Honam Mathematical J. **35** (2013), No. 2, pp. 303–310
http://dx.doi.org/10.5831/HMJ.2013.35.2.303

MINIMAL CLOZ-COVERS OF kX

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Abstract. In this paper, we first show that $z_{kX} : E_{cc}(kX) \longrightarrow kX$ is $z^{\#}$-irreducible and that if $\mathcal{G}(E_{cc}(\beta X))$ is a base for closed sets in βX, then $E_{cc}(kX)$ is C^*-embedded in $E_{cc}(\beta X)$, where kX is the extension of X such that $\upsilon X \subseteq kX \subseteq \beta X$ and kX is weakly Lindelöf. Using these, we will show that if $\mathcal{G}(\beta X)$ is a base for closed sets in βX and for any weakly Lindelöf space Y with $X \subseteq Y \subseteq kX$, $kX = Y$, then $kE_{cc}(X) = E_{cc}(kX)$ if and only if $\beta E_{cc}(X) = E_{cc}(\beta X)$.

1. Introduction

All spaces in this paper are Tychonoff spaces and βX(υX, resp.) denotes the Stone-Čech compactification(Hewitt realcompactification, resp.) of a space X .

Iliadis constructed the absolute of Hausdorff spaces, which is the minimal extremally disconnected cover of Hausdorff spaces and they turn out to be the perfect onto projective covers([5]). To generalize extremally disconnected spaces, basically disconnected spaces, quasi-F spaces and cloz-spaces have been introduced and their minimal covers have been studied by various authors([3]). In these ramifications, minimal covers of compact spaces can be nisely characterized.

In particular, Henriksen, Vermeer and Woods ([3]) introduced the notion of cloz-spaces and they showed that every compact space X has a minimal cloz-cover $(E_{cc}(X), z_X)$. In [6], it was shown that every space has a minimal cloz-cover.

In this paper, we first show that $z_{kX} : E_{cc}(kX) \longrightarrow kX$ is $z^{\#}$-irreducible and that if $\mathcal{G}(E_{cc}(\beta X))$ is a base for closed sets in βX, then

Received April 14, 2013. Accepted May 13, 2013.
2010 Mathematics Subject Classification. 54D80, 54D20, 54G05, 54C10.
Key words and phrases. Stone-space, weakly Linelöf space, cloz-space, covering map.
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$E_{cc}(kX)$ is C^*-embedded in $E_{cc}(\beta X)$, where kX is the extension of X such that $\upsilon X \subseteq kX \subseteq \beta X$ and kX is weakly Lindelöf. Using these, we will show that if $\mathcal{G}(\beta X)$ is a base for closed sets in βX and for any weakly Lindelöf space Y with $X \subseteq Y \subseteq kX$, $kX = Y$, then $kE_{cc}(X) = E_{cc}(kX)$ if and only if $\beta E_{cc}(X) = E_{cc}(\beta X)$.

For the terminology, we refer to [1] and [9].

2. Minimal cloz-covers of kX

The set $R(X)$ of all regular closed sets in a space X, when partially ordered by inclusion, becomes a complete Boolean algebra, in which the join, meet, and complementation operations are defined as follows : for any $A \in R(X)$ and any $\{A_i \mid i \in I\} \subseteq R(X)$,

$\vee\{A_i \mid i \in I\} = cl_X(\cup\{A_i \mid i \in I\})$,

$\wedge\{A_i \mid i \in I\} = cl_X(int_X(\cap\{A_i \mid i \in I\}))$, and

$A' = cl_X(X - A)$

and a sublattice of $R(X)$ is a subset of $R(X)$ that contains $\emptyset$, X and is closed under finite joins and meets.

Recall that a map $f : Y \longrightarrow X$ is called *a covering map* if it is a continuous, onto, perfect, and irreducible map.

