ISSN 1598-4095 (Print)
ISSN 2287-7401 (Online)
http://dx.doi.org/10.9712/KASS.2013.13.2.075

Journal of the Korean Association for Spatial Structures
Vol. 13, No. 2 (통권 52호), pp.75~82, June, 2013

다층 케이블 돔형 구조물의 자기평형을 위한 부재력 비율 결정

Determination of Member Force Ratios for Self-equilibrium State of Multi-Layered Cable Dome Type Structures

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Abstract

For each cable component in a cable dome structure, pre-tension is needed for stability of whole the structure. The summation of these pre-tension at each joint should be zero to achieve the self equilibrium structure. The first step in cable dome structure analysis is to find the ratio of pre-tension in each member which can produce a stable and structure on self-equilibrium. In this paper, a new method based on the basic principle of closed force polygon for equilibrium system is proposed for the determination of self-equilibrium mode of cable dome structure. A single layer cable dome and two multi layer type domes have been analyzed. The ratios of cable members are determined by the presented method, and check the validation of the results by numerical calculation.

Keywords : pre-tension, single layer dome structures, multi layer dome structures, self-equilibrium state, summation of vector

1. Introduction

Cable dome is a kind of tensegrity(tension +integrity) structures. Outer ring, inner ring, radial member and mast member are the main components of a cable dome structure. In a cable dome structure, the mast and the outer ring members are in compression so these members should keeps rigidity sufficiently.

The radial and inner ring members are in tension and they are made by cables. These cables should be in pre-tension since the structure is not stable without this pre tension. Self equilibrium stress state is the main characteristic of a cable dome structure. The basic theory for a self equilibrium of structure is that the summation of forces at each joint without any external load should be zero.

One of the main groups of cable dome structure is single layer dome or bicycle wheel like cable dome. The single layer cable domes have just one layer of inner ring (an upper ring and a lower ring) and one outer ring. The inner ring layer is inside of the outer ring. If the cable dome structure has more than one inner ring layer, the cable dome is multi inner ring cable dome structure and there are two types. The multi layer is a kind of multi inner ring cable dome where the joints of each inner ring's layer are connected to the next and previous layer of inner ring's joints except at the first and the last ones. For the first layer there is no previous layer and for the last layer there is outer ring instead of next inner ring layer. Multi layer cable dome has two types. If

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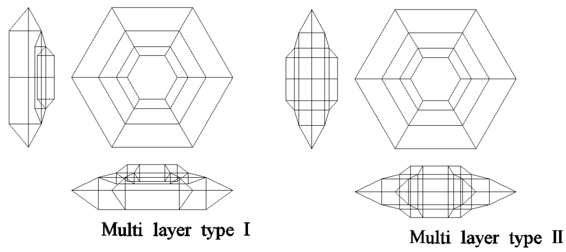
the upper inner ring or lower inner ring is connected to previous layer of inner ring, the structure is multilayer type I. If the upper inner and lower inner rings are connected to the previous layer of inner ring, then the cable dome structure is called multilayer type II.



〈Fig 1〉 The multi layer type dome

All this classification can be for single layer or a multi inner ring layer of cable dome. For example a cable dome can be with two openings, one of them is multi layer type I and another one is multi layer type II. Also in multi layer type I the number of first ring's member is less than the number of second ring's member. In the multi layer type two and one component of outer ring is used around the opening.

