

# A Study on Multi-Period Inventory Clearance Pricing in Consideration of Consumer's Reference Price Effect

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(Received: January 31, 2013 / Revised: April 30, 2013 / Accepted: April 30, 2013)

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## ABSTRACT

It is difficult to determine an appropriate discount price for daily perishable products to increase profit from a long-term standpoint. Even if the discount pricing is efficient to increase profit of the day, consumers memorize the sales price and they might hesitate to purchase the product at a regular price the following day. The authors discussed the inventory clearance pricing for a single period in our previous study by constructing a mathematical model to derive an optimal sales price to maximize the expected profit by considering the reference price effect of demand. This paper extends the discussion to handle the discount pricing for multiple periods. A mathematical analysis is first conducted to reveal the properties on an objective function, which is the present value of total expected profits for multiple periods. An algorithm is then proposed to derive an optimal price for asymmetric consumers. Numerical experiments investigate the characteristics of the objective function and optimal pricings.

Keywords: Clearance Pricing, Inventory Control, Multiple Periods, Optimization, Reference Price Effect

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## 1. INTRODUCTION

Many retail stores which deal in fresh goods or takeout goods often make a discount sale for daily perishable products when they are likely to be unsold. It is because the stores judge that the discount sale gains profit of the day greater than the disposal of the unsold products. Even if the profit of the day is increased by the discount, however, long-term profit might be decreased. Consumer's reference price is declined by the discount sale, and the consumers are not willing to purchase the products at their regular prices. The effect on decreased demand is referred to as a reference price effect on demand (Kahneman and Tversky, 1979).

Revenue management has been well-known as the

most successful research field in operations research during the last two decades (Talluri and van Ryzin, 2004). Revenue management is the research field aiming to maximize expected profit by determining sales conditions for perishable products appropriately such as sales price, volume of sales, and sales period. Markdown strategy is one of the active research areas in revenue management. The research targets perishable products such as airline tickets, hotel rooms, and seasonal goods. The influence by reference price effect is small for such types of products since it is hard to find the same sales condition for airline tickets and hotel rooms. In the case of seasonable goods, consumers usually purchase the same type of products only once. Compared with these types of products, daily perishable products have greater in-

fluence from reference price effect. The consumers who purchase products at a discount price memorize the purchase price, and their declined reference price has influences on the demand of the next day on the same product when it is sold at a regular price.

The reference price effect has been researched mathematically in some studies (Greenleaf, 1995; Kopalle *et al.*, 1996; Fibich *et al.*, 2003; Popescu and Wu, 2007). Simply speaking, the objective of the studies is to estimate the value of sales promotion. The target products are not perishable and their inventory level is not considered. A discount sale is conducted as a sales promotion in order to stimulate demand, not to reduce the volume of disposal products. It is significant to consider reference price effect as well as inventory if target products are daily perishable. Gimpl-Heersink *et al.* (2008) proposed a model which integrates inventory control into pricing in consideration of the reference price effect. Their model determines both an additional order amount and a selling price in every period. Since optimums in their model are sophisticatedly derived due to some assumptions with respect to replenishment policy, their model cannot be applied to our target problem where no replenishment is considered.

In our previous study (Koide and Sandoh, 2012), an optimal clearing pricing for daily perishable products in a single period was discussed. A mathematical model was constructed to estimate an expected profit function in a single period analytically and numerically. It has been shown that the expected profit function is concave for loss-neutral consumers and an optimal clearance price is simply derived through a first order condition. For loss-averse and loss-seeking consumers, the function is concave or bimodal.

In this study, our previous model is extended to deal with the discount pricing problem to maximize expected profit for multiple periods. Mathematical analysis shows that an expected profit function is concave with respect to sales prices for loss-neutral consumers while the function is sometimes bimodal for asymmetric consumers. A numerical experiment in a two-period case reveals visually the bimodality of an expected profit function. Further experiments indicate the significance to take into account the reference price effect to determine an inventory clearance price.

The rest of this paper is structured as follows. The notations adopted in this paper are listed and our target problem is established in Section 2. Section 3 briefly describes some results in a single period case shown in our previous paper. Section 4 proves the concavity of the expected profit function for loss-neutral consumers even in the multi-period case. An algorithm to explore an optimal pricing is proposed which is applicable to the target problem for asymmetric consumers. Section 5 executes numerical experiments to investigate the objective function and optimal pricings for the multi-period problem. Section 6 concludes the resulting knowledge in this study and mentions future works.