Lemma 2.1. ([3])

(1) *Let $f : Y \longrightarrow X$ be a covering map. Then the map $\psi : R(Y) \longrightarrow R(X)$, defined by $\psi(A) = f(A)$, is a Boolean isomorphism and the inverse map ψ^{-1} of ψ is given by $\psi^{-1}(B) = cl_Y(f^{-1}(int_X(B))) = cl_Y(int_Y(f^{-1}(B)))$.*

(2) *Let X be a dense subspace of a space K. Then the map $\phi : R(K) \longrightarrow R(X)$, defined by $\phi(A) = A \cap X$, is a Boolean isomorphism and the inverse map ϕ^{-1} of ϕ is given by $\phi^{-1}(B) = cl_K(B)$.*

Definition 2.2. Let X be a space.

(1) A cozero-set C in X is said to be *a complemented cozero-set in X* if there is a cozero-set D in X such that $C \cap D = \emptyset$ and $C \cup D$ is a dense subset of X. In case, $\{C, D\}$ is called *a complemented pair of cozero-sets in X*.

(2) Let $\mathcal{G}(X) = \{cl_X(C) \mid C$ is a complemented cozero-set in $X\}$.

Let X be a space and $Z(X)^\# = \{cl_X(int_X(A)) \mid A$ is a zero-set in $X\}$. Suppose that $\{C, D\}$ is a complemented pair of cozero-sets in X. Then $cl_X(C) = cl_X(X-D)$ and since $cl_X(X-D) \in Z(X)^\#$, $cl_X(C) \in Z(X)^\#$. Hence $\mathcal{G}(X) = \{A \in Z(X)^\# \mid A' \in Z(X)^\#\}$ and $\mathcal{G}(X)$ is a Boolean subalgebra of $R(X)$.

Since X is C^*-embedded in βX, by Lemma 2.1., $\mathcal{G}(X)$ and $\mathcal{G}(\beta X)$ are Boolean isomorphic.

Definition 2.3. ([3]) A space X is called *a cloz-space* if every element of $\mathcal{G}(X)$ is a clopen set in X.

A space X is a cloz-space if and only if βX is a cloz-space([3]).

Definition 2.4. Let X be a space.
(1) A pair (Y, f) is called *a cloz-cover of* X if Y is a cloz-space and $f : Y \longrightarrow X$ is a covering map.
(2) A cloz-cover (Y, f) of X is called *a minimal cloz-cover of* X if for any cloz-cover (Z, g) of X, there is a covering map $h : Z \longrightarrow Y$ with $f \circ h = g$.

Let $\mathcal{B}$ be a Boolean subalgebra of $R(X)$. Let $S(\mathcal{B}) = \{\alpha \mid \alpha$ is a $\mathcal{B}$-ultrafilter$\}$ and for any $B \in \mathcal{B}$, let $\Sigma_B^{\mathcal{B}} = \{\alpha \in S(\mathcal{B}) \mid B \in \alpha\}$. Then the space $S(\mathcal{B})$, equipped with the topology for which $\{\Sigma_B^{\mathcal{B}} \mid B \in \mathcal{B}\}$ is a base, called *the Stone-space of* $\mathcal{B}$. Then $S(\mathcal{B})$ is a compact zero-dimensional space([9]).

Henriksen, Vermeer and Woods showed that every compact space X has the minimal cloz-cover $(E_{cc}(X), z_X)$. Let X be a compact space, $\mathcal{S}(\mathcal{G}(X))$ the Stone-space of $\mathcal{G}(X)$ and $E_{cc}(X) = \{(\alpha, x) \mid x \in \cap\{A \mid A \in \alpha\}\}$ the subspace of the product space $\mathcal{S}(\mathcal{G}(X)) \times X$. Then $(E_{cc}(X), z_X)$ is the minimal cloz-cover of X, where $z_X((\alpha, x)) = x$([3]). It was shown that every space has a minimal cloz-cover([7]).

