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ON A PRODUCT-SYMMETRIC RECURRENT-METRIC CONNECTION IN AN ALMOST HERMITIAN MANIFOLD

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Abstract. In the present paper, we define a product-symmetric recurrent-metric connection in an almost Hermitian manifold and study some properties of this connection, in particular, its curvature properties.

1. Introduction

Let $M^n = (M^n, g)$ be a Riemannian manifold of dimension n with a metric tensor g. A linear connection ∇ on M^n satisfies

$$(i)\ \nabla_{fX+gY} = f\nabla_X + g\nabla_Y, \quad (ii)\ \nabla_X(fY) = (Xf)Y + f\nabla_X Y,$$

where f, g are smooth functions on M^n and X, Y are smooth vector fields on M^n. The torsion tensor T of ∇ is given by

$$T(X,Y) = \nabla_X Y - \nabla_Y X - [X,Y].$$

If the torsion tensor T vanishes, then ∇ is said to be symmetric, otherwise it is nonsymmetric. If the metric tensor g of M^n satisfies $\nabla g = 0$, then ∇ is said to be a metric connection, otherwise it is nonmetric. It is well known that a linear connection is symmetric and metric if and only if it is the Levi-Civita connection. In 1930, Hayden [9] introduced a metric connection with a nonzero torsion on a Riemannian manifold.

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In particular, a nonsymmetric connection ∇ is called semi-symmetric if the torsion tensor T of ∇ satisfies

$$T(X,Y) = u(Y)X - u(X)Y,$$

where u is a 1-form on M^n. In 1924, Friedmann and Schouten [7] introduced the idea of a semi-symmetric linear connection in a differential manifold. In case of a semi-symmetric metric connection, Yano [19] considered such a connection and studied some of its properties. In fact, Yano proved that a Riemannian manifold is conformally flat if and only if it admits a semi-symmetric metric connection whose curvature tensor vanishes. Later on such a connection on a Riemannian manifold equipped with certain geometric structures was extensively studied by several authors [4,5,10,14,20]. In 1975, Golab [8] defined and studied quarter-symmetric linear connections in manifolds. A linear connection is said to be a quarter-symmetric connection if its torsion tensor T is of the form

$$T(X,Y) = u(Y)\phi X - u(X)\phi Y,$$

where u is a 1-form and ϕ is a tensor of type (1,1). In case of quarter-symmetric metric connections, several authors investigate their properties in [6,11,12,13,15,16,17,21] among others. In 2003, Sengupta and Biswas [18] defined quarter-symmetric nonmetric connection in a Sasakian manifold and studied its properties. Recently Chaubey and Ojha [3] defined a quarter-symmetric nonmetric connection on an almost Hermitian manifold and studied its geometric properties. In series, the properties of quarter-symmetric nonmetric connection in a Kähler manifold have been studied in [2]. The purpose of this paper is to introduce a different kind of nonsymmetric nonmetric connection (namely, product-symmetric recurrent-metric connection) into an almost Hermitian manifold and to study some properties of such a connection. In particular, we investigate the various symmetries and properties of the curvature with respect to the product-symmetric recurrent-metric connection under certain conditions.

2. A product-symmetric recurrent-metric connection in an almost Hermitian manifold

Let $M^n = (M^n, g, J)$ be an almost Hermitian manifold of dimension $n(= 2m)$ with almost complex structure J and compatible Riemannian metric g, i.e., $J^2X = -X$ and $g(JX, JY) = g(X,Y)$. We define a linear connection $\bar{\nabla}$ in an almost Hermitian manifold M^n as follows:

$$\bar{\nabla}_X Y = \nabla_X Y - u(X)Y - u(JX)JY, \tag{2.1}$$

where ∇ denotes the Levi-Civita connection. Using (2.1), the torsion tensor $\bar{T}$ of $\bar{\nabla}$ is given by

$$\bar{T}(X,Y) = u(Y)X - u(X)Y + u(JY)JX - u(JX)JY.$$

Further, using (2.1), we have

$$(\bar{\nabla}_X g)(Y,Z) = 2u(X)g(Y,Z).$$

The linear connection $\bar{\nabla}$ of the type defined by (2.1) is said to be a product-symmetric recurrent-metric connection (briefly, PSRM connection) in an almost Hermitian manifold. For instance, we find a non-trivial PSRM connection in a product almost Hermitian manifold as follows:

Example. Let (M^n, g_{M^n}, J_{M^n}) be an almost Hermitian manifold and (T^2, g_{T^2}, J_{T^2}) a flat torus with standard complex structure. Then it is easy to see that the product space $(M^{n+2}, g, J) = (M^n \times T^2, g_{M^n} + g_{T^2}, J_{M^n} + J_{T^2})$ is an almost Hermitian manifold. Since T^2 has a nowhere vanishing vector field, we can choose such a vector field U tangent to T^2 at each point in $M^n \times T^2$ and so we obtain a non-trivial 1-form u associated with U on M^{n+2} by $g(U,X) = u(X)$. Now we can define a non-trivial PSRM connection $\bar{\nabla}$ in (M^{n+2}, g, J) by using the 1-form u mentioned above as follows:

$$\bar{\nabla}_X Y = \nabla_X Y - u(X)Y - u(JX)JY.$$

The Riemannian curvature tensor R has the following well known $SO(n)$-decomposition [1];

$$R(X,Y,Z,V) = \frac{s}{2n(n-1)} g \bullet g(X,Y,Z,V) +$$

$$+ \frac{1}{n-2}\left(r - \frac{s}{n} g\right) \bullet g(X,Y,Z,V) + W(X,Y,Z,V),$$

where s,r,W are the scalar, Ricci, Weyl curvature tensor, respectively. Here the symbol $\bullet$ is the Nomizu-Kulkarni product of symmetric (0,2)-tensors generating a curvature type tensor:

$$h \bullet k(X,Y,Z,W) = h(X,Z)k(Y,W) + h(Y,W)k(X,Z)$$

$$-h(X,W)k(Y,Z) - h(Y,Z)k(X,W).$$

Note that $W = 0$ if and only if (M^n, g) is conformally flat. The Weyl curvature tensor depends only on the conformal class of (M^n, g). Moreover, it satisfies the curvature symmetries and so we can treat it as a conformal curvature tensor. In particular, the Weyl curvature tensor is traceless. Analogous to the definition of curvature tensor R, Ricci tensor r, scalar curvature s and Weyl curvature tensor W, we define the curvature tensor $\bar{R}$, Ricci tensor $\bar{r}$, scalar curvature $\bar{s}$ and Weyl curvature tensor $\bar{W}$ with respect to $\bar{\nabla}$ by

$$\begin{aligned} \bar{R}(X,Y,Z,V) &= g(\bar{R}(X,Y)Z,V) \\ &= g(\bar{\nabla}_X\bar{\nabla}_Y Z - \bar{\nabla}_Y\bar{\nabla}_X Z - \bar{\nabla}_{[X,Y]}Z, V), \end{aligned} \tag{2.2}$$

$$\begin{aligned} \bar{r}(Y,Z) &= \sum_{i=1,\dots,n} \bar{R}(e_i, Y, Z, e_i), \\ \bar{s} &= \sum_{i=1,\dots,n} \bar{r}(e_i, e_i), \end{aligned} \tag{2.3}$$

and

$$\begin{aligned} \bar{W}(X,Y,Z,V) = \bar{R}(X,Y,Z,V) &- \frac{\bar{s}}{2n(n-1)} g \bullet g(X,Y,Z,V) \\ &- \frac{1}{n-2}(\bar{r} - \frac{\bar{s}}{n} g) \bullet g(X,Y,Z,V), \end{aligned}$$

where $\{e_i\}_{i=1,..,n}$ is an orthonormal frame. The Riemannian manifold (M^n, g) is called Einstein if the Ricci tensor r of M^n is proportional to the metric tensor g , i.e., $r = \frac{s}{n} g$. A 1-form v on M^n is said to be double-recurrent if it satisfies

$$\nabla_X v = v(X)v.$$

It is easy to see that a double-recurrent 1-form v is closed. Concerning the PSRM connection in an almost Hermitian manifold, we have the following result:

Theorem 2.1. *Let $M^n = (M^n, g, J)$ be an almost Hermitian manifold equipped with PSRM connection $\bar{\nabla}$.*
(i) Then we have

$$\bar{R}(X,Y,Z,W) = -\bar{R}(Y,X,Z,W).$$

(ii) If the 1-form u of $\bar{\nabla}$ is double-recurrent, then we have

$$\bar{R}(X,Y,Z,W) = -\bar{R}(X,Y,W,Z).$$

(iii) If the 1-form u of $\bar{\nabla}$ is double-recurrent and in addition, M^n is Kähler, then we have

$$\bar{r}(Y,Z) = \bar{r}(Z,Y).$$

(iv) If the 1-form u of $\bar{\nabla}$ is double-recurrent and M^n is Kähler and in addition $\bar{s} \geq s$, then the connections $\bar{\nabla}$ and ∇ coincide.