## 2. PRELIMINARIES

### 2.1 Notations

*Constants and coefficients:*

|              |                                       |
|--------------|---------------------------------------|
| $T$          | number of planning periods            |
| $p_H$        | regular sales price                   |
| $p_L$        | lower limit of discounted sales price |
| $q_t$        | average of $Q_t$                      |
| $\mathbf{q}$ | vector $\{q_1, q_2, \dots, q_T\}$     |
| $z_t$        | $q_t - d(p_t, r_t)$                   |
| $c$          | unit procurement cost                 |
| $h$          | unit disposal/salvage cost            |
| $s$          | unit opportunity loss cost            |
| $\alpha$     | smoothing parameter                   |
| $\beta$      | discount factor                       |

*Variables:*

|                 |   |
|-----------------|---|
| $p_t$           | sales price in period $t$   |
| $\mathbf{p}$    | vector $\{p_1, p_2, \dots, p_T\}$   |
| $r_t$           | reference price of consumers in period $t$                                |
| $Q_t$           | stochastic remaining inventory level at a prescheduled time in period $t$ |
| $\varepsilon_q$ | stochastic variation on inventory level                                   |
| $\varepsilon_d$ | stochastic variation on demand level                                      |
| $\varepsilon$   | $\varepsilon_q - \varepsilon_d$   |

*Functions:*

|                                  |   |
|----------------------------------|---|
| $D(p, r)$                        | stochastic demand                                 |
| $d(p, r)$                        | deterministic demand; average of $D(p, r)$        |
| $f(z)$                           | probabilistic density function of                 |
| $F(z)$                           | probabilistic distribution function of            |
| $\Pi(p, q, r)$                   | expected profit in a single period                |
| $V(\mathbf{p}, \mathbf{q}, r_1)$ | present value of expected profits for $T$ periods |

### 2.2 Problem Descriptions

Consider a firm under monopoly which deals in a type of products. The target time horizon is consecutive discrete  $T$  periods. The firm procures a certain amount of the product before opening time at a unit cost  $c$  based on forecasted demand in every period. The product is sold at a regular price  $p_H$ . The firm can discount the product only once at a prescheduled time if the firm judges that many products would be unsold at the end of the period. The discount price is bounded below by its lower limit  $p_L$ . The unsold products at the end of the period are disposed or salvaged at a unit cost  $h$ . The cost  $h$  is negative and positive when the products are salvaged and disposed, respectively. Assume that  $-h < p_L$ . The opportunity loss cost  $s$  is incurred when a consumer cannot purchase the product because of shortage. This study focuses on the discount pricing  $\mathbf{p}$  at the prescheduled time for  $T$  periods.

Assume that target consumers are so homogeneous that they have a common reference price  $r_t$  in period  $t$ . The reference price is updated by successive sales prices:

$$r_t = \alpha r_{t-1} + (1 - \alpha)p_{t-1} \quad (1 < t \leq T). \quad (1)$$

The smooth parameter  $\alpha$  implies the degree of retention on past sales prices. Let  $r_1$  exist in the price range  $[p_L, p_H]$  and then it concludes that  $r_t$  also exists in  $[p_L, p_H]$  for all  $t$ .

The inventory level  $Q_t$  is expressed as a random variable since the demand until the prescheduled time in a period is uncertain. Assume that  $Q_t$  is given by  $Q_t = q_t + \varepsilon_q$ , where the mean of  $\varepsilon_q$  is 0 and the range of  $\varepsilon_q$  is  $[q_L, q_H]$ . The assumption indicates that the firm estimates a common variation on the amount of inventory for all periods.

Let  $D(p, r)$  be the demand function for the products after the prescheduled time till closing time in a single period, where  $p$  is the sales price and  $r$  is the reference price of consumers. Assume that  $D(p, r)$  is independent of period  $t$  and includes both the reference price effect and stochastic variation:

$$D(p, r) = d(p, r) + \varepsilon_d, \quad (2)$$

$$d(p, r) = \begin{cases} d_G(p, r) & \text{if } p < r \\ d_L(p, r) & \text{otherwise,} \end{cases} \quad (3)$$

$$d_G(p, r) = \beta_0 - \beta_1 p + \beta_{2G}(r - p), \quad (4)$$

$$d_L(p, r) = \beta_0 - \beta_1 p + \beta_{2L}(r - p). \quad (5)$$

The positive parameters  $\beta_{2G}$  and  $\beta_{2L}$ , respectively, represent the degree of the reference price effect when consumers recognize an asking price as a gain ( $p < r$ ) and as a loss ( $p \geq r$ ). The parameters characterize the consumer's response toward a current sales price. The consumers with  $\beta_{2G} < \beta_{2L}$ ,  $\beta_{2G} = \beta_{2L}$  and  $\beta_{2G} > \beta_{2L}$  are called loss-averse (LA), loss-neutral (LN), and loss-seeking (LS), respectively. It is assumed that  $\varepsilon_d$  distributes within the range  $[d_L, d_H]$  and its mean is 0.