For any space X, let $z_\beta = z_{\beta X}$.

Lemma 2.5. ([6]) *Let X be a space. If $z_\beta^{-1}(X)$ is a cloz-space, then $(z_\beta^{-1}(X), z_{\beta_X})$ is the minimal cloz-cover of X.*

A z-filter $\mathcal{F}$ on a space X is called *real* if $\mathcal{F}$ is closed under countable intersections.

For any space X, let $kX = \upsilon X \cup \{p \in \beta X - \upsilon X \mid$ there is a real z-filter $\mathcal{F}$ on X such that $\cap\{cl_{\upsilon X}(F) \mid F \in \mathcal{F}\} = \emptyset$ and $p \in \cap\{cl_{\beta X}(F) \mid F \in \mathcal{F}\}\}$. Then kX is an extension of a space X such that $\upsilon X \subseteq kX \subseteq \beta X$([8]).

A space X is called *a weakly Lindelöf space* if for any open cover $\mathcal{U}$ of X, there is a countable subfamily $\mathcal{V}$ of $\mathcal{U}$ such that $\cup\{V \mid V \in \mathcal{V}\}$ is a dense subset of X. It is well-known that a space X is weakly Lindelöf if and only if for any $R(X)$-filter $\mathcal{F}$ with the countable meet property, $\cap\{F \mid F \in \mathcal{F}\} \neq \emptyset$.

Lemma 2.6. ([8]) *Let X be a space. Then kX is a weakly Lindelöf space and for any continuous map $f : X \longrightarrow Y$, there is a continuous map $f_k : kX \longrightarrow kY$ such that $f_k \circ k_X = k_Y \circ f$.*

Let X be a space. For any $B \in \mathcal{G}(\beta X)$, let $\Sigma_B^{\mathcal{G}(\beta X)} = \Sigma_B$ and $(\Sigma_B \times X) \cap E_{cc}(kX) = \sigma_B$. Then for any $B \in \mathcal{G}(\beta X)$, $z_\beta(\Sigma_B \times X) = B$.

Let $z_k = z_{\beta_{kX}} : z_\beta^{-1}(kX) \longrightarrow kX$ be the restriction and corestriction of z_β with respect to $z_\beta^{-1}(kX)$ and kX, respectively. Clearly, we have the following lemma :

We recall that a covering map $f : Y \longrightarrow X$ is called *$z^{\#}$-irreducible* if $f(Z(Y)^{\#}) = Z(X)^{\#}$. Let $f : Y \longrightarrow X$ be a covering map and Z a zero-set in X. By Lemma 2.1, $f(cl_Y(int_Y(f^{-1}(Z)))) = cl_X(int_X(Z))$ and $cl_Y(int_Y(f^{-1}(Z))) \in Z(X)^{\#}$. Hence $Z(X)^{\#} \subseteq f(Z(Y)^{\#})$ and so $f : Y \longrightarrow X$ is $z^{\#}$-irreducible if and only if $f(Z(Y)^{\#}) \subseteq Z(X)^{\#}$.

Lemma 2.7. *Let $f : Y \longrightarrow X$ and $g : W \longrightarrow Y$ be covering maps. Then $f \circ g : W \longrightarrow X$ is $z^{\#}$-irreducible if and only if $f : Y \longrightarrow X$ and $g : W \longrightarrow Y$ are $z^{\#}$-irreducible.*

Let X be a space such that $\mathcal{G}(X)$ is a base for closed sets in X. Then $E_{cc}(X) = \{\alpha \in S(\mathcal{G}(X)) \mid \cap\alpha \neq \emptyset\}$ is the subspace of $S(\mathcal{G}(X))$([6]).

