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A study on didactic transposition of mathematics textbooks and lessons in Korea and the U.S.[1]

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Didactic transposition refers to an adaptive treatment of mathematical knowledge into knowledge to be taught. This study intends to investigate how mathematical knowledge was modified in mathematics textbooks and lessons. This study identified examples of didactic transposition in mathematics textbooks and lessons in Korea and those in the U.S., The examples identified were FOIL method, trigonometry using s, c, t in writing style, order of operations(PEMDAS), area of a circle and circumference, order of prefixes in the metric system, trigonometry(SOH, CAH, TOA), operations on integer, and regular polyhedra. These examples were classified into the two categories, one for mnemonics, and one for concreteness and intuitiveness. Then a survey was conducted for in-service teachers in Korea and those in the U.S. to evaluate the appropriateness and the necessity of didactic transposition. Lastly, the potential didactic phenomena, meta-cognitive shift which may occur with these examples were discussed.

Key Words : Didactic Transposition, Mathematics Textbook, Mathematics Lesson, Meta-Cognitive Shift

Ⅰ. Introduction

The relationship between academic mathematics as practiced by mathematicians at universities and classroom mathematics is one of fundamental research questions in

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mathematics education. In this regards, didactic transposition which refers to any modification of knowledge under instructional purposes, is a crucial concept to understand mathematics education in general.

Didactical transposition deals with transposition of academic knowledge into practical knowledge intended to teach. Thus didactic transposition delves into the multi-dimensioned variation process that begins with the 'advanced mathematics as a discipline' to 'mathematics as a school subject', and 'mathematics for mathematicians' to 'mathematics for students'. Brousseau(1997), one of the pioneers in didactical transposition theory stated as follows.

> To make teaching easier, it isolates certain notions and properties, taking them away from the network of activities which provide their origin, meaning, motivation and use. It transposes them into a classroom context. Epistemologists call this *didactical transposition* (Brousseau, 1997, p. 21).

The purpose of this study is to investigate how mathematical knowledge is modified in mathematics textbooks and lessons. To pursue the aims of this study, the following two research questions were addressed.

What are the categories of examples of didactic transposition found in the textbooks and lessons in Korea and those in the U.S.?

How are the examples of didactic transposition evaluated by the in-service teachers in Korea and those in the U.S.?

Ⅱ. Backgrounds

In this section, the previous studies on didactic transposition were reviewed, and then one of typical examples of didactic transposition was explained and discussed in detail.

1. Previous studies

Various studies have been conducted regarding didactic transposition. One of the most representative works was done by Kang(1990). He considered didactic transposition as a temporary, hypothetical contextualization and personalization of knowledge to fit the students' situation. Kang reviewed three algebra textbooks and identified examples of didactic transpositions, and then categorized these examples into four groups: localization of mathematical concepts, real-world models for mathematical concepts, word problem types, and bodies of extra-mathematical knowledge.

Klisinska(2009) investigated the relationship between 'academic mathematics' and 'classroom mathematics' focusing on the Fundamental Theorem of Calculus (FTC) and its proof. The study comprises a historical account of the invention of the FTC and its proof, and interviews with mathematicians. The outcomes of the historical account and interview data about the mathematicians' views are discussed from the perspective of the theory of didactic transposition.

Winslow(2007) presented some key ideas and examples of work in the didactics of mathematics with the intention of identifying the didactics approach to education. Three crucial features of didactics were discussed: 1) its epistemological character, i.e. the central focus on specific knowledge in different forms, and the conditions for enabling pupils to acquire it; 2) its relations to mathematics as a science; and 3) its actual and potential roles in relation to teaching and teachers.

Bosch and Gascon(2006) reflected a twenty-five years of researches on the didactic transposition, and investigated how the theory on didactic transposition is diffused in different countries, for example in France, Spanish-speaking countries, and English-speaking countries. Also they summarized the three main contributions of the theory of didactic transpositions in the progress of mathematics education as a field of research: 1) an enlargement of the empirical unit of analysis; 2) the description of mathematical and didactic activities; and 3) constraints at different levels of determination.

Han(2004) investigated on didactic transposition and generalization of the Ceva theorem. He suggested inverse of the Ceva theorem, different forms of the Ceva theorem (oriented segment form, trigonometric form, and vector form), and generalized the Ceva theorem of polygon and tetrahedron from the perspective of didactic transposition. Lee and Shin(2009) investigated how trigonometric function is didactically transposed in high school textbooks. They classified transpositions into three types, one for easier use of didactical manipulative, one for more practical, and one for more principle based. Kim(2008) observed and analyzed textbooks and actual classes to discern how the didactic transpositions on the teaching of the ratio of circumference and the area of a circle are occurring in elementary mathematics. Park(2002) investigated how mathematical knowledge has been modified in classroom. Three mathematics classes in the 3rd grade were video-taped and analyzed. He concluded that the teachers tried to deliver more detailed context and background, and more frequently used creative materials when teaching geometry than arithmetic. Lee(2010) investigated the process of knowledge construction by metaphor and analogy, and their roles in didactic transposition. She elaborated three kinds of models using metaphor and analogy in didactic transposition; metaphor as a part of analogy, analogy as a connecting tool between metaphors, and analogy as a tool to expand and enrich metaphors.

