{0.7, 0.1, 0.5, 0.5\}$ in asymmetric scenario. The required rates at S_1 and S_2 are set as $R_{th1} = 1$ and $R_{th2} = 2$, respectively. In **Fig. 3**, it is seen that the proposed algorithm decreases outage probability by more than 3dB compared with the algorithm in [15], which provides a 1-2dB gain by using power allocation. Furthermore, Fig.2 and Fig.3 compare our lower bound calculated in (20) with the actual results for different cooperative modes and average channel gains. One can see that our lower bound is quite close to the exact outage probability in the entire SNR region.

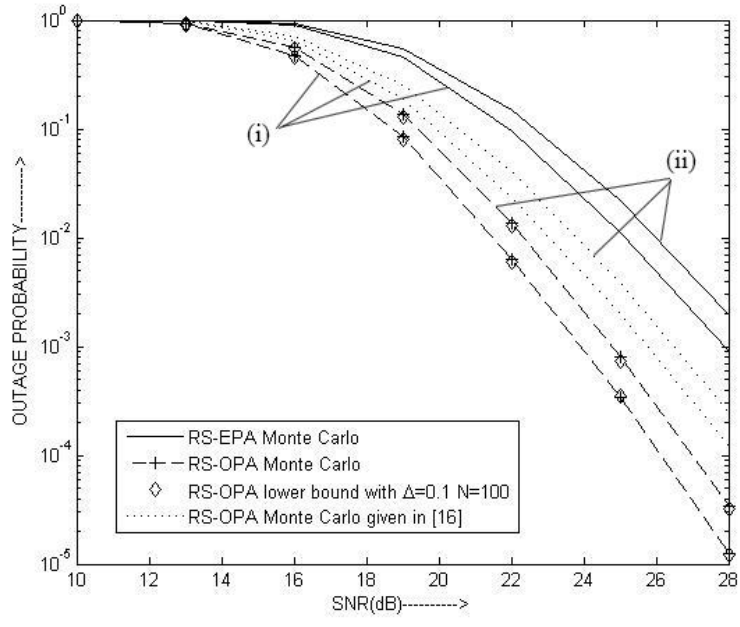

Fig. 3. outage probability in multi-relay system
with (i) symmetric channels and (ii) asymmetric channels

In **Fig. 4**, we compare lower bound we derived with actual results obtained by simulation under different value of N and Δ. We observe the gap between actual results and theoretical results calculated by (20) under the situations (i) N=100, $\Delta = 0.1$ (ii) N=2, $\Delta = 0.5$ (iii) N=0. Particularly, N=0 is a special case of formula (20), which is equivalent to calculating the lower bound by using inequality $\frac{xy}{x+y} < \min(x, y)$. As shown in the figure, the widely used inequality $\frac{xy}{x+y} < \min(x, y)$ in other articles is

unsuitable in this paper since the use of power allocation and relay selection increases the performance gap in low SNR region. In the entire SNR region, formula (20) with parameters N=2, $\Delta = 0.5$ provides a fairly accurate estimation of the outage probability with reasonable computation complexity, which proved that the approximation in *Theorem 2* can improve accuracy comparing with conventional approximation even if $N >> 0$ is unsatisfied. This can provide a trade-off between approximation accuracy and computation complexity.

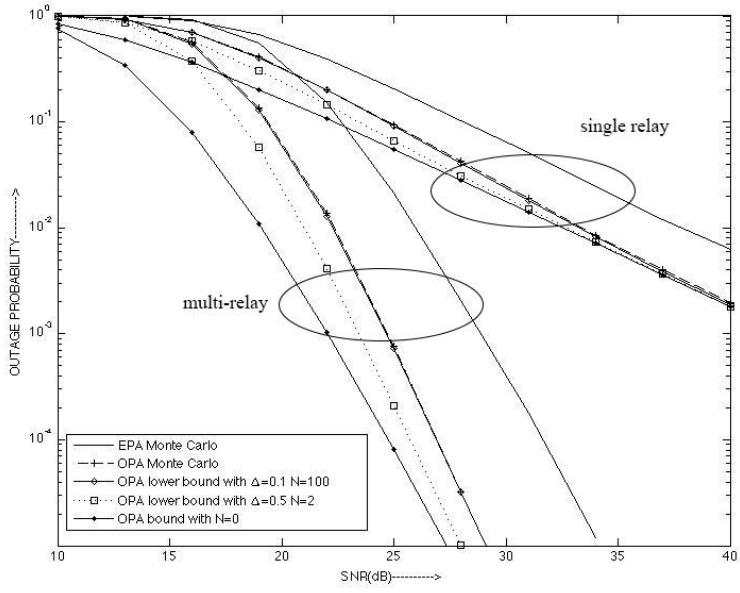

Fig. 4. comparison of lower bound of outage probability in different relay scenarios

Fig. 5 shows the outage probabilities as a function of relay number K at SNR=20dB. The gaps between Monte Carlo results and theoretic results become larger by the number of relays increasing. The power allocation algorithm we proposed maintains a distinct performance gain compared with other algorithms or algorithm without power allocation. Meanwhile, the lower bound we proposed is still tight, and scarcely affected by the number of relays.