Theorem 2.8. *Let X be a space. Then we have the following :*

(1) *$z_\beta^{-1}(kX)$ is a weakly Lindelöf space,*

(2) *z_k is $z^{\#}$-irreducible,*

(3) *$z_k(\mathcal{G}(z_\beta^{-1}(kX))) = \mathcal{G}(kX)$,*

(4) *$(z_\beta^{-1}(kX), z_k)$ is the minimal cloz-cover of kX, and*

(5) *if $\mathcal{G}(E_{cc}(\beta X))$ is a base for closed sets in βX, then $E_{cc}(kX)$ is C^*-embedded in $E_{cc}(\beta X)$*

Proof. (1) Let $T = z_\beta^{-1}(kX)$. Suppose that there is an $R(T)$-filter $\mathcal{F}$ with the countable meet property such that $\cap\{F \mid F \in \mathcal{F}\} = \emptyset$.

We first claim that $\cap\{z_k(F) \mid F \in \mathcal{F}\} = \emptyset$. Suppose that $\cap\{z_k(F) \mid F \in \mathcal{F}\} \neq \emptyset$. Pick $x \in \cap\{z_k(F) \mid F \in \mathcal{F}\}$. Note that for any $\mathcal{A} \subseteq R(T)$, $\wedge\mathcal{A} \subseteq \cap\mathcal{A}$. Since $\mathcal{F}$ has the countable meet property, $\mathcal{F}$ has the finte intersection property. Hence $\{F \cap z_k^{-1}(x) \mid F \in \mathcal{F}\}$ is a family of closed sets in $z_k^{-1}(x)$ with the finite intersection property. Since $z_k^{-1}(x)$ is a compact subset in T, $\cap\{F \cap z_k^{-1}(x) \mid F \in \mathcal{F}\} \neq \emptyset$ and so $\cap\{F \mid F \in \mathcal{F}\} \neq \emptyset$. This is a contradiction.

Hence $\cap\{z_k(F) \mid F \in \mathcal{F}\} = \emptyset$. Since kX is a weakly Lindelöf space, there is a sequence (F_n) in $\mathcal{F}$ such that $cl_{kX}(\cup\{kX - z_k(F_n) \mid n \in N\}) = kX$. Let $F \in \mathcal{F}$. Then $z_k^{-1}(z_k(T - F)) \supseteq T - F$ and hence

$z_k(F') \supseteq z_k(T-F) \supseteq kX - z_k(F)$. Thus $cl_{kX}(\cup\{z_k(F'_n) \mid n \in N\}) = kX$. Note that

$$\begin{aligned} kX &= cl_{kX}(\cup\{z_k(F'_n) \mid n \in N\}) \\ &= cl_{kX}(z_k(\cup\{F'_n \mid n \in N\})) \\ &= z_k(cl_T(\cup\{F'_n \mid n \in N\})) \\ &= z_k(\vee\{F'_n \mid n \in N\}). \end{aligned}$$

Since z_k is an irreducible map, by Lemma 2.1, $\vee\{F'_n \mid n \in N\} = T$ and so $(\vee\{F'_n \mid n \in N\})' = \wedge\{F_n \mid n \in N\} = \emptyset$. Since $\mathcal{F}$ has the countable meet property, it is a contradiction. Hence T is a weakly Lindelöf space.

(2) Take any $Z \in Z(T)^{\#}$. By (1), T is a weakly Lindelöf space and hence there is a sequence (A_n) in $Z(E_{cc}(\beta X))^{\#}$ such that $T - Z = cl_{E_{cc}(\beta X)}(\cup\{T - A_n \mid n \in N\}) \cap (T - Z)$ and for any $n \in N$, $T - A_n \subseteq T - Z$. Then clearly, $Z \subseteq cl_{E_{cc}(\beta X)}(int_{E_{cc}(\beta X)}(\cap\{A_n \mid n \in N\})) \cap T = \wedge\{A_n \mid n \in N\} \cap T$. Futher,