As shown in the above, there have been a variety of studies in this topic including theoretical investigation on didactic transposition theory, and empirical studies to explore the didactic transpositions in textbooks and lessons as well as the role of metaphor and analogy in didactic transposition.

2. An examples of didactic transposition

One of the most referred examples of didactic transposition in school mathematics is the definition of 'convergence of sequence' using $\epsilon - N$ method in *Analysis* and its intuitive definition in high school textbook.

Sequence $\{x_n\}$ converges into x
For every $\epsilon > 0$, there is an integer N such that $n \geq N$ implies that $\mid x_n - x \mid < \varepsilon$
⇓ didactic transposition
If n becomes larger and larger, then $\{x_n\}$ goes closer and closer to x

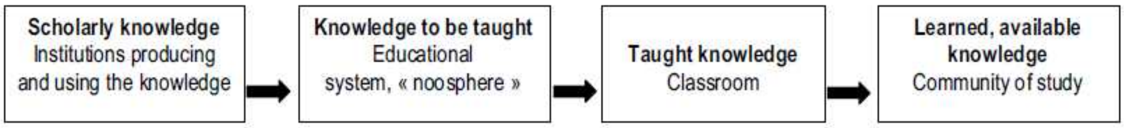
Intuitive definition (Hwang et al, 2009) in high school mathematics textbooks and formal definition in *Analysis* (Bartle & Sherbert, 2011) has several distinctive features. First is a contrast between dependency perspective and correspondence perspective on function. Intuitive definition reflects a dependency perspective on function since x_n varies according to n varies. However, formal definition adopts a correspondence perspective on function because x_n exits for every n. Second is a contrast between potential infinity and actual infinity. Intuitive definition is based on potential infinity because $\{x_n\}$ goes closer and closer, yet never reaches the value. Formal definition assumes a limit value and thus accepts actual infinity. Third is a contrast between finding and justification. Intuitive definition focuses on the process of finding a limit value while formal definition emphasizes on the justification of a limit value which was already found. Fourth is about the order of cause and effect. Intuitive definition, independent variable n is a cause of dependent variable x_n, and consequently the order is from cause to result. On contrary, formal definition requires to find N which ensures if $n \geq N$, then $|x_n - x| < \epsilon$, thus the order is from result to cause. Thus, a formal definition is against natural thinking process.

The above example shows that didactic transposition of a certain content causes changes of the nature of the content.

Ⅲ. Research Methodology

1. Conceptual Framework

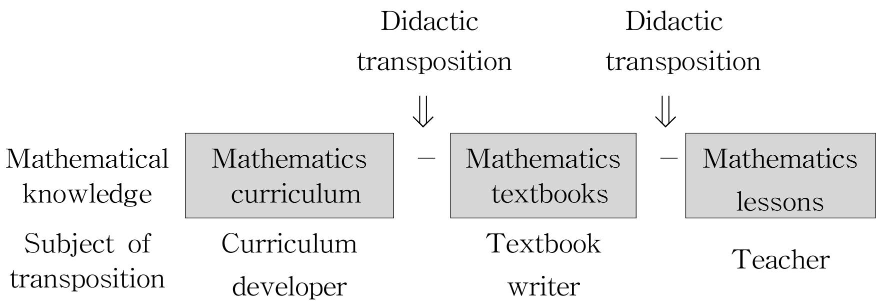
Chevallard(1991, p. 43) defines the didactic transposition as the work done during the transformation from *scholarly knowledge* via *knowledge to be taught* and the *actual knowledge taught* to *learnt knowledge*. Bosch and Gascon(2006) suggested the following process of the didactic transposition by reflecting Chevallard's idea.



[Figure 1] The didactic transposition process (Bosch and Gascon, 2006, p. 56)

In the above diagram, Bosch and Gascon made a clear distinction. First is the scholarly mathematical knowledge as it is produced by mathematicians. Second is the mathematical knowledge to be taught as it is officially designed by curricular. Third is the mathematical knowledge as it is actually taught by teachers in their classrooms. And the fourth is the mathematical knowledge as it is actually learnt by students.

Among the processes illustrated by Bosch and Gascon(2006), this study focuses on the second and the third, i.e., didactic transposition process from 'knowledge to be taught' to 'taught knowledge'. Thus this study divides this process into the two, didactic transposition from mathematics curriculum to textbooks, and that from textbooks to lessons.



[Figure 2] Phases of didactic transposition

2. Data Collection

To collect the examples of didactic transposition, mathematics classes in the 7th grade were chosen and observed for one semester. Korean classes were observed in School S, which was located in Seoul with mostly middle SES. The female teacher who taught the class had 9 years of teaching experiences. This school adopted mathematics textbooks published by company D which had one of the largest market shares in Korea.