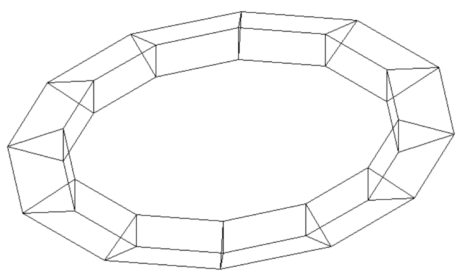
In previous researches for the determination of self-equilibrium mode, the ratios of each cable member are calculated by solving a set of equilibrium equations of joint.[1),2),3),4)] As one of the different method for the determination of ratios, SHIM et al.[5),6)] and KIM[7)] has been proposed a basic concept of geometric method by using CAD. The method based on the basic principle of closed force polygon for equilibrium system is proposed for the determination of self-equilibrium mode of cable dome structure. The proposed method which is called geometric method has the unique characteristic of visualization of the force mode needed for maintenance of self-quilibrium. Kim and Mohammad[8)] also has been presented the determination method of the ratios with the same method. Three numerical examples of dome structures are analyzed to illustrate the applicability of the method. The first one is a simple example where step-by-step procedures of geometric method are shown. The two last examples are two dome structures with newly introduced configurations not previously considered.

For these previous researches, all analysis models were very simple symmetric single layer type structures. In this paper, a single layer type dome and two multi layer type domes have been analyzed. The ratios of cable members are determined by the presented method, and check the validation of the results by numerical calculation. This is then followed by a section on the step-by-step procedures of geometric method as proposed by SHIM et al.

2. Analysis Example

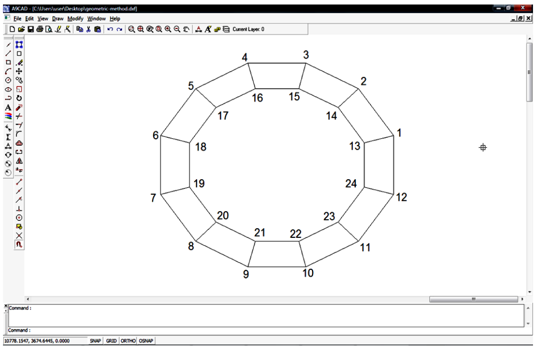
2.1 Single Layer Circular Cable Dome

An example which was solved by Chong[9)] with the use of Moore-Penrose generalized inverse method is selected for comparison propose. This cable dome structure has 36 joints and 60 members. (Example I)



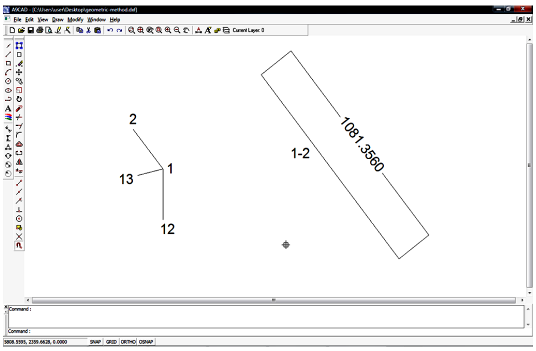
〈Fig 2〉 3D view of example I

At first, the 3D model data(Fig.2) is converted to 2D model(Fig. 3)



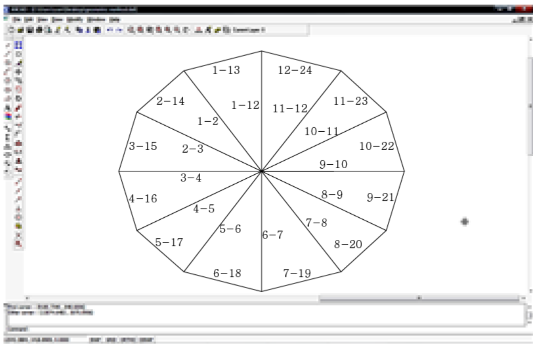
〈Fig 3〉 2D model of Example I

Next, select to a joint for starting the analysis and assumption of the ratio of first member. The first member is radial member 1-2 and the ratio is assumed as 1081.3560 (Fig. 4). This amount is the same as with result calculated by Chong[9]. And next, draw a close polygon for the first joint. After The close polygon for the first joint selected, drawing of the ratio polygon for the subsequent joints. The close polygon for the second joint (joint 3) is drawn as shown in Fig. 5.