$$\begin{aligned} Z &= (T - cl_{E_{cc}(\beta X)}(\cup\{T - A_n \mid n \in N\})) \cup Z \\ &= int_{E_{cc}(\beta X)}(\cap\{A_n \cap T \mid n \in N\}) \cup Z \end{aligned}$$

and hence $\wedge\{A_n \cap T \mid n \in N\} \subseteq Z$. Thus $Z = (\wedge\{A_n \mid n \in N\}) \cap T$. Note that $z_k(Z) = z_\beta(\wedge\{A_n \mid n \in N\}) \cap kX = (\wedge\{z_\beta(A_n) \mid n \in N\}) \cap kX$. Since $\wedge\{A_n \mid n \in N\} \in Z(E_{cc}(\beta X))^{\#}$ and z_β is $z^{\#}$-irreducible, $z_k(Z) \in Z(kX)^{\#}$.

(3) Clearly, $\mathcal{G}(kX) \subseteq z_k(\mathcal{G}(T))$. Let $B \in \mathcal{G}(T)$. Then $B, B' \in Z(T)^{\#}$. By (2) z_k is $z^{\#}$-irreducible and so $z_k(B), z_k(B)' \in Z(kX)^{\#}$. Hence $z_k(B) \in \mathcal{G}(kX)$ and thus $z_k(\mathcal{G}(T)) \subseteq \mathcal{G}(kX)$.

(4) Let $B \in \mathcal{G}(T)$. By (3), there is an $A \in \mathcal{G}(\beta X)$ such that $A \cap kX = z_k(B)$. Then $cl_{E_{cc}(\beta X)}(int_{E_{cc}(\beta X)}(z_\beta^{-1}(A)))$ is a clopen set in $E_{cc}(\beta X)$ and $cl_{E_{cc}(\beta X)}(int_{E_{cc}(\beta X)}(z_\beta^{-1}(A))) \cap T = B$. Hence B is clopen in T and so T is a cloz-space. By Lemma 2.5, $(z_\beta^{-1}(kX), z_k)$ is the minimal cloz-cover of kX.

(5) Suppose that $\mathcal{G}(E_{cc}(\beta X))$ is a base for closed sets in βX. Then $E_{cc}(\beta X) = S(\mathcal{G}(\beta X))$([3])and $E_{cc}(\beta X)$ is a zero-dimensional space. Since $z_\beta^{-1}(kX)$ is the minimal cloz-cover of kX, $\beta E_{cc}(kX)$ and $S(z_k(\mathcal{G}(T)))$

are homeomorphic ([7]). By (3), $S(z_k(\mathcal{G}(T)))$ and $S(\mathcal{G}(kX))$ are homeomorphic. By Lemma 2.1, $\mathcal{G}(kX)$ and $\mathcal{G}(\beta X)$ are Boolean isomorphic and so $\beta E_{cc}(kX)$ is homeomorphic to $E_{cc}(\beta X)$. □

Let X be a space. Then there is a covering map $g : \beta E_{cc}(X) \longrightarrow E_{cc}(\beta X)$ such that $z_\beta \circ g \circ \beta_{E_{cc}(X)} = \beta_X \circ z_X$. By Lemma 2.6, there is a unique continuous map $z^k_X : kE_{cc}(X) \longrightarrow kX$ such that $z^k_X \circ k_{E_{cc}(X)} = k_X \circ z_X$. Since $k_{E_{cc}(X)}$ is a dense embedding, $\beta_{kX} \circ z^k_X = z_\beta \circ g \circ \beta_{kE_{cc}(X)}$. Hence there is a continuous map $l : kE_{cc}(X) \longrightarrow E_{cc}(kX)$ such that $j \circ l = g \circ \beta_{kE_{cc}(X)}$ and $z_k \circ l = z^k_X$([9]), where $j : E_{cc}(kX) \longrightarrow E_{cc}(\beta X)$.