The expected profit function in a single period  $\Pi(p, q, r)$  is expressed as

$$\Pi(p, q, r) = \begin{cases} \Pi_G(p, q, r) & \text{if } p < r \\ \Pi_L(p, q, r) & \text{otherwise,} \end{cases} \quad (6)$$

Here,  $\Pi_G(p, q, r)$  and  $\Pi_L(p, q, r)$  are the expected profit functions where  $d(p, r) = d_G(p, r)$  and  $d(p, r) = d_L(p, r)$ , respectively. The functions  $\Pi_G(p, q, r)$  and  $\Pi_L(p, q, r)$  have a common point on  $p = r$ . The function  $\Pi(p, q, r)$  is the objective function in a single period case.

On the other hand, the objective function in multi-period case is, the present value of expected profit for multiple  $T$  periods, which is given by

$$V(p, q, r_1) = \sum_{t=1}^T \gamma^{t-1} \Pi(p_t, q_t, r_t). \quad (7)$$

The vector  $p$  is the decision variables in this study. The reference price  $r_t$  in period  $t$  is calculated by Eq. (1).

### 3. OPTIMAL PRICING IN A SINGLE PERIOD

Koide and Sandoh (2012) showed some theorems and lemmas associated with both the configuration of the expected profit function and the optimal pricing in a single period. This section summarizes the results briefly.

#### 3.1 Optimal Pricing for LN Consumers

For LN consumers, it holds that  $\beta_{2G} = \beta_{2L} \equiv \beta_2$  and the expected profit function  $\Pi(p, q, r)$  is a smooth and differentiable with respect to  $p$ . From the definition of  $z$  and  $\varepsilon$ , in accordance with Petruzzi and Dada (1999), the stochastic amount of unsold products is expressed as follows:

$$Q - D(p, r) = q - d(p, r) + \varepsilon_q - \varepsilon_d = z + \varepsilon. \quad (8)$$

The expected profit  $\Pi(p, q, r)$  in a single period is then expressed in the followings.

$$\Pi(p, q, r) = \Psi(p, r) - L(p, z), \quad (9)$$

$$\Psi(p, r) = (p - c)d(p, r), \quad (10)$$

$$L(p, z) = (c + h)\Lambda(z) + (p - c + s)\Theta(z), \quad (11)$$

$$\Lambda(z) = \int_{-z}^{z_H} (z + u)f(u)du, \quad (12)$$

$$\Theta(z) = -\int_{z_H}^{-z} (u - z)f(u)du. \quad (13)$$

In Eq. (11),  $\Lambda(z)$  and  $\Theta(z)$ , respectively, represent the expected volumes of excess and deficiency of inventory. The following theorem has been proved which derives the optimal price to maximize the expected profit  $\Pi(p, q, r)$ .

**Theorem 1.** (Theorem 3 in Koide and Sandoh (2012)) *When target consumers are LN, the expected profit function  $\Pi(p, q, r)$  is concave and the optimal price  $p^*(q, r)$  which maximizes  $\Pi(p, q, r)$  is derived by the following equation:*

$$p^*(q, r) = \min[\max\{\hat{p}(q, r), p_L\}, p_H], \quad (14)$$

where  $\hat{p}(q, r)$  is the unique solution of  $g(p, q, r) = 0$ :

$$g(p, q, r) = B_0(r) - (2p + h)B_1 + (p + h + s)B_1F(-z) - \Theta(z). \quad (15)$$

Note that  $B_0(r) = \beta_0 + \beta_2 r$  and  $B_1 = \beta_1 + \beta_2$ . Theorem 1 has proven that the optimal pricing is simply obtained by a computational method using a first order condition such as the gradient method.

#### 3.2 Optimal Pricing for Asymmetry Consumers

For the asymmetry consumers, namely, LA and LS

consumers, the expected profit function  $\Pi(p, q, r)$ , given by Eq. (6), is not smooth generally. Since the two functions  $\Pi_G(p, q, r)$  and  $\Pi_L(p, q, r)$  are proved to be concave by Theorem 1,  $\Pi(p, q, r)$  is at least piecewise concave, namely, bimodal or concave. The function is piecewise smooth and the optimal price cannot be computed simply by the first order condition.

