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Reconstruction of the Electron Density Profile in O-mode Ultrashort Pulse Reflectometry using a Two-dimensional Finite Difference Time Domain

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Abstract

The two-dimensional finite difference time domain algorithm is used to numerically reconstruct the electron density profile in O-mode ultrashort pulse reflectometry. A Gaussian pulse is employed as the source of a probing electromagnetic wave. The Gaussian pulse duration is chosen in such a manner as to have its frequency spectrum cover the whole range of the plasma frequency. By using a number of numerical band-pass filters, it is possible to compute the time delays of the frequency components of the reflected signal from the plasma. The electron density profile is reconstructed by substituting the time delays into the Abel integral equation. As a result of simulation, the reconstructed electron density profile agrees well with the assumed profile.

Key Words : Finite Difference Time Domain, Electron Density, Ultrashort Pulse Reflectometry, Abel Integral Equation

1. Introduction

Microwave reflectometry has a number of merits in measuring various plasma parameters including electron density profiles [1-2]. In reflectometry, the reflection of an incident electromagnetic (EM) signal from the cutoff layers is examined to reconstruct the electron density profile. Reflectometry can be operated in either phase or time measurement. As a type of time measurement, ultrashort pulse reflectometry (USPR) utilizes an ultrashort pulse with frequency components covering the full range of the electron density. The frequency components of the pulse are sequentially reflected from the corresponding cutoff layers of the plasma. By performing a time delay measurement for each frequency component of the reflected signal, it is possible to obtain the electron density profile with a single ultrashort pulse source. Therefore, USPR can considerably reduce the complexity of conventional reflectometry systems. With this advantage, USPR has been employed to measure electron density profiles and fluctuations in several fusion devices

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[3-4].

In this paper, a two-dimensional (2D) full-wave simulation is performed on the ordinary mode (O-mode) wave propagation through plasma to study the reconstruction of electron density profiles in USPR. The simulation method is based on a finite difference time domain (FDTD). It is well known that the FDTD method has gained remarkable popularity as a powerful tool for obtaining numerical solutions to numerous types of EM problems [5-7]. However, the application of the FDTD method to the O-mode wave propagation through the plasma is somewhat complicated because of the dispersive nature of the plasma.

This paper is organized as follows. In the next section, the outline of the FDTD algorithm for the O-mode wave is described. A detailed description of the 2D simulation for O-mode USPR is explained in Section 3. Simulation results are also presented. Finally, the conclusion of the paper is given in Section 4.

2. Description of FDTD Algorithm

Maxwell's equations for the propagation of O-mode waves in cold plasma is expressed in MKS units as follows [8].

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \tag{2}$$

where $\vec{E}$ is the electric field strength, $\vec{B}$ is the magnetic flux density, $\vec{J}$ is the plasma current density. μ_0 and ε_0 are the magnetic permeability and the electric permittivity in free space, respectively. All plasma effects are included in the response of the plasma current to the electric field. The plasma current equation and the electron motion equation are given by

$$\vec{J} = -en_e \vec{v_e} \tag{3}$$

$$m_e \frac{\partial \vec{v_e}}{\partial t} = -e(\vec{E} + \vec{v_e} \times \vec{B}) \tag{4}$$

where $\vec{v_e}$ is the velocity of the electron, $-e$ is the electron charge, n_e is the electron density, and m_e is the electron mass. Assuming O-mode wave propagation in the x-y plane and no spatial gradient in the z direction, i.e., $\vec{E} = \hat{a}_z E_z(x, y)$, $\vec{B} = \hat{a}_x B_x(x, y) + \hat{a}_y B_y(x, y)$, then,

$$\frac{\partial B_x}{\partial t} = -\frac{\partial E_z}{\partial y} \tag{5}$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial E_z}{\partial x} \tag{6}$$

$$\varepsilon_0 \mu_0 \frac{\partial E_z}{\partial t} = \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} - \mu_0 J_z \tag{7}$$

$$\frac{\partial J_z}{\partial t} = \varepsilon_0 \omega_p^2 E_z \tag{8}$$

where $\omega_p = \sqrt{n_e e^2 / (\varepsilon_0 m_e)}$ is the plasma frequency. For the numerical computation of O-mode wave propagation, equations (5)~(8) are solved using the FDTD algorithm. The placement of the electric and magnetic field vectors on a two-dimensional Yee [9] grid is shown in Fig. 1. The FDTD equations for the electric and magnetic fields can be obtained by applying the central difference formula to both the time and space derivatives in equations (5)~(8) as follows.