Corollary 2.9. *Let X be a space such that $\mathcal{G}(\beta X)$ is a base for closed sets in βX and $kE_{cc}(X) = E_{cc}(kX)$, that is, $l : kE_{cc}(X) \longrightarrow E_{cc}(kX)$ is a homeomeorphism. Then $\beta E_{cc}(X) = E_{cc}(\beta X)$, that is, $g : \beta E_{cc}(X) \longrightarrow E_{cc}(\beta X)$ is a homeomeorphism ([1]).*

Proof. Since $l : kE_{cc}(X) \longrightarrow E_{cc}(kX)$ is a homeomorphism, by Theorem 2.8, $kE_{cc}(X)$ is C^*-embedded in $E_{cc}(\beta X)$. Hence $\beta E_{cc}(X) = \beta kE_{cc}(X) = E_{cc}(\beta X)$. Thus g is a homeomorphism. □

Let X be a space such that $\beta E_{cc}(X) = E_{cc}(\beta X)$. Then $g : \beta E_{cc}(X) \longrightarrow E_{cc}(\beta X)$ is a homeomorphsim such that $j \circ l = g \circ \beta_{kE_{cc}(X)}$. Since $g \circ \beta_{kE_{cc}(X)}$ is an embedding, l is an embedding.

Theorem 2.10. *Let X be a space such that $\mathcal{G}(\beta X)$ is a base for closed sets in βX and for any weakly Lindelöf space Y with $X \subseteq Y \subseteq kX$, $kX = Y$. Then $kE_{cc}(X) = E_{cc}(kX)$ if and only if $\beta E_{cc}(X) = E_{cc}(\beta X)$.*

Proof. Suppose that $\beta E_{cc}(X) = E_{cc}(\beta X)$. Then cleraly, $z_\beta^{-1}(X) = E_{cc}(X)$. Let $m = z^k_X : kE_{cc}(X) \longrightarrow kX$.

We first claim that $m(kE_{cc}(X))$ is a weakly Lindelöf space. Take any open cover $\mathcal{U}$ of $m(kE_{cc}(X))$. Then $\mathcal{V} = \{m^{-1}(U) \mid U \in \mathcal{U}\}$ is an open cover of $kE_{cc}(X)$. Since $kE_{cc}(X)$ is a weakly Lindelöf space, there is a countable subfamily $\mathcal{U}_0$ of $\mathcal{U}$ such that $\cup\{m^{-1}(U) \mid \mathcal{U} \in \mathcal{U}_0\} = m^{-1}(\cup\{U \mid U \in \mathcal{U}\})$ is dense in $m(kE_{cc}(X))$. Since m is continuous, $\cup\{U \mid \mathcal{U} \in \mathcal{U}_0\}$ is dense in $m(kE_{cc}(X))$. Hence $m(kE_{cc}(X))$ is weakly Lindelöf.

Since $X \subseteq m(kE_{cc}(X)) \subseteq kX$, by the assumption, $m(kE_{cc}(X)) = kX$ and so m is onto. Take any $x \in kX$. Since m is an onto map and z_X is a covering map, $m(kE_{cc}(X) - E_{cc}(X)) = kX - X$([9]). Since $\beta_{kX} \circ m = z_\beta \circ g \circ \beta_{k\Lambda X}$, $m^{-1}(x) = (z_\beta \circ g)^{-1}(x) \subseteq kE_{cc}(X) - E_{cc}(X)$.

Since $z_\beta \circ g$ is a covering map, $m^{-1}(x)$ is a compact subset of $kE_{cc}(X)$ and hence m is a compact map.