If a condition holds, which is proved as Theorem 4 in Koide and Sandoh (2012), the function  $\Pi(p, q, r)$  is concave even for LA or LS consumers. Otherwise, the optimal pricing for asymmetry consumers is obtained by comparison between two optimums for  $\Pi_G(p, q, r)$  and  $\Pi_L(p, q, r)$  within the ranges  $[p_L, r]$  and  $[r, p_H]$ , respectively.

#### 4. OPTIMAL PRICING IN MULTIPLE PERIODS

##### 4.1 Optimal Pricing for LN Consumers

As in the single period case, the concavity of the objective function  $V(\mathbf{p}, \mathbf{q}, r_1)$  is proved by the following theorem.

**Theorem 2.** *When the target consumers are LN, the objective function  $V(\mathbf{p}, \mathbf{q}, r_1)$  is concave.*

**Proof.** Eq. (1) can be rewritten as follows:

$$r_i = \alpha^{i-1}r_1 + (1-\alpha)\sum_{t=1}^{i-1}\alpha^{i-1-t}p_t \quad (i > 1). \quad (16)$$

For  $1 \leq t \leq T$ , the next equation is obtained:

$$\frac{\partial r_i}{\partial p_t} = \begin{cases} (1-\alpha)\alpha^{i-1-t} & \text{if } i > t, \\ 0 & \text{otherwise.} \end{cases} \quad (17)$$

Due to  $z_t = q_t - d(p_t, r_t)$ , it holds

$$\frac{\partial z_t}{\partial r_t} = \frac{\partial}{\partial r_t} \{q_t - (B_0(r_t) - B_1p_t)\} = -\beta_2, \quad (18)$$

$$\frac{\partial \Pi(p_t, q_t, r_t)}{\partial r_t} = \beta_2 \{(p_t + h + s)\bar{F}(-z_t) - s\}, \quad (19)$$

$$\frac{\partial^2 \Pi(p_t, q_t, r_t)}{\partial r_t^2} = -\beta_2^2 (p_t + h + s)f(-z_t) < 0. \quad (20)$$

Hence, for  $1 \leq t \leq T$ ,

$$\begin{aligned} \frac{\partial V(\mathbf{p}, \mathbf{q}, r_1)}{\partial p_t} &= \frac{\partial}{\partial p_t} \gamma^{t-1} \Pi(p_t, q_t, r_t) \\ &+ \sum_{i=t+1}^T \gamma^{i-1} \frac{\partial r_i}{\partial p_t} \frac{\partial \Pi(p_i, q_i, r_i)}{\partial r_i}, \end{aligned} \quad (21)$$

$$\frac{\partial^2 V(\mathbf{p}, \mathbf{q}, r_1)}{\partial p_t^2} = \gamma^{t-1} \{-2B_1\bar{F}(-z_t) - (p_t + h + s)B_1^2 f(-z_t)\}$$

$$-(1-\alpha)^2 \beta_2^2 \sum_{i=t+1}^T \gamma^{i-1} (p_i + h + s) \alpha^{2(i-1-t)} f(-z_t). \quad (22)$$

Eq. (22) shows that  $V(\mathbf{p}, \mathbf{q}, r_1)$  is concave with respect to  $p_t$  for  $t = 1, 2, \dots, T-1$ .

The expected profit function  $\Pi(p_t, q_t, r_t)$  is independent of  $p_T$  for  $t = 1, 2, \dots, T-1$  and  $\Pi(p_T, q_T, r_T)$  is concave with respect to  $p_T$ , proved by Theorem 1. It concludes therefore that  $V(\mathbf{p}, \mathbf{q}, r_1)$  is concave with respect to  $p_T$  and the theorem has been proved.  $\square$

Theorem 2 reveals that the optimal price  $\mathbf{p}^*(\mathbf{q}, r_1)$  which maximizes  $V(\mathbf{p}, \mathbf{q}, r_1)$  is derived through the first order conditions with respect to  $\mathbf{p}$ .

##### 4.2 Optimal Pricing for Asymmetry Consumers

The expected profit function  $\Pi(p, q, r)$  in a single period is bimodal or concave for LA and LS consumers and it naturally holds that  $V(\mathbf{p}, \mathbf{q}, r_1)$  in multiple periods is not always concave. The problem to maximize  $V(\mathbf{p}, \mathbf{q}, r_1)$ , however, can be divided into partial problems where  $V(\mathbf{p}, \mathbf{q}, r_1)$  is certainly concave within the segmented search space. In the following, we propose an algorithm to derive an optimal pricing for multiple periods.