$$\left[B_x\right]_{i,j+1/2}^{n+1/2} = \left[B_x\right]_{i,j+1/2}^{n-1/2} - \frac{\Delta t}{\Delta y}\left(\left[E_z\right]_{i,j+1}^{n} - \left[E_z\right]_{i,j}^{n}\right) \quad (9)$$

$$\left[B_y\right]_{i+1/2,j}^{n+1/2} = \left[B_y\right]_{i+1/2,j}^{n-1/2} - \frac{\Delta t}{\Delta x}\left(\left[E_z\right]_{i+1,j}^{n} - \left[E_z\right]_{i,j}^{n}\right) \quad (10)$$

$$\begin{aligned}\left[E_z\right]_{i,j}^{n+1} &= \left[E_z\right]_{i,j}^{n} - \left[J_z\right]_{i,j}^{n+1} \\ &+ \frac{\Delta t}{\varepsilon_0\mu_0\Delta x}\left(\left[B_y\right]_{i+1/2,j}^{n+1/2} - \left[B_y\right]_{i-1/2,j}^{n+1/2}\right) \\ &- \frac{\Delta t}{\varepsilon_0\mu_0\Delta y}\left(\left[B_x\right]_{i,j+1/2}^{n+1/2} - \left[B_x\right]_{i,j-1/2}^{n+1/2}\right) \quad (11)\end{aligned}$$

$$\left[J_z\right]_{i,j}^{n+1} = \left[J_z\right]_{i,j}^{n} + \Delta t\varepsilon_0\omega_p^2\left[E_z\right]_{i,j}^{n} \quad (12)$$

Note that superscript and subscript indices of a field component denote the sampling time and the position in the Yee grid, respectively. In the FDTD algorithm the electric and magnetic fields are not sampled at the same time. For time step Δt, the electric field component is sampled at nΔt (n is integer), but the magnetic field components are sampled at (n+1/2)Δt.

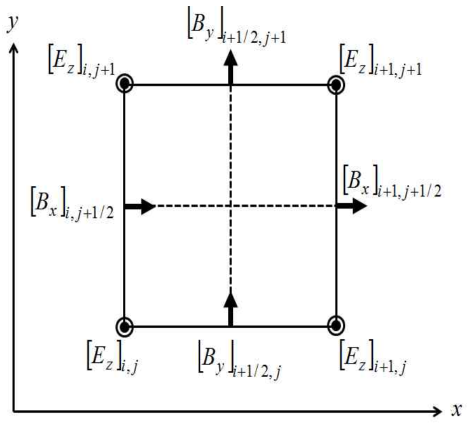

Fig. 1. Two-dimensional Yee grid on O-mode wave propagation

3. Two-dimensional Simulation of O-mode USPR

3.1 Simulation parameters

Fig. 2 depicts the schematic domain to compute the FDTD equations. The domain is composed of N_x(=1200)×N_y(=201) grid cells, where N_x and N_y are the number of cells in x and y directions, respectively. The use of a perfectly matched layer (PML) for the boundary is essential to avoid spurious reflections from the boundaries [10-11]. The PML condition is applied to the left, lower and upper boundaries. It is not necessary to satisfy the PML condition on the right boundary because all frequency components of an incident USPR pulse reflect from the critical density layers of the plasma and propagate toward the left side.

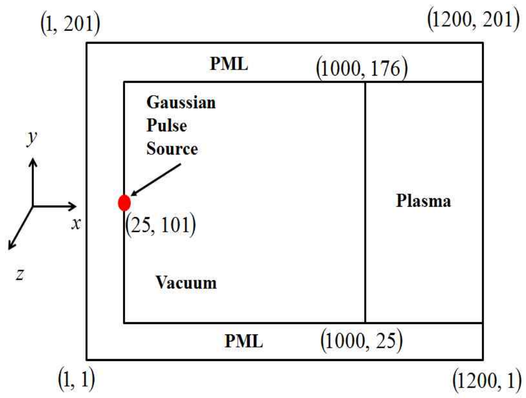

Fig. 2. The schematic domain of simulation

The plasma is located at the box defined by 1000 ≤N_x≤1200 and 26≤N_y≤175. The thickness of PML is 25. The grid size is assumed to be Δ_x=Δ_y=0.001m. To ensure the Courant-Friedrichs-Lewy (CFL) stability condition of the time-stepping FDTD algorithm, the temporal increment Δ_t is selected to be Δ_t=Δ_x/(2c) where c is the speed of light in vacuum [12].