Let F be a closed set in $kE_{cc}(X)$ and $x \in kX - m(F)$. Then $m^{-1}(x) \cap F = \emptyset$. Since $m^{-1}(x)$ is a comact space and $E_{cc}(\beta X)$ is the Stone space of $\mathcal{G}(\beta X)$, there is a $B \in \mathcal{G}(\beta X)$ such that $m^{-1}(x) \subseteq \Sigma_B$ and $F \subseteq \Sigma_{B'}$. Since $z_\beta(\Sigma'_B) = B'$ and $z_\beta^{-1}(x) \cap \Sigma_{B'} = m^{-1}(x) \cap \Sigma_{B'} = \emptyset$, $x \notin B'$. Since $cl_{kX}(m(F)) \subseteq B'$, $x \notin cl_{kX}(m(F))$. Thus m is a closed map and so m is a perfect map.

Since $z_\beta \circ g \circ \beta_{kE_{cc}(X)} = \beta_{kX} \circ m$ and $z_\beta \circ g$ is a covering map, m is a covering map. Since $kE_{cc}(X)$ is a cloz-space, there is a covering map $t : kE_{cc}(X) \longrightarrow E_{cc}(kX)$ such that $z_k \circ t = m$. Since $kE_{cc}(X)$ is C^*-embedded in $\beta E_{cc}(X)$ and $z_\beta \circ g : \beta E_{cc}(X) \longrightarrow \beta X$ is $z^\#$-irreducible, m is $z^\#$-irreducible. Hence by Lemma 2.7, t is $z^\#$-irreducible.

Take any $\delta_1 \neq \delta_2$ in $kE_{cc}(X)$. Note that $\beta E_{cc}(X) = S(\mathcal{G}(\beta X))$ and $kE_{cc}(X) \subseteq \beta E_{cc}(X)$. Then there are $A, B \in \mathcal{G}(\beta X)$ such that $\delta_1 \in \sigma_A$, $\delta_2 \in \sigma_B$ and $\sigma_A \cap \sigma_B = \emptyset$. Since m is $z^\#$-irreducible, $m(\sigma_A) = z_k(t(\sigma_A)) \in \mathcal{G}(kX)$. Hence $cl_{E_{cc}(kX)}(z_k^{-1}(z_k(t(\sigma_A)))) = t(\sigma_A)$ is a clopen set in $E_{cc}(kX)$. Similarly, $t(\sigma_B)$ is a clopen set in $E_{cc}(kX)$. Since $t(\sigma_A) \wedge t(\sigma_A) = \emptyset$, $t(\sigma_A) \cap t(\sigma_A) = \emptyset$. Since $t(\delta_1) \in t(\sigma_A)$ and $t(\delta_2) \in t(\sigma_B)$, t is one-to-one and hence t is a homeomorphism. □

References

[1] L. Gillman and M. Jerison, *Rings of continuous functions*, Van Nostrand, Princeton, New York, 1960.

[2] M. Henriksen, J. Vermeer, and R. G. Woods, *Quasi-F covers of Tychonoff spaces*, Trans. Amer. Math. Soc. **303** (1987), 779–804.

[3] M. Henriksen, J. Vermeer, and R. G. Woods, *Wallman covers of compact spaces*, Dissertationes Math. **283** (1989), 5-31.

[4] M. Henriksen and R. G. Woods, *Cozero complement spaces; When the space of minimal prime ideals of a $C(X)$ is compact*, Topology Appl. **141** (2004), 147-170.

[5] S. Iliadis, *Absolute of Hausdorff spaces*, Sov. Math. Dokl. **2** (1963), 295-298.

[6] C. I. Kim, *Cloz-covers of Tychonoff spaces*, J. Korea Soc. Math. Educ. Ser. B Pure Appl. Math. **18** (2011), 361-386.

[7] C. I. Kim, *Minimal cloz-covers and Boolean Algebras*, Korean J. Math. **20** (2012), 517-524.

[8] Y. S. Yun and C. I. Kim, *An extension which is a weakly Lindelöf space*, J. Korea Soc. Math. Educ. Ser. B Pure Appl. Math. **19** (2012), 273-279.

[9] J. Porter and R. G. Woods, *Extensions and Absolutes of Hausdorff Spaces*, Springer, Berlin, 1988.

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