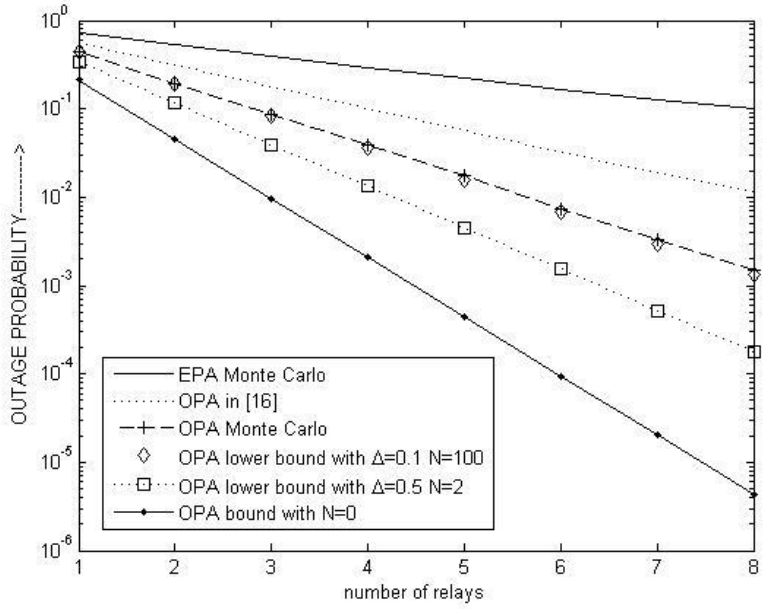

Fig. 5. outage probability against relay number K

5. Conclusions

In this paper, we studied outage performance in AF-based two-way relay networks with asymmetric traffic requirements. We proposed a joint relay selection and optimal power allocation algorithm based on instantaneous CSI, and derived a tight lower bound of outage probability in closed-form. We also proposed a simplified criterion which uses the harmonic mean of the channel gains between relays and sources to select the best relay. Simulation results verified our algorithm has much performance improvement, and proved that the derived bound in this paper was exactly fitting the actual results obtained by Monte-Carlo simulations in the entire SNR region.

Appendix A:

We let $E_1 = 3\alpha E$, $E_2 = 3\beta E$ and $E_r = 3(1-\alpha-\beta)E$, where $0<\alpha<1$, $0<\beta<1$ and $\alpha+\beta \le 1$. We can derive the outage probability expression as

$$\Pr(\gamma_{i1} < \mathrm{r}_{th1}\, or\, \gamma_{i2} < \mathrm{r}_{th2}) = \Pr(\frac{\gamma_{i1}}{\mathrm{r}_{th1}} < 1 or \frac{\gamma_{i2}}{\mathrm{r}_{th2}} < 1) . \tag{23}$$

Based on the above equation, we can express the equivalent SNR as

$$\gamma_1' = \frac{\gamma_1}{r_{th1}} \approx \frac{3E}{r_{th1}} \frac{|hf|^2 \beta(1-\alpha-\beta)}{|h|^2 (1-\beta) + |f|^2 \beta} , \tag{24}$$

$$\gamma_2' = \frac{\gamma_2}{r_{th2}} \approx \frac{3E}{r_{th2}} \frac{|hf|^2 \alpha(1-\alpha-\beta)}{|f|^2 (1-\alpha) + |f|^2 \alpha}. \tag{25}$$

We solve the max-min problem with the method in Section II.C of [18]. Thus, we define a new function as

$$M(\pi_0, \alpha, \beta) = \pi_0 \gamma_1' + (1-\pi_0)\gamma_2'. \tag{26}$$

where $0 \le \pi_0 \le 1$. Let $\alpha(\pi_0)$ and $\beta(\pi_0)$ denote the values of α and β, which maximize $M(\pi_0, \alpha, \beta)$ for a fixed π_0. According to [18], the solution to (26) must belong to the set formed by $\alpha(\pi_0)$ and $\beta(\pi_0)$, or we can say must maximize $M(\pi_0, \alpha, \beta)$.