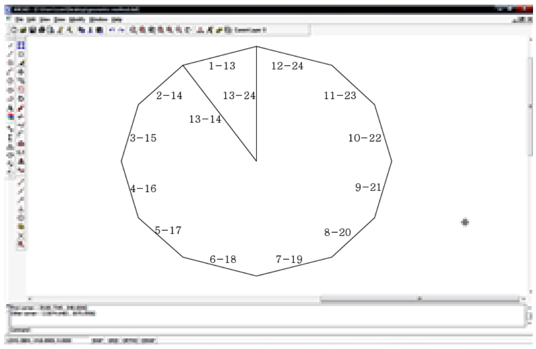
〈Fig 4〉 First joint and assumption for analysis

With the subsequent drawing to the last joint, the entire closed ratio polygon of outer ring is shown in Fig. 5.



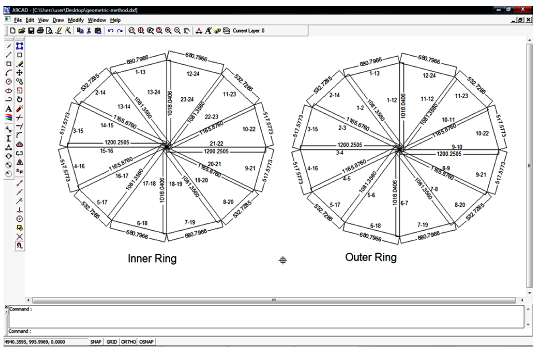
〈Fig 5〉 Ratio polygon of outer ring

As the same procedures to the inner ring, the joint 13 is selected as the first joint and its polygon is shown in Fig. 6.



〈Fig 6〉 Closed polygon ratio of inner ring

As a results, the member ratios of inner rings and outer rings in 2D are determined as shown in Fig. 7.



〈Fig 7〉 The ratios members of example I

As the last step, the 2D ratio data should be converted to 3D models. These ratios after conversion to 3D model are shown in the Table 1. The comparison results of Chong[9] are also shown. The minus ratios are the ratio of pre-tension members.

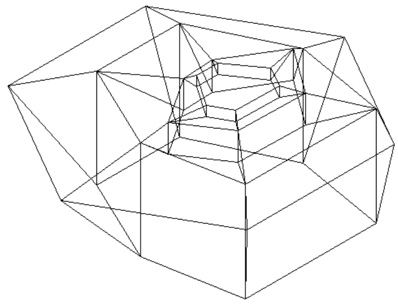
〈Table 1〉 The ratios of members in 3D

member	Computer software	Presented method
1-2	2162.741	2162.712
2-3	2231.784	2331.752
6-7	2036.111	2036.082
7-8	2162.748	2162.712
8-9	2331.792	2331.752
9-10	2400.540	2400.502
10-11	2331.790	2331.752
1-13	-715.6367	-715.636
6-18	-715.6354	-715.636
7-19	-715.6527	-715.636
12-24	-715.6367	-715.636

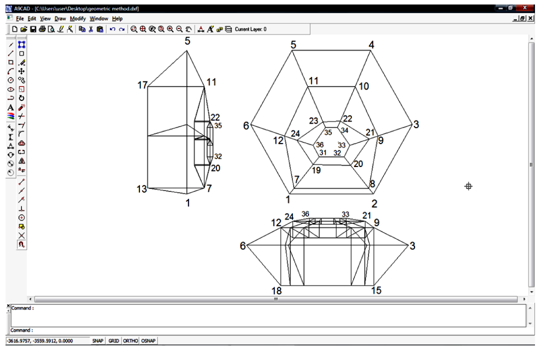
To the check the validation of method, the data are check by means of the equilibrium of joints. The ratio of assumed member is the same as result calculated by Chong[7]. The result of geometric method has shown better accuracy since it is predicted that symmetrical components should have the same ratio. Such results are not achieved in the analysis results by Chong[7]. For example members 1-13, 6-18, 7-19 and 12-24 are symmetrical. The ratios of all of them from geometric method are 715.636 but the results of computer are 715.6367, 715.6354, 715.6527 and 715.6367.

