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The Spectrum of the Opertator $D(r, 0, 0, s)$ over the Sequence Spaces c_0 and c

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Abstract. In this paper we have examined the spectra of the operator $D(r, 0, 0, s)$ on sequence spaces c_0 and c.

1. Introduction

Spectral theory is an important branch of mathematics due to its application in other branches of science. It has been proved to be a standard tool of mathematical sciences because of its usefulness and application oriented scope in different fields. In numerical analysis, the spectral values may determine whether a discretization of a differential equation will get the right answer or how fast a conjugate gradient iteration will converge. In aeronautics, the spectral values may determine whether the flow over a wing is laminar or turbulent. In electrical engineering, it may determine the frequency response of an amplifier or the reliability of a power system. In quantum mechanics, it may determine atomic energy levels and thus, the frequency of a laser or the spectral signature of a star. In structural mechanics, it may determine whether an automobile is too noisy or whether a building will collapse in an earthquake. In ecology, the spectral values may determine whether a food web will settle into a steady equilibrium. In probability theory, they may determine the rate of convergence of a Markov process.

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Sequences spaces and series have been investigated from different aspects in the recent past. In summability theory, different classes of matrices have been investigated. Rath and Tripathy [10], Tripathy [11], Tripathy and Sen [25] and many others have characterized different class of matrices transforming from one class of sequences into another class of sequences. There are particular types of summability methods like Nörlund mean, Riesz mean, Euler mean, Abel transformation etc. Matrix methods have been studied from different aspects recently by Altin et.al [4], Tripathy and Baruah [14] and others.

Functional analysis methods have been applied for studying different classes of sequences by Tripathy and Mahanta [22], Tripathy and Sarma [24] and others. The spectra of difference operator have also been investigated on some classes of sequences. Altay and Basar ([1], [2], [3]) studied the spectra of difference operator $\triangle$ and generalized difference operator on c_0, c and ℓ_p. Okutoyi [8] has studied the spectra of Cesàro operator on bv_0. Rath and Tripathy [9] have investigated the spectra of the operator Schur matrices. Still there is a lot to be explored on spectra of some matrix operators transforming one class of sequences into another class of sequences.

Throughout N denote the set of non-negative integers. Throughout the paper w, ℓ_∞, c and c_0 denote the space of all, bounded, convergent and null sequences $x = (x_k)$ with complex terms, respectively, norm by $||x|| = \sup_k |x_k|$. The zero sequence is denoted by $\theta = (0, 0, 0, ...)$. Kizmaz [7] defined the difference sequence spaces $\ell_\infty(\triangle)$, $c(\triangle)$ and $c_0(\triangle)$ as follows:

$Z(\triangle) = \{x = (x_k) : (\triangle x_k) \in Z\}$, for $Z = \ell_\infty, c$ and c_0, where $\triangle x = (\triangle x_k) = (x_k - x_{k+1})$.

The above spaces are Banach spaces, normed by $||x||_\triangle = ||x_1|| + \sup_k ||\triangle x_k||$.

Different classes of sequence spaces using the difference operator have been introduced and investigated in the recent past by Tripathy, Altin and Et [12], Tripathy and Baruah [13], Tripathy and Borgohain [16], Tripathy and Chandra [17], Tripathy, Choudhary and Sarma [18], Tripathy and Dutta [19], Tripathy and Mahanta [21], Tripathy and Sarma [23] and many others. The idea of Kizmaz [7] was applied to introduce a new type of generalized difference operator on sequence spaces by Tripathy and Esi [20].

Let $m \in N$ be fixed, then Esi and Tripathy [20] have introduced the following type of difference sequence spaces

$Z(\triangle_m) = \{x = (x_k) : (\triangle_m x_k) \in Z\}$, for $Z = \ell_\infty, c$ and c_0, where $\triangle_m x = (\triangle_m x_k) = (x_k - x_{k+m})$.

Taking $m = 1$, we have the sequence spaces $\ell_\infty(\triangle)$, $c(\triangle)$ and $c_0(\triangle)$ studied by Kizmaz [7].

2. Preliminaries and definition

Let X be a linear space. By $B(X)$, we denote the set of all bounded linear operators on X into itself. If $T \in B(X)$, where X is a Banach space, then the adjoint operator T^* of T is a bounded linear operator on the dual X^* of X defined by $(T^*\phi)(x) = \phi(Tx)$ for all $\phi \in X^*$ and $x \in X$.