As a source of USPR, the following Gaussian pulse is positioned at the node (25, 101).

$$G(n\Delta t) = \exp\left(-\frac{(n\Delta t - t_0)^2}{2\sigma}\right) \tag{13}$$

where σ is a parameter to determine the pulse duration (full with at half maximum: FWHM). σ is equal to $FWHM/(2\sqrt{2\ln 2})$. The Fourier transform of G is also a Gaussian function and its frequency bandwidth Δf is expressed as follows [13].

$$\Delta f = \frac{2\sqrt{2\ln 2\ln 10}}{\pi \times FWHM} \tag{14}$$

In the simulation, FWHM is chosen to be 44 ps and the maximum frequency component of the Gaussian pulse becomes ~18GHz.

Since the plasma current depends on the electron density, the electron density profile should be known to solve the FDTD equations. For simplicity of calculation in the simulation, the electron density profile is assumed to be a linearly increasing function in the x-direction.

$$n_e(N_x, N_y) = n_0 \frac{N_x \Delta x - x_0}{200\Delta x} \tag{15}$$

where $1000 \le N_x \le 1200$ and the plasma edge location x_0 is $1000\Delta_x$. The maximum electron density n_0 is assumed to be $4\times10^{18}\ m^{-3}$ so that the maximum cutoff frequency is 18GHz.

3.2 Simulation results

To perform the 2D simulation for O-mode wave propagation, the FDTD algorithm has been implemented in MATLAB. Fig. 3 illustrates the 2D images of the incident Gaussian pulse in vacuum. There is no dispersive effect in vacuum; thus the pulse propagates toward all directions in the x-y plane while the pulse shape remains constant. As stated earlier, PML plays a crucial role in the computation of the pulse propagation. Without the PML boundary, as shown in Fig. 4, the spurious reflections interfere with the original EM waves.

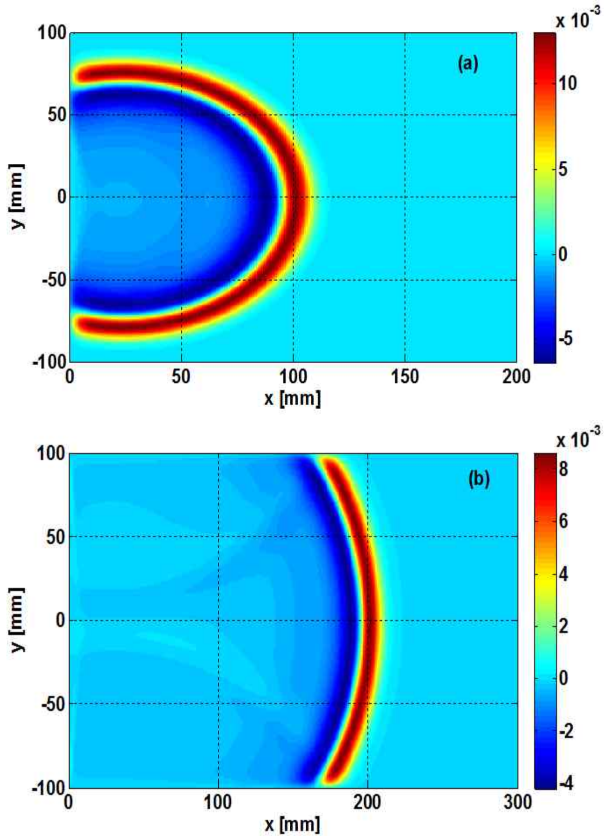

Fig. 3. 2D image of the incident Gaussian pulse, $E_z(x,y)$ at the time step of (a) 200 (b) 400

Fig. 5 (a) shows the 2D image of the reflected signal from the plasma during propagation of the Gaussian pulse inside the plasma. The propagation characteristics of an O-mode wave depend on the electron density profile that determines the cutoff frequency. The frequency component of the Gaussian pulse, which is lower than the cutoff