Our target problem can be written as an optimization formulation as follows:

**Problem MP:**

$$\text{Max}_{\mathbf{p}} V(\mathbf{p}, \mathbf{q}, r_1) = \sum_{t=1}^T \gamma^{t-1} \Pi(p_t, q_t, r_t) \quad (23)$$

s. t.

$$\Pi(p_t, q_t, r_t) = \begin{cases} \Pi_G(p_t, q_t, r_t) & \text{if } p_t < r_t \\ \Pi_L(p_t, q_t, r_t) & \text{otherwise} \end{cases} \quad (1 \leq t \leq T), \quad (24)$$

$$r_t = \alpha r_{t-1} + (1-\alpha)p_{t-1} \quad (1 < t \leq T), \quad (1)$$

$$p_L \leq p_t \leq p_H \quad (1 \leq t \leq T). \quad (25)$$

By dividing the search region, Problem MP is transformed to the following Problem MP' as a discrete optimization problem:

**Problem MP':**

$$\text{Max}_{\mathbf{x}} \phi(\mathbf{x}) \quad (26)$$

s. t.

$$x_t \in [0, 1], \quad (27)$$

$$\mathbf{x} = (x_1, x_2, \dots, x_T). \quad (28)$$

The function  $\phi(\mathbf{x})$  in Eq. (26) is the optimal value of the following Problem SP(x):

**Problem SP(x):**

$$\phi(\mathbf{x}) = \text{Max}_p \left[ \sum_{t=1}^T \gamma^{t-1} \left\{ \begin{array}{l} x_t \Pi_G(p_t, q_t, r_t) \\ + (1-x_t) \Pi_L(p_t, q_t, r_t) \end{array} \right\} \right] \quad (29)$$

s. t.

$$r_t = \alpha r_{t-1} + (1-\alpha)p_{t-1} \quad (1 < t \leq T), \quad (30)$$

$$\begin{aligned} x_t p_L + (1-x_t)r_t &\leq p_t \\ &\leq x_t r_t + (1-x_t)p_H \quad (1 \leq t \leq T). \end{aligned} \quad (31)$$

Problem SP(x) restricts the search regions on  $p$  in Problem MP. If  $x_t = 1$  in Problem SP(x), the search region on  $p_t$  is restricted within the gain pricing range  $[p_L, r_t]$ , which can be ascertained by substituting  $x_t = 1$  into Eq. (31), and the expected profit  $\Pi(p_t, q_t, r_t)$  becomes  $\Pi_G(p_t, q_t, r_t)$ . On the other hand, if  $x_t = 0$  in Problem SP(x),  $p_t$  exists within the loss pricing range  $[r_t, p_H]$  and the profit function leads to  $\Pi_L(p_t, q_t, r_t)$ . Even though the target consumer is LA or LS, the function  $\Pi(p_t, q_t, r_t)$  is concave if the variable  $p_t$  exists within either  $[p_L, r_t]$  or  $[r_t, p_H]$ . The profit functions  $\Pi_G(p_t, q_t, r_t)$  and  $\Pi_L(p_t, q_t, r_t)$  in Eq. (29) are all concave for a given value of  $x$  and the optimal price for Problem SP(x) can be computed through the first order condition. By exploring the optimums for Problem SP(x) for all settings of  $x$ , the optimal pricing for Problem MP' can be obtained. It is mentionable that the proposed algorithm eventually creates  $2^T$  sub-problems and the total computational time increases exponentially with respect to  $T$ , the number of planning period.

### 5. NUMERICAL EXAMPLES

This section investigates the characteristics with respect to the objective function  $V(p, q, r)$  and estimates the influence of consumer's characteristics to optimal pricings. Throughout this section, we focus on the following situation. The remaining products are likely to be unsold at the prescheduled time in the first period. The amounts of inventory in the following periods are supposed to be as expected, namely, no discount sales are planned in the following periods. The manager has to determine if the sales price of the products is to be discounted or not by considering the reference price effect in the future.

The following common parameters in the experiments conducted in this section are set as follows:  $\beta_0 = 100$ ,  $\beta_1 = 0.1$ ,  $c = 250$ ,  $s = 50$ ,  $h = -50$ ,  $p_H = 500$ ,  $p_L = 250$ ,  $\alpha = 0.5$ ,  $\gamma = 0.95$ . The variation  $\varepsilon$  on  $z$  is assumed to follow a uniform distribution over  $[q_L - d_H, q_H - d_L] = [-20, 20]$ . Note that  $d(p_H, p_H) = 50$ , which means that if consumer's reference price is equal to a regular price, the average amount of demand for the product sold at the regular price is 50. Since  $h$  is negative, the firm has no incentive to conduct a discount unless the amount of the remaining product is greater than 50.