Let $T : D(T) \to X$ be a linear operator, defined on $D(T) \subset X$, where $D(T)$ denote the domain of T and X is a complex normed linear space. For $T \in B(X)$ we associate a complex number α with the operator $(T - \alpha I)$ denoted by T_α defined on the same domain $D(T)$, where I is the identity operator. The inverse $(T - \alpha I)^{-1}$, denoted by T_α^{-1} is known as the resolvent operator of T.

A regular value is a complex number α of T such that
(R_1) T_α^{-1} exists,
(R_2) T_α^{-1} is bounded and
(R_3) T_α^{-1} is defined on a set which is dense in X.

The resolvent set of T is the set of all such regular values α of T, denoted by $\rho(T)$. Its complement is given by $C \setminus \rho(T)$ in the complex plane C is called the spectrum of T, denoted by $\sigma(T)$. Thus the spectrum $\sigma(T)$ consist of those values of $\alpha \in C$, for which T_α is not invertible.

Classification of spectrum:

The spectrum $\sigma(T)$ is partitioned into three disjoint sets as follows:

(i) The point(discrete) spectrum $\sigma_p(T)$ is the set such that T_α^{-1} does not exist. Further $\alpha \in \sigma_p(T)$ is called the eigen value of T.

(ii) The continuous spectrum $\sigma_c(T)$ is the set such that T_α^{-1} exists and satisfies $(R3)$ but not $(R2)$ that is T_α^{-1} is unbounded.

(iii) The residual spectrum $\sigma_r(T)$ is the set such that T_α^{-1} exists (and may be bounded or not) but does not satisfy $(R3)$, that is, the domain of T_α^{-1} is not dense in X.

This is to note that in finite dimensional case, continuous spectrum coincides with the residual spectrum and equals to the empty set and the spectrum consists of only the point spectrum.

Let E and F be two sequence spaces and $A = (a_{nk})$ be an infinite matrix of real or complex numbers a_{nk}, where $n, k \in N = \{0, 1, 2, ...\}$. Then, we say that A defines a matrix mapping from E into F, denote by $A : E \to F$, if for every sequence $x = (x_n) \in E$ the sequence $Ax = \{(Ax)_n\}$ is in F where $(Ax)_n = \sum_{k=0}^{\infty} a_{nk}x_k$, provided the right hand side converges for every $n \in N$ and $x \in E$.

Our main focus in this paper is on the operator $D(r,0,0,s)$, where

$$D(r,0,0,s) = \begin{pmatrix} r & 0 & 0 & 0 & ... \\ 0 & r & 0 & 0 & ... \\ 0 & 0 & r & 0 & ... \\ s & 0 & 0 & r & ... \\ 0 & s & 0 & 0 & ... \\ . & . & . & . & ... \end{pmatrix}$$

Here we assume that r and s are complex parameters and $(s \neq 0)$.

Remark. In particular if we consider $r = -1$ and $s = 1$ then $D(-1,0,0,1) = \triangle_3$.

Lemma 2.1. *The matrix $A = (a_{nk})$ gives rise to a bounded linear operator $T \in B(c)$ i.e. from c to itself if and only if*
(1) *the rows of A are in ℓ_1 and their ℓ_1 norms are bounded,*
(2) *the columns of A are in c,*
(3) *the sequence of row sums of A is in c.*
The operator norm of T is the supremum of the ℓ_1 norms of the rows.

Corollary 2.1. *$D(r,0,0,s) : c \to c$ is a bounded linear operator and $||D(r,0,0,s)||_{(c,c)} = |r| + |s|$.*

Lemma 2.2. *The matrix $A = (a_{nk})$ gives rise to a bounded linear operator $T \in B(c_0)$ i.e from c_0 to itself if and only if*
(1) *the rows of A* are in *ℓ_1 and their ℓ_1 norms are bounded,*
(2) *the columns of A are in c_0.*
The operator norm of T is the supremum of the ℓ_1 norms of the rows.

Corollary 2.2. *$D(r,0,0,s) : c_0 \to c_0$ is a bounded linear operator and $||D(r,0,0,s)||_{(c_0,c_0)} = |r| + |s|$.*

Lemma 2.3. *Let $T \in B(X)$, where X is any Banach space. Then the spectrum of T^* is identical with the spectrum of T. Further $R_\lambda(T) = (T - \lambda I)^{-1}$ and $\rho(T) = \{\lambda \in C : (T - \lambda I)^{-1} exists\}$.*

Lemma 2.4. *T has a dense range if and only if T^* is one to one, where T^* denote the adjoint operator of T.*

3. Spectrum of the operator $D(r,0,0,s)$ on the sequence spaces c_0 and c.

Theorem 3.1. $\sigma(D(r,0,0,s), c_0) = \{\alpha \in C : |r - \alpha| \leq |s|\}$.