In the U.S., the classes was observed in school C. This school was a public school located in the suburb of metropolitan city in California with relative high SES. The achievement level of the school was one of the highest in California since the school had large proportion of Asian students. Seventh graders were supposed to take *Pre-Algebra* or *Algebra*. Among these two courses, *Algebra* class was observed. The female teacher who was responsible for the class had 13 years of teaching experiences. In addition to the observation of classes, the textbooks used in the class were reviewed as well

3. Research Method

The research was conducted by the following method. First, the examples of didactic transposition collected from mathematics textbooks and lessons were reviewed, and the categories were formed by grounded theory approach(Strauss & Corbin, 1999). Rather than beginning with a hypothesis, the collected examples were analyzed, and similar examples were grouped. In the process of repeating this analysis, the categories were came out. The result is shown in section IV.

Second, a small scale survey was conducted for the teachers in Korea and those in the U.S. to probe into their evaluation of the examples in didactic transposition. The Korean sample consisted of 56 in-service teachers who participate in a teacher training program as professional development during the vacation. The U.S. sample consisted of 31 in-service teachers attending the master's program in the State University. Before handing out the questionnaire to in-service teachers, the researcher explained the operational definition of didactic transposition and the examples of didactic transposition. Especially, more explanations on the U.S. examples were needed for Korean teachers, and the vice versa since they were not familiar with the examples derived from the counterpart. For example, the U.S. teachers could not figure out the alphabet writing styles of s, c, and t to denote sin, cos, and tan unless further explanation was provided. The results were listed in section V.

Ⅳ. Classification of the examples of didactic transposition

The examples of didactic transposition found in textbooks and lessons in Korea and those in the U.S. were listed and then reviewed repeatedly by the researcher and two mathematics teachers. After reviewing the examples of didactic transposition over and over again, and finally we came up with the two categories based on the nature and the intention of didactic transposition. The two categories were didactic transposition for mnemonics, and that for intuitiveness and concreteness.

<Table 1> The classification of examples of didactic transposition.

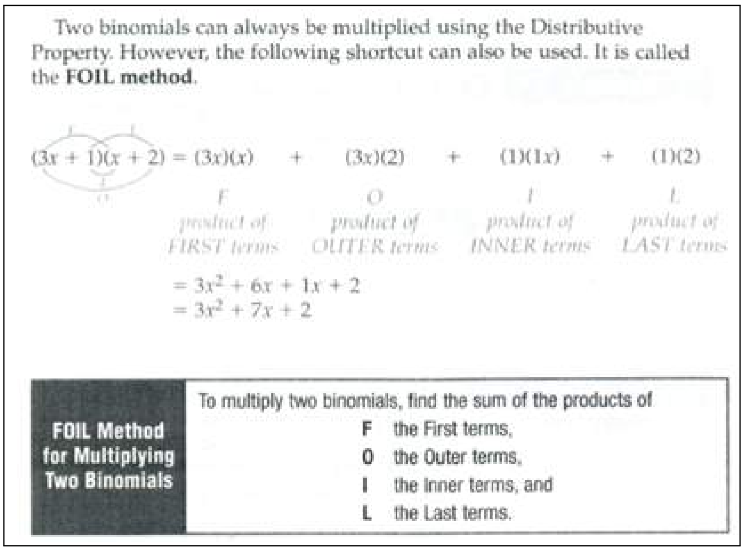
Category	Textbook	Lesson
Mnemonics	· FOIL · trigonometry (s, c, and t in writing style)	· order of operations · area of a circle and circumference · order of prefixes in the metric system · trigonometry (SOH, CAH, TOA)
Intuitiveness and concreteness	· operations on integer · regular polyhedra	· operations on integer

1. Didactic transposition for mnemonics

Mnemonics aim to translate information into a form that the human brain can retain better than its original form. Many examples of didactic transposition correspond to this category, in fact mnemonics is one of the most frequently identified intentions of didactic transposition from the textbooks and lessons. The examples in this category are the FOIL method in the U.S. textbooks, the definition of trigonometry using s, c, and t in Korean textbook, and several examples found in the U.S. lessons.

1) FOIL method (the U.S. textbook)

The FOIL method has been quoted as a typical example of didactic transposition found in the U.S. mathematics textbooks. The word FOIL is an acronym for the four terms of the product First, Outer, Inner, and Last. FOIL is a mnemonic which is devised to facilitate students′ understanding of binomial multiplication.


Two binomials can always be multiplied using the Distributive Property. However, the following shortcut can also be used. It is called the **FOIL method**.

$(3x+1)(x+2) = (3x)(x) + (3x)(2) + (1)(1x) + (1)(2)$

F: product of FIRST terms; O: product of OUTER terms; I: product of INNER terms; L: product of LAST terms

$= 3x^2 + 6x + 1x + 2$
$= 3x^2 + 7x + 2$

FOIL Method for Multiplying Two Binomials

To multiply two binomials, find the sum of the products of
- **F** the First terms,
- **O** the Outer terms,
- **I** the Inner terms, and
- **L** the Last terms.