When $\pi_0 \ne \frac{r_{th1}}{r_{th1}+r_{th2}}$, the values of $\alpha(\pi_0)$ and $\beta(\pi_0)$ are derived in the case of $M(\pi_0, \alpha, \beta)$ being maximized.

$$\begin{cases} \alpha(\pi_0) = 0, \beta(\pi_0) = \frac{|h|}{|h|+|f|}, \text{ when } 0 \le \pi_0 < \frac{r_{th1}}{r_{th1}+r_{th2}} \\ \beta(\pi_0) = 0, \alpha(\pi_0) = \frac{|h|}{|h|+|f|}, \text{ when } \frac{r_{th1}}{r_{th1}+r_{th2}} < \pi_0 \le 1 \end{cases}. \tag{27}$$

When either $\alpha = 0$ or $\beta = 0$, the outage probability of the system is one. This implies that the solution above cannot be the solution to (26).

Therefore, the solution to (26) can only be found at $\pi_0 = \frac{r_{th1}}{r_{th1}+r_{th2}}$. The value of $\alpha(\frac{r_{th1}}{r_{th1}+r_{th2}})$ depends on $\beta(\frac{r_{th1}}{r_{th1}+r_{th2}})$ can be expressed as

$$\alpha(\frac{r_{th1}}{r_{th1}+r_{th2}}) = \frac{|f|(|hf| - (|f|^2 \beta(\frac{r_{th1}}{r_{th1}+r_{th2}}) + |h|^2 (1-\beta(\frac{r_{th1}}{r_{th1}+r_{th2}}))))}{|h|(|f|^2 - |h|^2)}. \tag{28}$$

Meanwhile, the solution to (25) should satisfy $\gamma_1' = \gamma_2'$ according to [9], [10] and [18]. Based on this equation and (28), we can obtain

$$\alpha(\frac{r_{th1}}{r_{th1}+r_{th2}}) = \frac{r_{th2}|f|}{(r_{th1}+r_{th2})(|h|+|f|)}, \tag{29}$$

$$\beta(\frac{r_{th1}}{r_{th1}+r_{th2}}) = \frac{r_{th1}|h|}{(r_{th1}+r_{th2})(|h|+|f|)}. \tag{30}$$

By substituting (29) and (30) into the definition of E_1, E_2 and E_r, we get the power allo-

cation parameters (13)-(15).

Appendix B:

$$P_{outage}(R_i) = \Pr(\gamma_{i1} < \mathrm{r}_{th1}\, or \gamma_{i2} < \mathrm{r}_{th2}) = 1 - \Pr(\gamma_{i1} > \mathrm{r}_{th1}, \gamma_{i2} > \mathrm{r}_{th2})$$

$$= 1 - \Pr(\frac{3r_{th1}E}{r_{th1}+r_{th2}} \frac{|h_i f_i|^2}{(|h_i|+|f_i|)^2} > r_{th1}, \frac{3r_{th2}E}{r_{th1}+r_{th2}} \frac{|h_i f_i|^2}{(|h_i|+|f_i|)^2} > r_{th2})$$

$$= 1 - \Pr(\frac{|h_i f_i|^2}{(|h_i|+|f_i|)^2} > \Upsilon), \tag{31}$$

where $\Upsilon = \frac{r_{th1}+r_{th2}}{3E}$.

$$\Pr(\left(\frac{|h_i||f_i|}{|h_i|+|f_i|}\right)^2 > \Upsilon)$$

$$= \sum_{n=0}^{N-1} (\underbrace{\Pr(\left(\frac{|h_i||f_i|}{|h_i|+|f_i|}\right)^2 > \Upsilon, (1+n\Delta)|f_i| \le |h_i| \le (1+(n+1)\Delta)|f_i|)}_{\Pr_1(n)}$$

$$+ \underbrace{\Pr(\left(\frac{|h_i||f_i|}{|h_i|+|f_i|}\right)^2 > \Upsilon, (1+n\Delta)|h_i| \le |f_i| \le (1+(n+1)\Delta)|h_i|))}_{\Pr_2(n)}$$

$$+ \underbrace{\Pr(\left(\frac{|h_i||f_i|}{|h_i|+|f_i|}\right)^2 > \Upsilon, (1+N\Delta)|f_i| \le |h_i|)}_{\Pr_3}$$

$$+ \underbrace{\Pr(\left(\frac{|h_i||f_i|}{|h_i|+|f_i|}\right)^2 > \Upsilon, (1+N\Delta)|h_i| \le |f_i|)}_{\Pr_4}. \tag{32}$$