Proof. First, we prove that $(D(r,0,0,s) - \alpha I)^{-1}$ exits and is in (c_0, c_0) for $|r - \alpha| > |s|$ and then we show that the operator $(D(r,0,0,s) - \alpha I)$ is not invertible for $|r - \alpha| \leq |s|$.
Let $\alpha \notin \{\alpha \in C : |r - \alpha| \leq |s|\}$. Since $s \neq 0$ we have $\alpha \neq r$ and so $(D(r,0,0,s) - \alpha I)$ is triangle, hence $(D(r,0,0,s) - \alpha I)^{-1}$ exists.

Let,

$$\begin{pmatrix} r-\alpha & 0 & 0 & 0 & 0 & ... \\ 0 & r-\alpha & 0 & 0 & 0 & ... \\ 0 & 0 & r-\alpha & 0 & 0 & ... \\ s & 0 & 0 & r-\alpha & 0 & ... \\ 0 & s & 0 & 0 & r-\alpha & ... \\ . & . & . & . & . & ... \end{pmatrix} \begin{pmatrix} p_0 & 0 & 0 & 0 & 0 & ... \\ p_1 & p_0 & 0 & 0 & 0 & ... \\ p_2 & p_1 & p_0 & 0 & 0 & ... \\ p_3 & p_2 & p_1 & p_0 & 0 & ... \\ p_4 & p_3 & p_2 & p_1 & p_0 & ... \\ . & . & . & . & . & ... \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & ... \\ 0 & 1 & 0 & 0 & 0 & ... \\ 0 & 0 & 1 & 0 & 0 & ... \\ 0 & 0 & 0 & 1 & 0 & ... \\ 0 & 0 & 0 & 0 & 1 & ... \\ . & . & . & . & . & ... \end{pmatrix}.$$

Then we have
$p_0 = \frac{1}{r-\alpha}$
$p_1 = 0$
$p_2 = 0$
$p_3 = -\frac{s}{(r-\alpha)^2}$
$p_4 = 0$
$p_5 = 0$
$p_6 = \frac{s^2}{(r-\alpha)^3}$
- - -
we obtain
$p_{3k} = \frac{(-s)^k}{(r-\alpha)^{k+1}}$, $(k \geq 0)$
and
$p_{3k+1} = 0$, $(k \geq 0)$
and
$p_{3k+2} = 0$, $(k \geq 0)$.

Hence, we get

$$(D(r,0,0,s)-\alpha I)^{-1} = \begin{pmatrix} \frac{1}{r-\alpha} & 0 & 0 & 0 & 0 & ... \\ 0 & \frac{1}{r-\alpha} & 0 & 0 & 0 & ... \\ 0 & 0 & \frac{1}{r-\alpha} & 0 & 0 & ... \\ -\frac{s}{(r-\alpha)^2} & 0 & 0 & \frac{1}{r-\alpha} & 0 & ... \\ 0 & -\frac{s}{(r-\alpha)^2} & 0 & 0 & \frac{1}{r-\alpha} & ... \\ . & . & . & . & & ... \end{pmatrix}.$$

Clearly, columns of $(D(r,0,0,s)-\alpha I)^{-1}$ are in c_0 if $|r-\alpha| > |s|$.

Again, $||(D(r,0,0,s) - \alpha I)^{-1}||_{(c_0,c_0)} = \sup_n \sum_{k=1}^{n} |p_k| = \sum_{k=0}^{\infty} |p_k| = \sum_{m=0}^{\infty} |p_{3m}| +$

$\sum\limits_{m=0}^{\infty}|p_{3m+1}|+\sum\limits_{m=0}^{\infty}|p_{3m+2}|=\frac{1}{|r-\alpha|}\sum\limits_{m=0}^{\infty}|\frac{s}{(r-\alpha)}|^m+0+0<\infty$ if $|r-\alpha|>|s|$.

Thus, $(D(r,0,0,s)-\alpha I)^{-1}\in(c_0,c_0)$ if $|r-\alpha|>|s|$.