[Figure 3] FOIL method

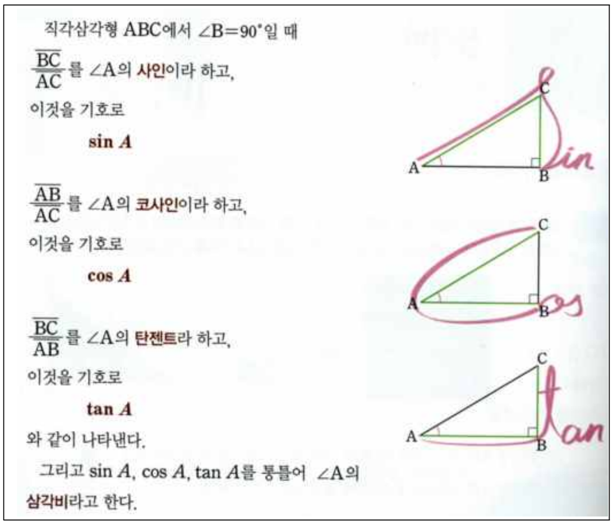
In fact, the formula $(a+b)(c+d) = ac+ad+bc+bd$ is also derived from the distributive law $a(b+c) = ab+ac$. In the perspective of mathematics, the only significance is on the distributive law, and the formula $(a+b)(c+d) = ac+ad+bc+bd$ as well as other multiplication formulae are the results of didactic transposition. Although multiplication formulae are the by-product of distributive law, these formulae are emphasized as a special mathematical knowledge with significance in mathematics textbooks.

Distributive law	Didactic transposition ⇓ ⇒	Multiplication formulae0	Didactic transposition ⇓ ⇒	Mnemonic
$a(b+c) = ab+ac$	⇒	$(a+b)(c+d) = ac+ad+bc+bd$ $(a+b)^2 = a^2+2ab+b^2$ $(a+b)(a-b) = a^2-b^2$	⇒	FOIL

[Figure 4] Didactic transposition of the FOIL method

2) Trigonometry (Korean textbook)

As Figure 5 shows, Korean mathematics textbooks present sin, cos, and tan as *s, c,* and *t* in writing style to use the visual images to facilitate students' understanding.



[Figure 5] S, c, and t in writing style to denote sin, cos, tan (Woo, et al, 2011, p. 226)

The explanation of sin, cos, and tan above is the result of didactic transposition. It is a kind of visual mnemonic system to help learners easily memorize. This explanation is not used in the U.S. textbooks because, whereas in Korea fraction is read 'denominator first and then numerator', in the U.S. it is read in the reverse order, 'numerator over denominator'.

3) Order of operations (the U.S. lesson)

The U.S. textbooks explicitly emphasize the rules for order of operations, as shown in Figure 6. Korean textbooks include the similar contents as well.

Order of Operations	1. Do all operations within grouping symbols first. 2. Multiply and divide in order from left to right. 3. Add and subtract in order from left to right.

[Figure 6] Order of operations (Collins, et al, 2001, p. 8)

In math classes in the U.S., a mnemonic system called PEMDAS, was introduced to teach order of operations. Here, PEMDAS stands for Parenthesis, Exponents, Multiplication and Division, and Addition and Subtraction. PEMDAS was transformed once again in order to help students memorize easily. PEMDAS can be retrieved from the sentence, "Please(P) Excuse(E) My(M) Dear(D) Aunt(A) Sally(S)." The American teacher explained that what PEMDAS stood for, and then introduced the sentence to facilitate to memorize PEMDAS. In addition, BOMDAS where Parenthesis and Exponents were replaced with Bracket(B) and Ordinals(O) was also mentioned in the class.

4) Area of a circle and circumference (the U.S. lesson)

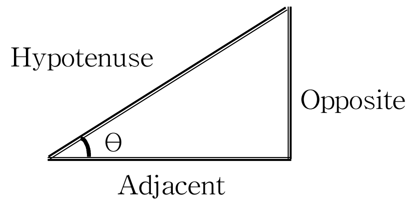
In math classes in the U.S. a mnemonic system was used to memorize the formula of the area and perimeter of a circle. The formula to find the area of a circle was $A=\pi r^2$, which can be transformed into apple(A) pie(π) are(r) square. Another mnemonic, apple (A) pie(π) are(r) round(r) which denoted $A=\pi rr$ was also mentioned. As circumference was π times diameter, the U.S. teacher transformed $C=\pi d$ into cherry (C) pie(π) delicious(d). All these examples were devised by the U.S. teachers to facilitate students' efficient memorization.

5) Order of prefixes in the metric system (the U.S. lesson)

In the metric system, 10^3 is Kilometer(K), 10^2 Hectometer(H), 10^1 Decameter(D), 10^0 Meter(M), 10^{-1} Decimeter(D), 10^{-2} Centimeter(C), and 10^{-3} Millimeter(M). In order to memorize these, the following sentence was used in the U.S. class; "King(K) Henry(H) Died(D) Monday(M) Drinking(D) Chocolate(C) Milk(M)." From this sentence, prefixes in the metric system were expected to be retrieved.