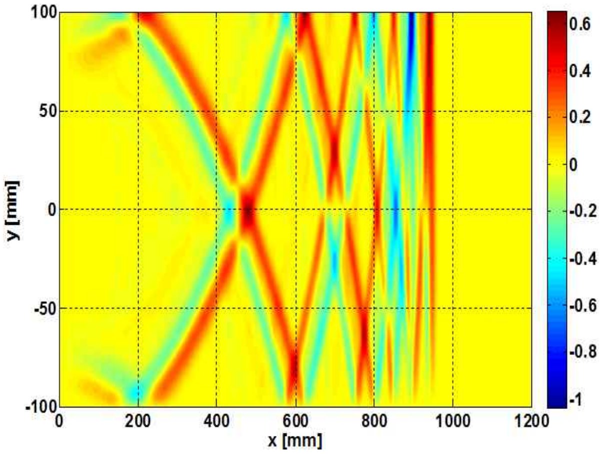

Fig. 4. 2D image of the incident Gaussian pulse, E_z(x,y) at the time step of 1900 without PML

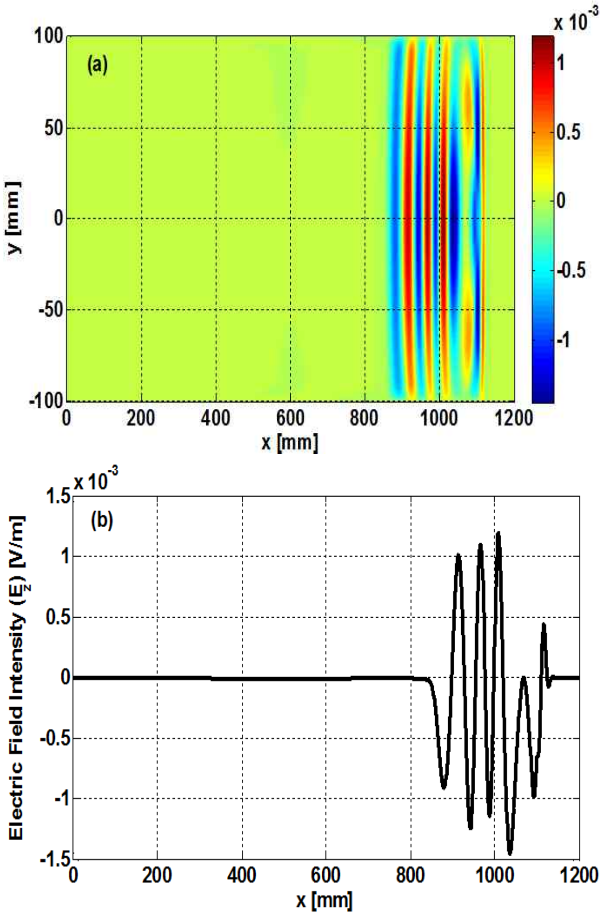

Fig. 5. Reflected signal at the time step of 2300. (a) the 2D image of E_z(x,y) (b) the 1D waveform of E_z(x,y=0)

frequency at a given density layer, reflects back to the vacuum side while the higher frequency components continue to propagate through the plasma. This phenomenon can be observed in the one-dimensional (1D) waveform of the reflected signal as shown in Fig. 5 (b).

Fig. 6 (a) depicts the 2D image of the reflected signal after all frequency components of the Gaussian pulse reflect from the plasma completely. It can be clearly seen in Fig. 6 (b) that the shape of the Gaussian pulse changes to a chirped waveform due to the dispersive property of the plasma.