Conversely let, $\alpha\in\{\alpha\in C:|r-\alpha|\le|s|\}$ and $r\neq\alpha$. Since $(D(r,0,0,s)-\alpha I)$ is a triangle, $(D(r,0,0,s)-\alpha I)^{-1}$ exists but $||(D(r,0,0,s)-\alpha I)^{-1}||=\infty$, if $|r-\alpha|<|s|$ that is, $(D(r,0,0,s)-\alpha I)^{-1}\notin B(c_0)$.

If $r=\alpha$, then the operator

$$(D(r,0,0,s)-\alpha I)=\begin{pmatrix}0&0&0&0&0&...\\0&0&0&0&0&...\\s&0&0&0&0&...\\0&s&0&0&0&...\\.&.&.&.&&...\end{pmatrix}=D(0,0,0,s).$$

Since $\overline{R(D(0,0,0,s))}\neq c_0$, so $D(0,0,0,s)$ is not invertible. This completes the proof.□

Theorem 3.2. $\sigma_p(D(r,0,0,s),c_0)=\emptyset$.

Proof. Suppose that $D(r,0,0,s)x=\alpha x$ for $x\neq\theta=(0,0,0,...)$ in c_0. Then by solving the system of linear equations we have

$rx_0=\alpha x_0$
$rx_1=\alpha x_1$
$rx_2=\alpha x_2$
$sx_0+rx_3=\alpha x_3$
$sx_1+rx_4=\alpha x_4$
- - -
$sx_k+rx_{k+3}=\alpha x_{k+3}$, $(k\ge0)$.

If $x_{n_0}\neq0$ is the first non-zero entry of the sequence $x=(x_n)$, then $\alpha=r$ and $x_{n_0+k}=0$ for all $k\in N$. This contradicts the fact that $x_{n_0}\neq0$. This completes the proof. □

If $T:c_0\to c_0$ is a bounded linear operator with the matrix A, then it is well known that its adjoint operator $T^*:c_0^*\to c_0^*$ is defined by transpose of the matrix A.

It should be noted that the dual space c_0^* of c_0 is isometrically isomorphic to the Banach space ℓ_1 of absolutely summable sequences normed by $||x||=\sum\limits_{k=0}^{\infty}|x_k|$.

Theorem: 3.3. $\sigma_p(D(r,0,0,s)^*,c_0^*)=\{\alpha\in C:|r-\alpha|<|s|\}$.

Proof. Suppose that $D(r,0,0,s)^*x=\alpha x$ for $x\neq\theta$ in $c_0^*\cong\ell_1$. Then by solving the system of linear equations we have

$rx_0 + sx_3 = \alpha x_0$
$rx_1 + sx_4 = \alpha x_1$
$rx_2 + sx_5 = \alpha x_2$
$---$
$rx_k + sx_{k+3} = \alpha x_k,\ (k \geq 0).$

Now, from above system of equations we have,

$$\sum |x_n| = |x_0| \left\{1 + |\tfrac{\alpha-r}{s}| + |\tfrac{\alpha-r}{s}|^2 + ...\right\} + |x_1| \left\{1 + |\tfrac{\alpha-r}{s}| + |\tfrac{\alpha-r}{s}|^2 + ...\right\} + |x_2| \left\{1 + |\tfrac{\alpha-r}{s}| + |\tfrac{\alpha-r}{s}|^2 + ...\right\} < \infty, \text{ if } |\alpha - r| < |s|.$$

That is $x \in \ell_1$ if and only if $|\alpha - r| < |s|$. □

Theorem 3.4. $\sigma(D(r,0,0,s)^*, c_0^*) = \{\alpha \in C : |r-\alpha| \leq |s|\}$.

Proof. We have $\sigma(D(r,0,0,s)) = \sigma(D(r,0,0,s)^*)$. Now the proof follows from Lemma 2.3 and Theorem 3.1. □

Theorem 3.5. $\sigma_r(D(r,0,0,s), c_0) = \{\alpha \in C : |r-\alpha| < |s|\}$.

Proof. For $|r-\alpha| < |s|$, the operator $(D(r,0,0,s) - \alpha I)$ is one to one and hence has an inverse. But Theorem 3.3 implies that $(D(r,0,0,s)^* - \alpha I)$ is not one to one for $|r-\alpha| < |s|$. Now using Lemma 2.4 we can conclude that the range of $(D(r,0,0,s) - \alpha I)$ is not dense in c_0, that is $\overline{R(D(r,0,0,s) - \alpha I)} \neq c_0$. This completes the proof. □

Theorem 3.6. $\sigma_c(D(r,0,0,s), c_0) = \{\alpha \in C : |r-\alpha| = |s|\}$.