Let $X = |h_i|^2$, $Y = |f_i|^2$ X and Y subject exponential distribution with parameters $\lambda_{h_i} = \frac{1}{\Omega_{h_i}}$, $\lambda_{f_i} = \frac{1}{\Omega_{f_i}}$. To calculate $\Pr_1(n)$, $\forall \Delta > 0$ we have

$$\frac{|h_i||f_i|}{|h_i|+|f_i|}=\frac{|h_i||f_i|}{(|h_i|-(1+n\Delta)|f_i|)+(2+n\Delta)|f_i|}<\frac{|h_i||f_i|}{(2+n\Delta)|f_i|}=\frac{|h_i|}{(2+n\Delta)}, \quad (33)$$

since $(1+n\Delta)|f_i|\le|h_i|\le(1+(n+1)\Delta)|f_i|)$.

$\Pr_1(n)$ can be calculated as

$$\Pr_1(n)=\Pr\left(\left(\frac{|h_i||f_i|}{(|h_i|-(1+n\Delta)|f_i|)+(2+n\Delta)|f_i|}\right)^2>\Upsilon(1+n\Delta)|f_i|\le|h_i|\le(1+(n+1)\Delta)|f_i|)\right)$$

$$<\tilde{\Pr}_1(n)=\Pr\left(\left(\frac{|h_i|}{(2+n\Delta)}\right)^2>\Upsilon,(1+n\Delta)|f_i|\le|h_i|\le(1+(n+1)\Delta)|f_i|)\right)$$

$$=\Pr(X>(1+b_n)^2\Upsilon,b_n^2Y\le X\le a_n^2Y)$$

$$=\Pr(Y>\frac{(1+b_n)^2}{b_n^2}\Upsilon,b_n^2Y\le X\le a_n^2Y)+\Pr(\frac{(1+b_n)^2}{a_n^2}\Upsilon<Y<\frac{(1+b_n)^2}{b_n^2}\Upsilon,b_n^2Y\le X\le a_n^2Y)$$

$$=\frac{a_n^2\lambda_{h_i}}{a_n^2\lambda_{h_i}+\lambda_{f_i}}\exp((-(b_n+1)^2\lambda_{h_i}+\frac{(b_n+1)^2}{a_n^2}\lambda_{f_i})\Upsilon)$$

$$-\frac{b_n^2\lambda_{h_i}}{b_n^2\lambda_{h_i}+\lambda_{f_i}}\exp((-(b_n+1)^2\lambda_{h_i}+\frac{(b_n+1)^2}{b_n^2}\lambda_{f_i})\Upsilon). \quad (34)$$

where $a_n=1+(n+1)\Delta$, $b_n=1+n\Delta$.

Similarly, the second probability can be evaluated as

$$\tilde{\Pr}_2(n)<\Pr_2(n)$$

$$=\frac{a_n^2\lambda_{f_i}}{\lambda_{h_i}+a_n^2\lambda_{f_i}}\exp((-\frac{(b_n+1)^2}{a_n^2}\lambda_{h_i}+(b_n+1)^2\lambda_{f_i})\Upsilon)$$

$$-\frac{b_n^2\lambda_{f_i}}{\lambda_{h_i}+b_n^2\lambda_{f_i}}\exp((-\frac{(b_n+1)^2}{b_n^2}\lambda_{h_i}+(b_n+1)^2\lambda_{f_i})\Upsilon). \quad (35)$$

When $N>>0$, we can obtain $\frac{|h_i||f_i|}{|h_i|+|f_i|}<\frac{|h_i||f_i|}{|h_i|}=|f_i|$ because $(1+N\Delta)|f_i|\le|h_i|$.

Then $\Pr_3$ can be calculated as

$$\Pr_3<\tilde{\Pr}_3=\Pr(Y>\Upsilon,\mathrm{b}_N^2Y<X)=\frac{\lambda_{f_i}}{b_N^2\lambda_{h_i}+\lambda_{f_i}}\exp((-b_N^2\lambda_{h_i}+\lambda_{f_i})\Upsilon). \quad (36)$$

Similarly, we obtain lower bound of $\Pr_4$. In summary, the outage probability can be bounded as

$$P_{outage}(R_i) > 1 - \left(\sum_{n=0}^{N-1} (\tilde{\Pr}_1(n) + \tilde{\Pr}_2(n)) + \tilde{\Pr}_3 + \tilde{\Pr}_4 \right). \tag{37}$$

The lower outage bound of the whole system can be expressed as

$$P_{outage}(R) > \prod_{i=1}^{K} \left(P_{outage}(R_i) \right). \tag{38}$$

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