6) Trigonometry (the U.S. lesson)

When the American teacher introduced trigonometry(sin, cos, and tan), SOH, CAH, and TOA were used. Here, SOH denotes that Sine is Opposite over Hypotenuse, CAH that Cosine is Adjacent over Hypotenuse, and TOA that Tangent is Opposite over Adjacent. In reality, SOH, CAH, and TOA are pronounced as [so], [ka], and [toe], respectively, that have no meanings. Therefore, the teachers suggested "soak a toe," which had similar pronunciation for easy memorization.



As mentioned earlier, *s, c,* and *t* in writing style is commonly used in Korean textbooks and lessons. But it can not be used for memorizing in English-speaking classrooms, thus the U.S. lesson adopted different mnemonic system.

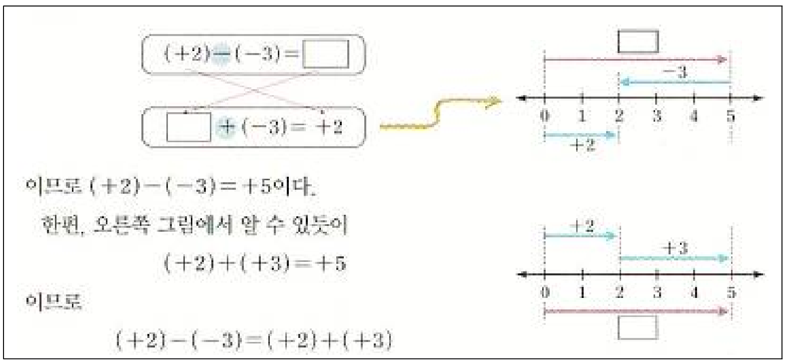
2. Didactic transposition for intuitiveness and concreteness.

Other than mnemonics, the most common intention of didactic transposition was to make mathematics more intuitive and concrete. Mathematically rigorous definitions and explanations are converted into more intuitive and concrete ones for promoting students' understanding. Operations on integer and regular polyhedra correspond to this category.

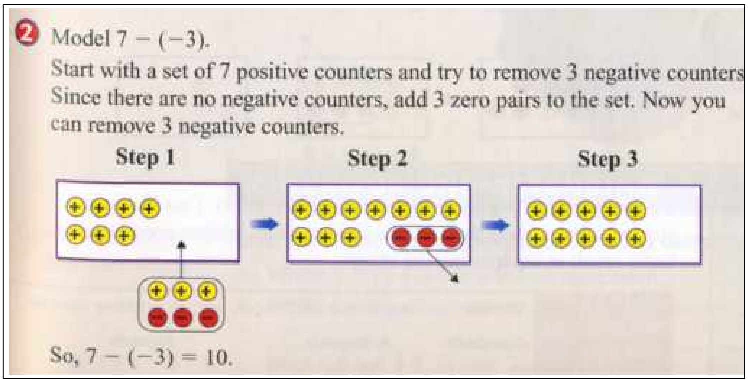
1) Operations on integers (Korean and the U.S. textbooks)

In relation to operations on integers, 'a negative times a negative is a positive' causes cognitive difficulties among students. This can be explained in a mathematically rigorous way using algebraic properties in mathematics as a discipline.

To explain the operations on integers, the Korean mathematics textbook in the 7th grade provides a number line model (Figure 7) while the U.S. textbooks adopts white and black stone model (Figure 8).



[Figure 7] Number line model for operations on integers (Woo, et al, 2009, p. 59)



[Figure 8] White and black stone model for operations on integers (Collins, et al, 2001, p. 201)

Meanwhile, the Korean teacher proposed a model of double structure, where a postman delivers(+) or retrieves(−) a certificate(+) or a tax bill(−). In this model, if a tax bill(−) is retrieved(−), it has a positive effect. Therefore, it can be interpreted that a negative times a negative is a positive.