Proof. The proof immediately follows from the fact that the set of spectrum is the disjoint union of the point spectrum, residual spectrum and continuous spectrum, that is
$\sigma(D(r,0,0,s), c_0) = \sigma_p(D(r,0,0,s), c_0) \cup \sigma_r(D(r,0,0,s), c_0) \cup \sigma_c(D(r,0,0,s), c_0)$. □

Theorem 3.7. $\sigma(D(r,0,0,s), c) = \{\alpha \in C : |r-\alpha| \leq |s|\}$.

Proof. This is obtained in the similar way that is used in the proof of Theorem 3.1. □

Theorem 3.8. $\sigma_p(D(r,0,0,s), c) = \emptyset$.

Proof. The result can be established in a way similar to the proof of Theorem 3.2. □

If $T : c \to c$ is a bounded matrix operator with matrix A, then $T^* : c^* \to c^*$ acting on $c \bigoplus \ell_1$ has a matrix representation of the form $\begin{bmatrix} \chi & 0 \\ b & A^t \end{bmatrix}$ where χ is the limit of the sequence of row sums of A minus the sum of the columns of A, and b is the column vector whose k^{th} entry is the limit of the k^{th} column of A for each

$k \in N$. For $D(r,0,0,s) : c \to c$, the matrix $D(r,0,0,s)^* \in B(\ell_1)$ is of the form

$$D(r,0,0,s)^* = \begin{pmatrix} r+s & 0 \\ 0 & D(r,0,0,s)^t \end{pmatrix}.$$

Theorem 3.9. $\sigma_p(D(r,0,0,s)^*, c^*) = \{\alpha \in C : |r-\alpha| < |s|\} \cup \{r+s\}$.

Proof. We suppose that $D(r,0,0,s)^* y = \alpha y$, for $y(\neq \theta) \in c^*(= c \bigoplus \ell_1)$. We get the following system of equations

$(r+s)y_0 = \alpha y_0$

$ry_1 + sy_4 = \alpha y_1$

$ry_2 + sy_5 = \alpha y_2$

$ry_3 + sy_6 = \alpha y_3$

$ry_4 + sy_7 = \alpha y_4$

$---$

$ry_k + sy_{k+3} = \alpha y_k$, $(k \geq 1)$.

We obtain that,

(1) $\quad y_{4k} = (\frac{\alpha-r}{s})^k y_1, \quad (k \geq 1)$

and

(2) $\quad y_{4k+1} = (\frac{\alpha-r}{s})^k y_2$, $(k \geq 1)$

and

(3) $\quad y_{4k+2} = (\frac{\alpha-r}{s})^k y_3$, $(k \geq 1)$.

If $x_0 \neq 0$, then $\alpha = r+s$. So, $\alpha = r+s$ is an eigen value with the corresponding eigen vector $x = (x_0, 0, 0, ...)$.If $\alpha \neq r+s$, then $x_0 = 0$ and we observe that from (1), (2) and (3) $x \in \ell_1$ if and only if $|\alpha - r| < |s|$. □

Theorem 3.10. $\sigma_r(D(r,0,0,s), c) = \sigma_p(D(r,0,0,s)^*, c^*)$.

Proof. The proof can be obtained in a way analogous to the proof of theorem 3.5. □

Theorem 3.11. $\sigma_c(D(r,0,0,s), c) = \{\alpha \in C : |\alpha - r| = |s|\} \setminus \{r+s\}$.

Proof. The proof immediately follows from the fact that the set of spectrum is the disjoint union of the point spectrum, residual spectrum and continuous spectrum, that is $\sigma(D(r,0,0,s),c) = \sigma_p(D(r,0,0,s),c) \cup \sigma_r(D(r,0,0,s),c) \cup \sigma_c(D(r,0,0,s),c)$.

Conclusion : We can generalize our operator

$$(D(r,0,0,..(n-1)times, s) = \begin{pmatrix} r & 0 & 0 & 0 & 0 & 0 & . & . & . \\ 0 & r & 0 & 0 & 0 & 0 & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ s & 0 & . & . & r & 0 & . & . & . \\ 0 & s & 0 & . & . & r & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \end{pmatrix}$$

If we take $r = -1$ and $s = 1$, then the operator $(D(r, 0, 0, ..(n-1)times, s)$ will be the same as the generalized difference operator $\triangle_n$. Further on considering the operator $(D(r, 0, 0, ..(n-1)times, s)$ in place of $D(r, 0, 0, s)$, one can get parallel all our results obtained in this paper.

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