### 5.1 Configuration of Objective Function

Firstly, some introductory experiments have been conducted in order to explore an appropriate value of some remaining parameters such as  $\beta_2$ ,  $q_t$ , and  $r_1$ .

Figure 1 depicts the optimal prices in a single period for LN consumers with  $r_1 = p_H = 500$ . The optimal price is equal to the regular price 500 when the inventory level  $q$  is not greater than 67 with  $\beta_2 = 0.02$ . It is noticeable that the single period model does not consider the future demand influenced by current discount. Nevertheless, discount pricings are not effective to increase the profit of the day unless the inventory level is more than 67 in this case. When  $\beta_2 = 0.05$  and  $\beta_2 = 0.1$ , the firm should conduct a discount if the inventory level is greater than 60 and 52, respectively. The consumers with greater value of  $\beta_2$  react more sensitively to a discount price and less inventory level is needed for a profitable discount sale.

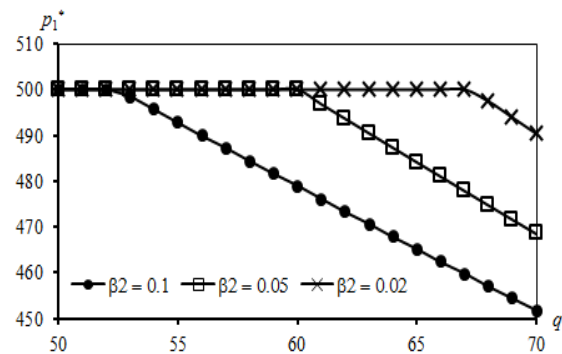


Figure 1. Optimal prices  $p_1^*$  in a single period.

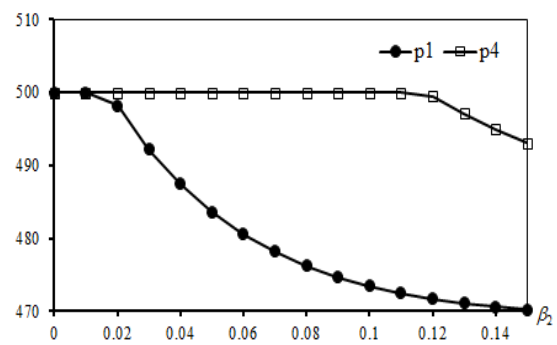


Figure 2. Optimal prices  $p_1^*$  and  $p_4^*$ .

Figure 2 shows the optimal prices  $p_1^*$  and  $p_4^*$  for LN consumers in four periods model with average inventory levels are  $q_1 = 70$ , and  $q_2 = q_3 = q_4 = 50$ . The initial reference price  $r_1$  is 500, equal to the regular sales price. The optimal prices  $p_2^*$  and  $p_3^*$  are omitted in Figure 2 since they are uniformly equal to 500 for any values of  $\beta_2$ . The optimal price  $p_1^*$  is higher than that in Figure 1 for  $\beta_2 = 0.02, 0.05$ , and  $0.1$ , which indicates that the optimal price is raised so as to relax the decrease of future demand caused by the reference price effect.



On the other hand, Figure 2 shows that the optimal price  $p_4^*$  is also less than the regular price for  $\beta_2 > 0.11$ . The discount price increases the demand and the profit in the fourth period, the last period in this case. The declined reference price caused by the discount in the last period has no negative effect on the objective function. In order to estimate optimal pricings for long-term profit by using short period models, the parameter  $\beta_2$  should be advisedly set so that such a discount in the last period does not occur.

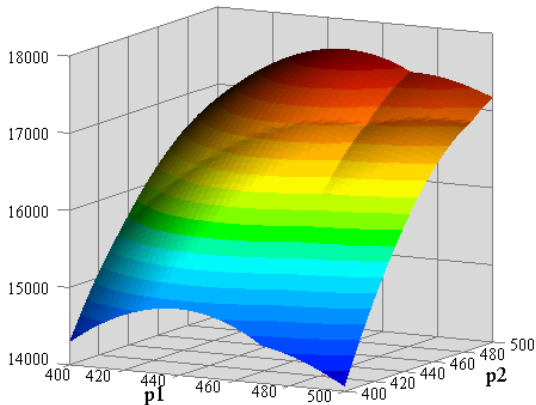


Figure 3. Three-dimensional image of  $V(p, q, r_1)$  for loss-seeking consumers.