2.2 Multi layer type I cable dome structure

This example, Example II, is a type I of multilayer cable dome structure with 3 inner ring layers and an outer ring. The number of members in each ring is 6 and the number of radial members in each inner ring layer is 12. There are 6 mast members in each inner ring layer. The total number of members is 96 and the number of joints is 42 (Fig. 8). The geometric shapes of all rings are not symmetrical and the protractions of radial members do not meet each other in a particular point. The top view and sides view of this example are in Fig. 9.



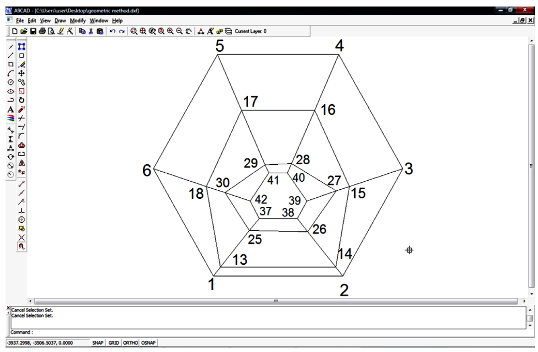
〈Fig 8〉 3D view of example II



〈Fig 9〉 Top and sides views of example II

Analysis procedures are same as Example I and here II-1 means step-1 of example II.

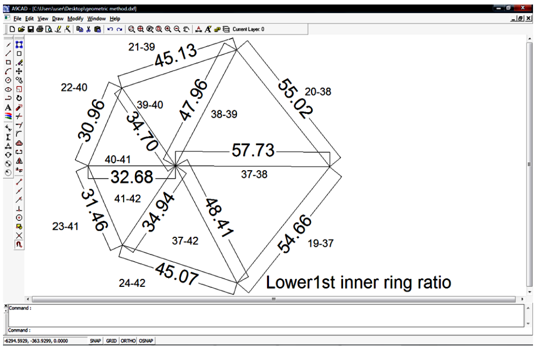
II-1 : Conversion from 3D to 2D is shown in Fig. 10.



〈Fig 10〉 2D model of example II

II-2:Drawing the 1st inner ring ratio polygon

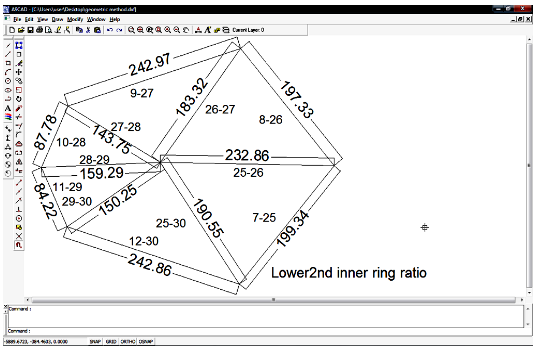
The ratio polygon of the first inner ring layer is shown in Fig. 11.



〈Fig 11〉 Ratio polygon of 1st inner ring layer

II-3: Drawing the 2nd inner ring ratio polygon

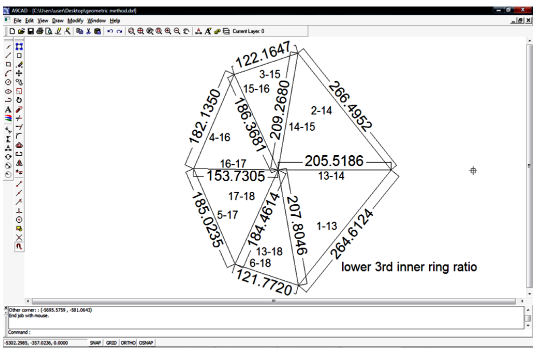
The ratio polygon of the second inner ring layer is shown in Fig.12.