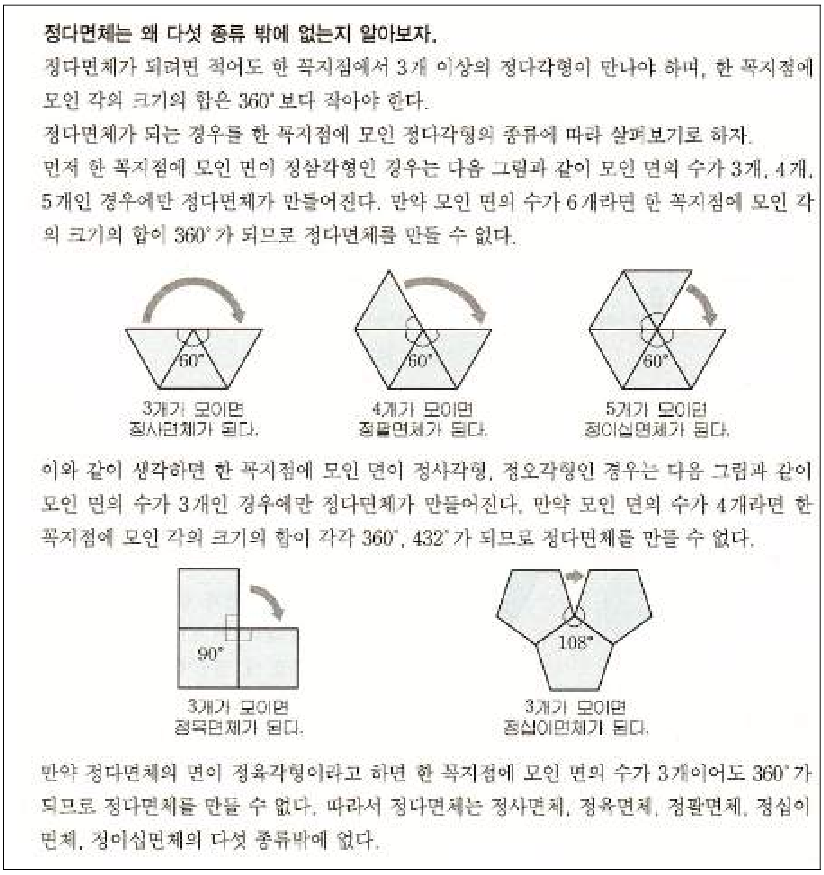
In the U.S. lesson, a different model was introduced. The American teacher adopted a metaphor of a friend and an enemy. A friend was treated as a positive and an enemy a negative. As an enemy of an enemy was treated as a friend, the conclusion became $(-)\times(-)=(+)$.

2) Regular polyhedra (Korean textbooks)

One of the important topics of solid geometry in the 7th grade in Korea is regular polyhedra. Most Korean textbooks in the 7th grade provide a discussion problem which asks students to answer why there are only five regular polyhedra shown in Figure 9.

[Figure 9] Discussion problem about five regular polyhedra (Woo, et al, 2009, p. 251)

The mathematics textbooks based on the previous curriculum include informal justification in Figure 10 as an enrichment reading.



정다면체는 왜 다섯 종류 밖에 없는지 알아보자.

정다면체가 되려면 적어도 한 꼭지점에서 3개 이상의 정다각형이 만나야 하며, 한 꼭지점에 모인 각의 크기의 합은 360°보다 작아야 한다.

정다면체가 되는 경우를 한 꼭지점에 모인 정다각형의 종류에 따라 살펴보기로 하자.

먼저 한 꼭지점에 모인 면이 정삼각형인 경우는 다음 그림과 같이 모인 면의 수가 3개, 4개, 5개인 경우에만 정다면체가 만들어진다. 만약 모인 면의 수가 6개라면 한 꼭지점에 모인 각의 크기의 합이 360°가 되므로 정다면체를 만들 수 없다.

이와 같이 생각하면 한 꼭지점에 모인 면이 정사각형, 정오각형인 경우는 다음 그림과 같이 모인 면의 수가 3개인 경우에만 정다면체가 만들어진다. 만약 모인 면의 수가 4개라면 한 꼭지점에 모인 각의 크기의 합이 각각 360°, 432°가 되므로 정다면체를 만들 수 없다.

만약 정다면체의 면이 정육각형이라고 하면 한 꼭지점에 모인 면의 수가 3개이어도 360°가 되므로 정다면체를 만들 수 없다. 따라서 정다면체는 정사면체, 정육면체, 정팔면체, 정십이면체, 정이십면체의 다섯 종류밖에 없다.

[Figure 10] Informal justification about five regular polyhedra (Kang, Jung, & Lee, 2000)

Mathematics for the 7th grade requires explanations of intuitive level. For example, five regular polyhedra can be explained based on the fact that more than three polygons should be at a vertex of a regular polyhedron and that the sum of interior angles of the polygons at a vertex should be less than 360°. In fact, explanation on five regular polyhedra is included in Euclid's *Elements*. The last part of the last book of *Elements*, which consists of 13 books, relates to regular polyhedra. Propositions from 13 to 17 describe constructions of tetrahedron, octahedron, cube, icosahedron, and dodecahedron, respectively. Proposition 18 offers the summary of propositions from 13 to 17 and explains that there are only five regular polyhedra in the remark. The textbook explanation in Figure 10 is an informal version of mathematically rigorous justification below.