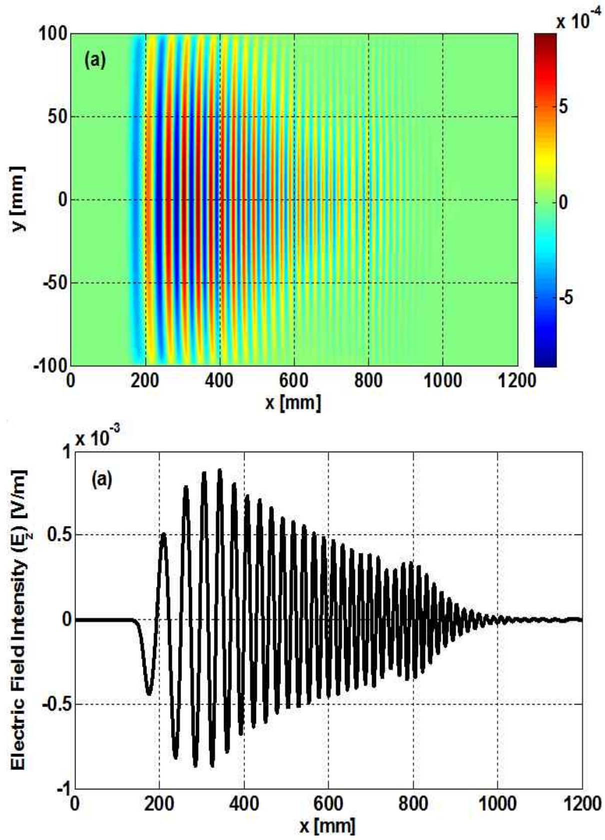

Fig. 6. Reflected signal at the time step of 3700. (a) the 2D image of E_z(x,y) (b) the 1D waveform of E_z(x,y=0)

Once the chirped waveform of the reflected signal in Fig. 6 (b) is obtained, it is possible to extract information on the time delays of frequency components. In this paper, thirteen numerical band-pass filters with integer center frequencies of 5~17GHz are used to measure the time delays. In O-mode USPR, the electron density profile can be reconstructed using the following Abel integral equation [3].

$$x(\omega_{pe}) = \frac{c}{\pi}\int_0^{\omega_{pe}} \frac{\tau_{pe}(\omega)d\omega}{\sqrt{\omega_{pe}^2 - \omega^2}} \qquad (16)$$

where $\tau_{pe}(\omega)$ is the double-pass time delay as a function of frequency. Note that $\tau_{pe}(\omega)$ is the double-pass time delay for the frequency component to travel from the plasma edge to the critical density layer. Therefore, the time delay in vacuum must be subtracted from the time delay in Fig. 6(b) to find $\tau_{pe}(\omega)$ in Fig. 7. By applying the trapezoidal integration technique to the Abel integral equation with $\tau_{pe}(\omega)$, the density profile can be reconstructed as shown in Fig. 8. Here, the assumed density profile is also presented to examine the performance of the reconstruction of the density profile. As can be seen, the reconstructed density profile agrees with the assumed profile.

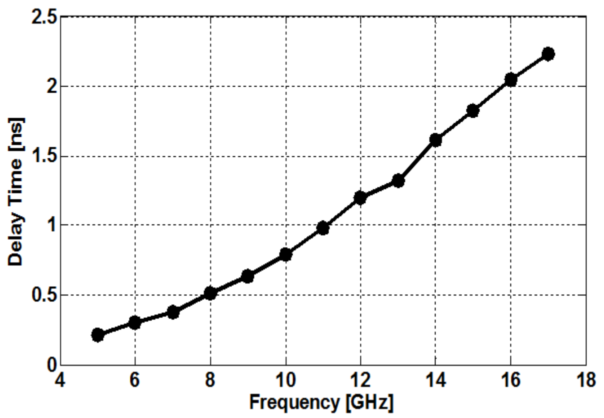

Fig. 7. Double-pass time delays of frequency components in the plasma

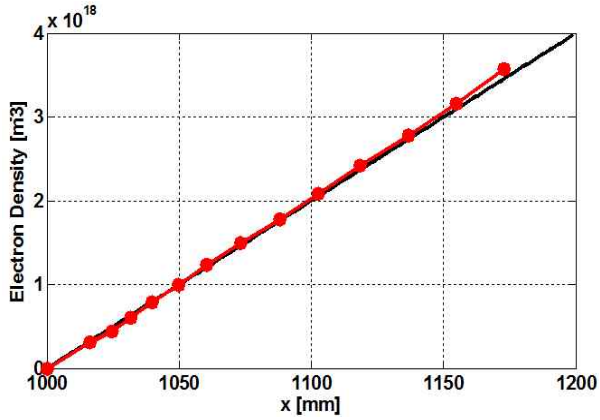

Fig. 8. Reconstructed density profile (red dot) and assumed density profile (black line)

4. Conclusion

The 2D FDTD algorithm was successfully employed to numerically solve Maxwell's equations for the propagation of O-mode USPR waves in the plasma. The PML condition is included in the algorithm to avoid spurious reflections from the boundaries. It was found from simulation results that the shape of the Gaussian pulse changes to a chirped waveform due to the dispersive property of the plasma. It was also demonstrated that the electron density profile can be accurately reconstructed by computing the Abel integral equation.

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