〈Fig 12〉 Ratio polygon of 2nd inner ring layer

II-4: Drawing the 3rd inner ring ratio polygon

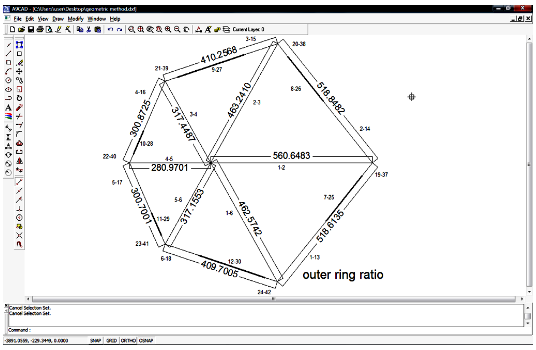
The ratio polygon of the third inner ring layer is shown in Fig.13.



〈Fig 13〉 Ratio polygon of 3rd inner ring layer

II-5: Drawing the outer ring ratio polygon ratio

The ratio polygon of the outer ring is shown in Fig.14.



〈Fig 14〉 Ratio polygon of outer ring

II-6: Determine the ratios of upper radial members

The ratios of upper radial members based on the ratio polygon of inner ring layers are obtained. Table 2 shows these ratios.

〈Table 2〉 The ratios of upper radial members in 2D

Upper 3rd layer radial	01-07	02-08	03-09
Ratio in 2D	772.61	771.19	698.36
Upper 2rd layer radial	07-19	08-20	09-21
Ratio in 2D	308.66	307.37	333.23
Upper 1rd layer radial	19-31	20-32	21-33
Ratio in 2D	54.66	55.02	45.13

II-7: Conversion of the ratio from 2D to 3D models

The results of force ratios after conversion are shown in Table 3.

〈Table 3〉 The ratios of members in 3D

Member	Ratio	Member	Ratio
01-02	1121.3	02-14	1837.38
03-09	**772.44**	**08-09**	**209.27**
09-10	**186.37**	**09-15**	**330.11**
09-21	**397.43**	20-32	57.00
09-27	**325.50**	20-38	57.00
31-32	57.73	08-14	1817.96
37-38	57.373	20-26	63.13
01-07	1976.73	32-38	14.87
01-13	1838.63	03-04	634.90

II-8: Check the equilibrium of a random joint

Forces of on members meeting at a random joint, here node 9, are decomposed in to X, Y and Z. It is shown that the summation of each direction equal to 0.

〈Table 4〉 Ratios of members connected to node 9

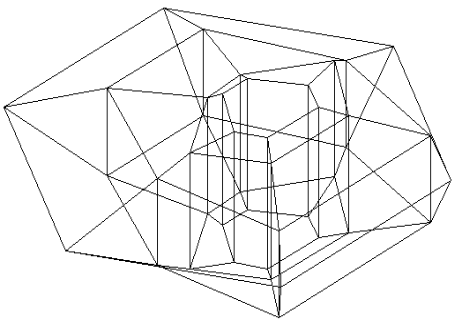
Member	X-D	Y-D	Z-D
03-09	664.45	214.92	330.111
08-09	-36.52	-206.06	0
09-10	-79.71	168.46	0
09-21	-317.05	-102.55	-216.61
09-27	-231.17	-74.77	-216.61
09-15	0	0	-330.11

$$\sum F_X = 664.45 - 36.52 - 79.71 - 317.05 - 231.17 + 0 = 0$$
$$\sum F_Y = 214.92 - 206.06 + 168.46 - 102.55 - 74.77 + 0 = 0$$
$$\sum F_Z = 330.11 - 0 - 0 - 216.61 + 216.61 - 330.11 = 0$$