Justification using indeterminate inequality

If m is the number of edges of each face, and n is the number of faces meeting at each vertex, then there is n regular m-gon in each vertex of regular polyhedron. Since an interior angle of a regular m-gon is $\frac{(m-2)}{m}\times 180°$, the sum of interior angles of n regular m-gon is $\frac{(m-2)}{m}\times 180°\times n$. To form a polyhedron, the sum of interior angles should be smaller than 360°. Thus, $\frac{(m-2)}{m}\times 180°\times n < 360°$, and it is reduced to $(m-2)(n-2) < 4$. The positive integer solutions of this indeterminate inequality are as follows

1) $m-2=1, n-2=1 \Rightarrow m=3, n=3 \Rightarrow$ three triangles in each vertex: tetrahedron
2) $m-2=1, n-2=2 \Rightarrow m=3, n=4 \Rightarrow$ four triangles in each vertex: octahedron
3) $m-2=2, n-2=1 \Rightarrow m=4, n=3 \Rightarrow$ three squares in each vertex: cube
4) $m-2=1, n-2=3 \Rightarrow m=3, n=5 \Rightarrow$ five triangles in each vertex: icosahedron
5) $m-2=3, n-2=1 \Rightarrow m=5, n=3 \Rightarrow$ three pentagons in each vertex: dodecahedron

Topological justification

According to Euler's polyhedron formula, $V-E+F=2$ where V, E, F are respectively the numbers of vertices, edges and faces in a given polyhedron. If m is the number of edges of each face and n is the number of faces meeting at each vertex, then $mF=2E=nV$.

Combining these two formulae, we can get $\frac{2E}{n}-E+\frac{2E}{m}=2$. This formula is simplified as $\frac{1}{n}+\frac{1}{m}=\frac{1}{2}+\frac{1}{E}$ and $\frac{1}{n}+\frac{1}{m}>\frac{1}{2}$ $(\because \frac{1}{E}>0)$. Since n and m should be equal or larger than 3, the possible solutions are (3, 3), (4, 3), (3, 4), (5, 3), (3, 5)

Five regular polyhedra included in mathematics textbooks use didactic transposition appropriate to the level of the 7th graders, instead of justification using indeterminate inequality or topological justification described above.

Ⅴ. Survey results

As described in the methodology section, a small scale survey was conducted for the teachers in Korea and those in the U.S. The in-service teachers were asked to fill out the questionnaire including 8 examples of didactic transposition. In each example of didactic transposition, teachers were asked to evaluate the appropriateness and the necessity in 5 Likert scale, and were also requested to freely write down anticipated positive and/or negative aspects that each didactic transposition may cause. The following is a questionnaire sample of the first example.

1. FOIL
$(a+b)(c+d)=ac+ad+bc+bd$
First Outer Inner Last

1 The appropriateness of didactic transposition
① very inappropriate ② inappropriate ③ neutral ④ appropriate ⑤ very appropriate

2 The necessity of didactic transposition
① very unnecessary ② unnecessary ③ neutral ④ necessary ⑤ very necessary

3 Write down positive and/or negative aspects of didactic transposition

The teachers were supposed to choose a single value between 1 to 5 to represent the level of appropriateness and necessity. The table 2 shows the results of the survey.

[Table 2] The results of the survey to evaluate the examples of didactic transposition[3)]

Category	Example	Korean teachers (N=56)				the U.S. teachers (N=31)			
		Appropriateness		Necessity		Appropriateness		Necessity	
		Mean	SD	Mean	SD	Mean	SD	Mean	SD

3) The t-test or ANOVA were not tried here because the main focus of this survey was not to compare the rating scores of Korean teachers and those of American. teachers. In addition, Korean teachers and American teacher have different experiences of teaching the topics included in the survey. Thus the survey involve uneven condition of these samples.

Mnemonics	The FOIL method	1.91	0.32	1.68	0.38	3.39	0.44	3.88	0.33
	Trigonometry (s, c, and t in writing style)	4.56	0.46	4.10	0.52	3.31	0.45	3.80	0.36
	Order of operations (PEMDAS)	1.83	0.43	1.61	0.37	3.48	0.49	3.73	0.42
	Area of a circle and circumference	2.25	0.35	2.13	0.24	2.82	0.38	2.56	0.37
	Order of prefixes in the metric system	1.89	0.29	1.26	0.36	2.25	0.42	2.41	0.30
	Trigonometry (SOH, CAH, and TOA)	1.82	0.33	2.42	0.43	2.33	0.41	3.57	0.41
Intuitiveness and concreteness	Operations on integer (number line)	3.93	0.41	3.37	0.39	3.61	0.32	3.72	0.30
	Operations on integer (white and black stone model)	3.58	0.36	3.52	0.33	3.66	0.28	3.83	0.28
	Regular polyhedra	3.27	0.32	3.55	0.29	3.83	0.46	3.59	0.40

In general, the teachers' responses were relatively favorable. The teachers admitted that the examples of didactic transposition were mostly appropriate and necessary. Also, the survey result showed that there were not much differences between the mean scores in appropriateness and necessity for the sampled teachers in the two countries. Yet there were a couple of items in which the differences were large. For example, the mean scores of the FOIL method and PEMDAS (as an order of operations) were relatively high in the U.S. teachers while the mean scores for these two examples were low in Korean teachers. The opposite case was the trigonometry(s, c, and t in writing style) found in Korean textbooks, in other words, the mean score of Korean teachers was higher than that of U.S. teachers. The teachers tended to prefer the examples of didactic transposition found in textbooks or lessons in their own country. Except these three examples, the results of the two countries were mostly synchronized.