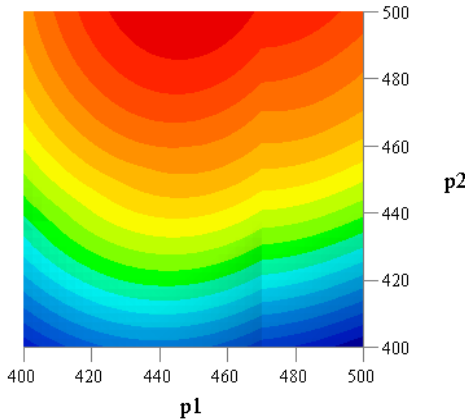


Figure 4. Contour of  $V(p, q, r_1)$  for loss-seeking consumers.

Figures 3 and 4 illustrate a three-dimensional image and a contour graph of the objective function  $V(p, q, r_1)$  of two-period problem for LS consumers, where  $q_1 = 70, q_2 = 50, r_1 = 470$ , and  $(\beta_{2G}, \beta_{2L}) = (0.1, 0.05)$ . The figure has two valley lines; the sharp vertical line  $p_1 = r_1$  and the blurred diagonal line

$$p_2 = r_2 = \alpha r_1 + (1 - \alpha)p_1. \quad (32)$$

The objective function  $V(p, q, r_1)$  is not smooth along the valley lines which make it hard to explore the optimal prices.

## 5.2 Optimal Pricings in Four Periods Against Different Consumer's Attitudes

This subsection investigates how consumer's attitudes toward an asking price affect optimal pricings and acquired profit. The consumer's attitudes are characterized by the parameters  $(\beta_{2G}, \beta_{2L})$  where  $(\beta_{2G}, \beta_{2L}) = (0.05, 0.1)$  for LA,  $(0.1, 0.05)$  for LS,  $(0.05, 0.05)$  and  $(0.1, 0.1)$  for LN. The LN consumer with the settings of  $(\beta_{2G}, \beta_{2L}) = (0.05, 0.05)$  and  $(0.1, 0.1)$  are named as LN005 and LN01, respectively. The average inventory levels for four periods are  $q_1 = 70$ , and  $q_2 = q_3 = q_4 = 50$ . The initial reference price is  $r_1 = 500$ , equal to the regular price. Under such a setting, optimal pricings in all periods except period 1 are 500 regardless of what the consumer's attitude is. The remarkable point in the followings is whether the firm should conduct discount sales or not in the first period.

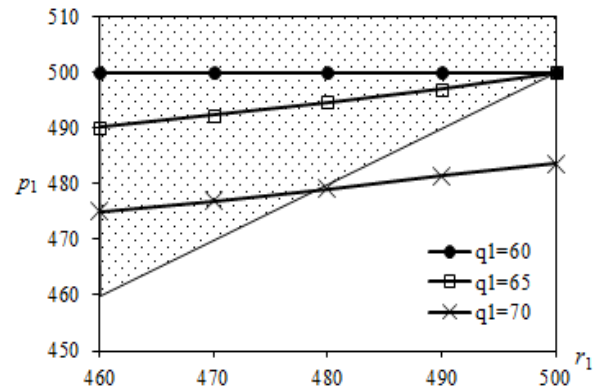


Figure 5. Optimal prices  $p_1^*$  for loss-neutral (LN005) consumers.

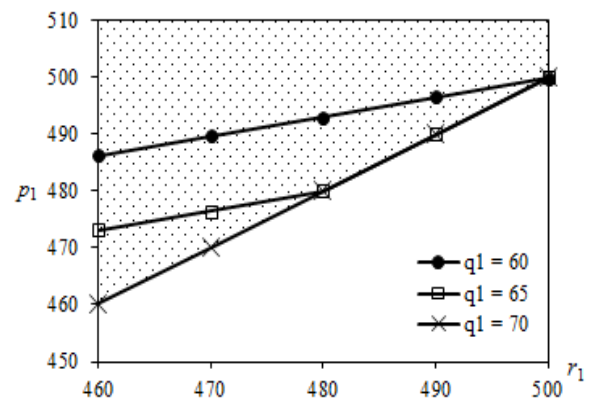


Figure 6. Optimal prices  $p_1^*$  for loss-averse consumers.

Figures 5 through 8 illustrate the optimal price in the first period for the four types of consumers. The dotted region in the figures represents a loss price range in which consumers recognize the price a loss, namely  $p_1 > r_1$ . The other region in the figures represents a gain price range. Firstly, it is confirmed that the optimal price-

es do not decline as inventory level  $q_1$  increases. It interprets the instance that when the firm has more excess products, more drastic discount is required to improve its long-term profit. Figures 5 and 8 also show that the optimal pricings for LN consumers are approximately linear with respect to  $r_1$ , and they are not affected from the loss and the gain region.