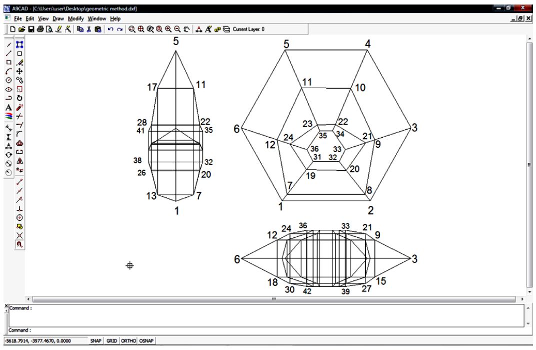
$$ratio\,03-09 = \sqrt{664.45^2 + 214.92^2 + 330.11^2} = 772.44$$
$$ratio\,09-27 = \sqrt{231.17^2 + 74.77^2 + 216.61^2} = 325.50$$

2.3 Multi layer type II cable dome structure

This example, Example III, is a multilayer type II cable dome structure (Fig.15). The number of joints and members in this example is same as the previous one. In both examples, the members connection and coordinates of joints in Upper inner ring of 3rd, 2nd and 1st layers, lower inner ring of 1st layer and outer ring are the same. The X and Y coordinates of all the joints in both examples are the same. Hence the 2D models of both examples are similar.

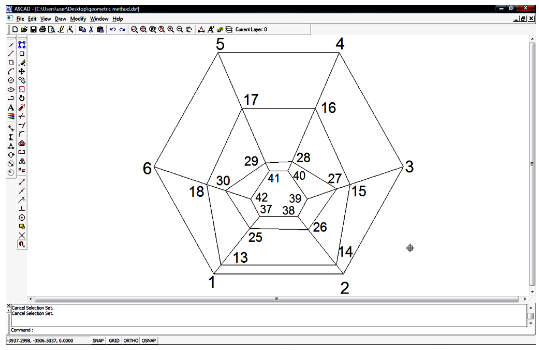


〈Fig 15〉 3D view of example III



〈Fig 16〉 Up and sides views of example III

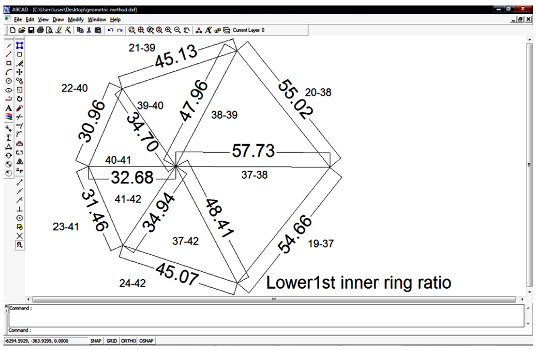
III-1 : Conversion from 3D to 2D



〈Fig 17〉 2D model of example III

III-2 : Drawing the 1st inner ring ratio polygon

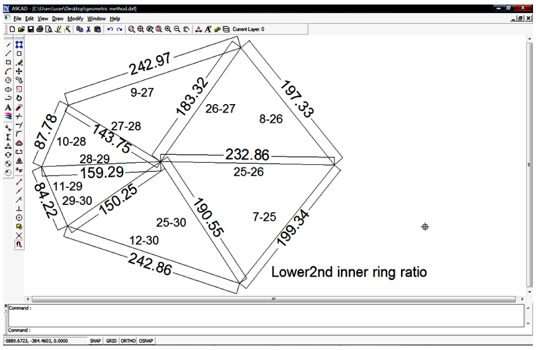
The ratio polygon of the first inner ring layer is shown in Fig.18



〈Fig 18〉 Ratio polygon of 1st inner ring layer

III-3 : Drawing the 2^{nd} inner ring ratio polygon

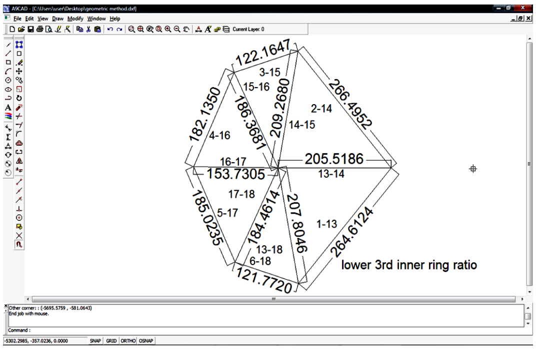
The ratio polygon of the second inner ring layer is shown in Fig.19.