In the survey, not many teachers answered for the 3rd question in each example, i.e., positive and/or negative aspects of didactic transposition. Korean teachers had some concerns about mnemonics found in the U.S. textbooks and lessons. They cautiously criticized that the FOIL method and PEMDAS had a possibility of making mathematics too trivial, There were also some concerns about mnemonics of trigonometry(SOH, CAH, TOA) and order of prefixes in the metric system.

One of the Korean teachers pointed out the drawbacks of mnemonics.

"The multiplication of two binomials is supposed to be learned through practicing various exercise problems, not through explicitly memorizing FOIL."

Another Korean teacher was also critical about mnemonics.

"The effectiveness of PEMDAS is questionable."

One of the American teachers indirectly mentioned the danger of hasty generalization.

"Not all the teachers in the U.S. are using non-canonical mathematics like FOIL and PEMDAS"

Another American teacher showed favorable response to trigonometry in Korean textbook.

"The visual s, c, t on a right triangle is neat. I think it works well in Korean"

Six out of 56 Korean teachers pointed out that these mnemonics may cause ″meta-cognitive shift″ (Brousseau, 1997). This will be discussed in more detain in the following section

Ⅵ. Summary and Discussion

Didactic transposition refers to an adaptive treatment of mathematical knowledge into knowledge to be taught. With the review of examples of didactic transposition, it was attempted to classify these examples into two categories. The first category, mnemonics includes the FOIL method, trigonometry using s, c, t in writing style, order of operations(PEMDAS), area of a circle and circumference, order of prefixes in the metric system, and trigonometry(SOH, CAH, TOA). These examples are designed to help learners easily memorize the concept or formula. The second category, concreteness and intuitiveness, includes operations on integer and regular polyhedra. These examples transform rigorous explanation into intuitive one to promote learners′ better understanding.

The main purpose of this survey was not to compare the mean scores of Korean teachers and those of the U.S. teachers, neither to generalize the result. Instead, this survey intended to figure out how the examples of didactic transposition were accepted by the sampled teachers in Korea and those in the U.S. In general, the mean scores of the category 'concreteness and intuitiveness' is higher than those of the category 'mnemonics'. Especially, there were a certain level of skepticism for mnemonics among Korean teachers.

Responses to the open-ended item in the survey indicated that the examples of didactic transposition may cause extreme didactic phenomenon such as meta-cognitive shift (Brousseau, 1997; Chevallard, 1991; Kang, 1990). Meta-cognitive shift is a phenomenon which takes place when the process of personalization and contextualization of mathematical knowledge is overemphasized. The didactical efforts of textbooks may shift students' attention from the mathematical knowledge to a didactical device. For instance, the FOIL method, there might be a meta-cognitive shift from binomial multiplication to mnemonic FOIL, and it is also the case of PEMDAS. The problem with a meta-cognitive shift is that students' interest may just linger on FOIL and PEMDAS, and they may not get to mathematical meaning.

Didactic transposition has positive implications, as it helps learners enhance mathematical understanding. However, it also has potential to cause negative impacts including meta-cognitive shift. In this regard, didactic transposition is a double-edged sword. One of the most important goals of mathematics education is to improve learners' mathematical understanding. To achieve this goal, didactic transposition plays a crucial role since didactic transposition deals with the mechanism of transformation of mathematical knowledge into knowledge to be taught. The mission of didactic transposition is preserving mathematical meaning and significance in the process of turning mathematics as a discipline into school mathematics.

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한국과 미국의 수학 교과서와 수업에 나타난 교수학적 변환에 대한 연구

박경미[4)]

초 록

교수학적 변환은 '학문 수학'을 '학교 수학'으로 변화시키는 다차원적인 변용의 과정을 다룬다. 본 연구는 학문 수학에서 출발하여 수학 교육과정, 수학 교과서, 수학 수업으로 이어지는 일련의 과정에서 나타날 수 있는 교수학적 변환의 예로 FOIL 방법, 삼각함수의 정의(한국 교과서), 연산의 순서, 원의 넓이와 원주, 미터법의 단위, 삼각함수의 정의(미국 수업), 정수의 연산, 정다면체에 대한 설명을 제시하고, 그 특징 및 변환의 의도에 따라 기억법, 직관화/구체화의 두 가지 유형으로 분류하였다. 또한 교수학적 변환에 대한 한국과 미국 수학 교사의 인식을 알아보기 위해 설문조사를 실시하고, 변환의 적절성과 필요성을 조사하였다. 마지막으로는 교수학적 변환과 관련하여 발생할 수 있는 극단적인 교수학적 현상인 '메타-인지 이동'을 설문조사 결과를 중심으로 논의하였다.

주요용어 : 교수학적 변환, 수학 교과서, 수학 수업, 메타인지 이동

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