It is noticeable that the optimal pricings for asymmetric consumers act differently in both regions as shown in Figures 6 and 7. In Figure 6, all prices exist inside the loss region. Since LA consumers are not stimulated so much by gain prices compared to their undesirable response to loss prices, the firm should raise their reference price to acquire more profit in the future. Unlike in the other figures, the optimal prices in Figure 7 do not increase monotonically with respect to  $r_1$ . In the case of  $q_1 = 65$ , the optimal price for LS consumers with  $r_1 = 480$  is greater than that with  $r_1 = 490$ . Figure 9 indicates the profits in each period in this case. When  $r_1 = 480$ , the firm should raise consumer's reference price to increase the profit in the remaining periods. When  $r_1 = 490$ , the gain pricing in the first period contributes to increase of ultimate total profit due to consumer's initial high reference price. Such a non-monotonicity occurs because of the bimodality of the objective function  $V(p, q, r_1)$ .

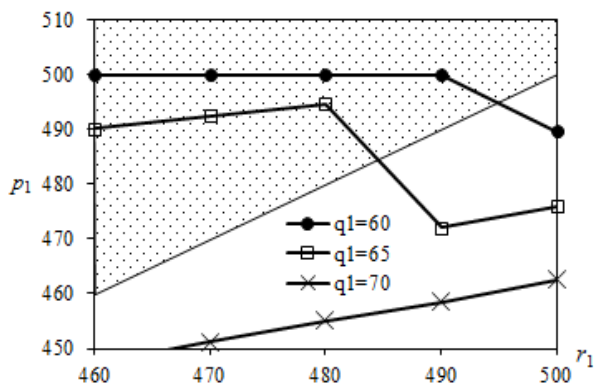


Figure 7. Optimal prices  $p_1^*$  or loss-seeking consumers.

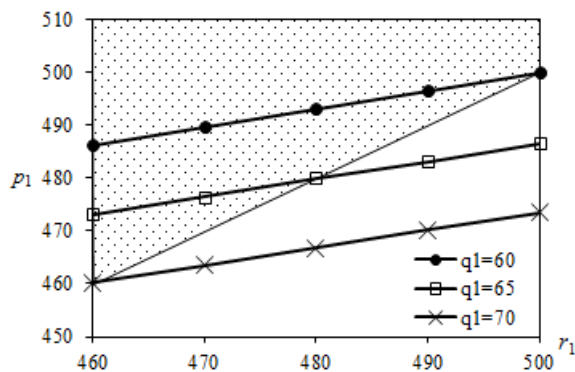


Figure 8. Optimal prices  $p_1^*$  or loss-neutral (LN01) consumers.

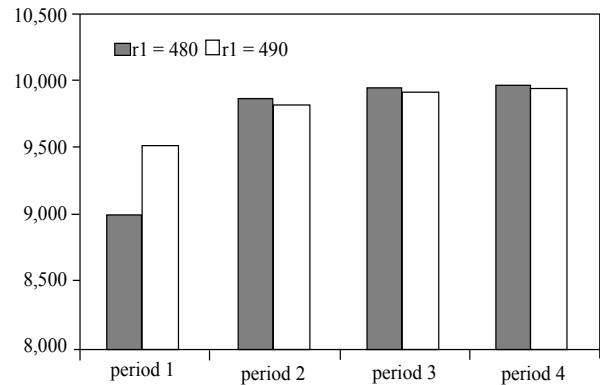


Figure 9. Transition of profits in the case of  $q_1 = 65$ .

## 6. CONCLUSION

This paper has investigated an optimal clearance pricing considering the consumer's reference price effect in multiple periods. A mathematical model proposed in our previous research is extended to handle the problem aiming for multiple periods. The objective function, the present value of the sum of expected profit during the multiple periods, is proved to be concave when target consumers are LN. An algorithm is proposed to explore the optimal pricing for LA and LS consumers where the objective function is not always concave. Numerical experiments reveal that optimal pricings are profoundly influenced by the consumer's reaction as well as the inventory level and initial reference price.

The proposed algorithm is in essence a complete enumeration method, and it requires computational time exponentially to the size of multiple periods. The algorithm should be revised to tackle the problem with larger size of planning periods. Some techniques in complete enumeration algorithms for combinatorial optimization problems might be helpful for the revision.

## ACKNOWLEDGMENTS

This work was partly supported by Grant-in-Aid for Scientific Research (C) No. 22510147 and MEXT, Japan.

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