〈Fig 19〉 Ratio polygon of 2nd inner ring layer

III-4 : Drawing the 3rd inner ring ratio polygon

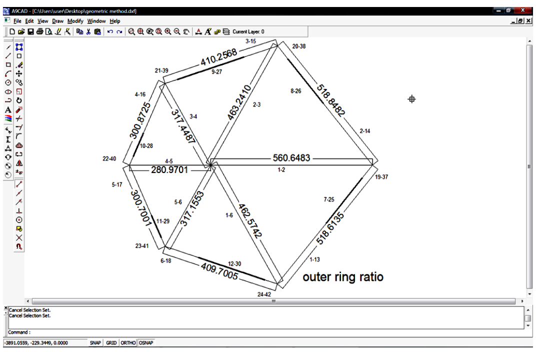
The ratio polygon of the third inner ring layer is shown in Fig.20.



〈Fig 20〉 Ratio polygon of 3rd inner ring layer

III-5:Drawing the outer ring polygon ratio

The ratio polygon of the outer ring is shown in Fig.21.



〈Fig 21〉 Ratio polygon of outer ring

III-6:Conversion of the ratio from 2D to 3D

The results of force ratios after conversion are shown in Table 5.

〈Table 5〉 The ratios of members in 3D of example III

Member	Ratio	Member	Ratio
01-02	1121.3	**03-09**	**453.78**
07-08	205.52	**08-09**	**209.27**
09-10	**186.37**	**09-21**	**343.61**
09-15	**6.66**	08-14	1171.25
25-26	232.86	04-05	561.94
31-32	57.73	10-11	153.73
37-38	57.73	22-23	159.29

III-7: Check the equilibrium of a random joint

Forces of on members meeting at a random joint, here node 9, are decomposed in to X, Y and Z. It is shown that the summation of each direction equal to 0.

〈Table 6〉 Ratios of members connected to node 9

Member	X-D	Y-D	Z-D
03-09	390.34	126.26	-193.93
08-09	-36.52	-206.06	0
09-10	-79.71	168.46	0
09-21	-274.11	-88.66	187.27
09-15	0	0	6.66

$$\sum F_X = 309.34 - 36.52 - 79.71 - 274.11 + 0 = 0$$
$$\sum F_Y = 126.6 - 206.06 - 168.46 - 88.66 + 0 = 0$$
$$\sum F_Z = -193.93 + 0 + 187.27 + 6.66 = 0$$

$$ratio\,08-09 = \sqrt{36.52^2 + 206.06^2 + 0^2} = 209.27$$
$$ratio\,09-21 = \sqrt{274.11^2 + 86.66^2 + 187.27^2} = 343.61$$

5. Conclusion

A geometric method which is a method for analysis of cable dome structure to determine self equilibrium stress mode has been proposed. Applicability of this new method has been proven by comparison with results obtained from previous method. The modified geometric method can easily be applied to multi inner ring layer of cable dome structure. From the ranges of models analyzed on the this study, there is a lot of possibilities for geometric shape of cable dome that the multi inner ring layer is mixed with different shape of single layer.

In this study, the mast members are vertically oriented and the ring members are horizontally. But, for possible study in the future, more variable types of cable dome type structures should be analyzed. A structure that the mast members are strain vertically oriented but the ring members are not oriented horizontally and the mast members are not vertical should be considered.

Acknowledgements

This research was supported by Basic Science Research Program through the National Research Foundation of Korea(NRF) funded by the Ministry of Education, Science and Technology (2011-0021548)

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(접 수 일 자 : 2013년 02월 22일)
(심사완료일자 : 2013년 04월 26일)
(게재확정일자 : 2013년 05월 22일)