Proof. Taking account of (2.1) and (2.2), we have

$$\begin{aligned}\bar{R}(X,Y,Z,W) = R(X,Y,Z,W) &- g(\nabla_X U, Y)g(Z,W)\\ &+ g(\nabla_Y U, X)g(Z,W) - g(\nabla_X U, JY)g(JZ,W)\\ &+ g(\nabla_Y U, JX)g(JZ,W) - g(U,(\nabla_X J)Y)g(JZ,W)\\ &+ g(U,(\nabla_Y J)X)g(JZ,W) + g(U,JX)g((\nabla_Y J)Z,W)\\ &- g(U,JY)g((\nabla_X J)Z,W),\end{aligned} \tag{2.4}$$

where U is a vector field associated with the 1-form u by $u(X) = g(U,X)$.
From (2.4), it follows that

$$\bar{R}(X,Y,Z,W) = -\bar{R}(Y,X,Z,W)$$

holds true. If we further assume that the 1-form u of $\bar{\nabla}$ is double-recurrent, then we have

$$\begin{aligned}\bar{R}(X,Y,Z,W) = R(X,Y,Z,W) &- u(X)u(JY)g(JZ,W)\\ &+ u(Y)u(JX)g(JZ,W) - u((\nabla_X J)Y)g(JZ,W)\\ &+ u((\nabla_Y J)X)g(JZ,W) + u(JX)g((\nabla_Y J)Z,W)\\ &- u(JY)g((\nabla_X J)Z,W).\end{aligned} \tag{2.5}$$

It is easy to see that when $M^n = (M^n, g, J)$ is an almost Hermitian manifold, the identity

$$g((\nabla_X J)Y, Z) = -g(Y, (\nabla_X J)Z) \tag{2.6}$$

holds. Therefore it follows from (2.5) and (2.6) that

$$\bar{R}(X,Y,Z,W) = -\bar{R}(X,Y,W,Z)$$

holds. If we further assume that M^n is Kähler, then (2.5) yields

$$\begin{aligned}\bar{R}(X,Y,Z,W) = R(X,Y,Z,W) &- u(X)u(JY)g(JZ,W)\\ &+u(Y)u(JX)g(JZ,W),\end{aligned}$$

from which follows that

$$\bar{r}(Y,Z) = r(Y,Z) - u(JY)u(JZ) - u(Y)u(Z). \tag{2.7}$$

The identity (2.7) immediately yields

$$\bar{r}(Y,Z) = \bar{r}(Z,Y).$$

Taking account of (2.7), we have

$$\bar{s} = s - 2||u||^2. \tag{2.8}$$

If we assume $\bar{s} \geq s$ on M^n, then from (2.8) we obtain $u = 0$, which tells us that the connections $\bar{\nabla}$ and ∇ coincide. This completes the proof of Theorem 2.1. □

Concerning Weyl curvature tensors, we have

Corollary 2.2. *Let $M^n = (M^n, g, J)$ be a Kähler manifold equipped with PSRM connection $\bar{\nabla}$. If the associated 1-form u of $\bar{\nabla}$ is double-recurrent and $\bar{s} \geq s$ on M^n, then the Weyl curvature tensors W and $\bar{W}$ coincide.*

On the other hand, the Kähler form Ω and the Nijenhuis tensor N of an almost Hermitian manifold M^n are defined by $\Omega(X,Y) = g(JX,Y)$ and $N(X,Y) = (\nabla_{JX}J)Y - (\nabla_{JY}J)X - J((\nabla_X J)Y) + J((\nabla_Y J)X)$, respectively. Recall that M^n is Kähler if and only if $\nabla J = 0$; M^n is Hermitian if and only if $N = 0$ (in fact, it follows from the Nirenberg-Newlander theorem that the vanishing of N is equivalent to the integrability of the almost complex structure J) ; M^n is almost Kähler if and only if $(\nabla_X \Omega)(Y,Z) + (\nabla_Y \Omega)(Z,X) + (\nabla_Z \Omega)(X,Y) = 0$ (i.e., the Kähler form Ω of M^n is closed). We call an almost Hermitian manifold $M^n = (M^n, g, J)$ Einstein with respect to $\bar{\nabla}$ if the Ricci tensor $\bar{r}$ with respect to $\bar{\nabla}$ is proportional to the metric tensor g on M^n, i.e., $\bar{r} = \frac{\bar{s}}{n} g$.

Analogous to the definition of the Nijenhuis tensor N, we define the Nijenhuis tensor $\bar{N}$ with respect to $\bar{\nabla}$ by

$$\begin{aligned}\bar{N}(X,Y) = (\bar{\nabla}_{JX}J)(Y) - (\bar{\nabla}_{JY}J)(X) - J((\bar{\nabla}_X J)(Y)) \\ + J((\bar{\nabla}_Y J)(X)).\end{aligned}$$

Then we obtain the following:

Theorem 2.3. *On an almost Hermitian manifold $M^n = (M^n, g, J)$ with PSRM connection $\bar{\nabla}$, the Nijenhuis tensors N and $\bar{N}$ coincide.*

Proof. Covariant derivative of JY with respect to $\bar{\nabla}$ gives

$$(\bar{\nabla}_X J)Y + J(\bar{\nabla}_X Y) = \bar{\nabla}_X (JY).$$

Taking account of (2.1) and the above identity, we have

$$(\bar{\nabla}_X J)Y = (\nabla_X J)Y. \tag{2.9}$$

From (2.9) and the definition of Nijenhuis tensor, it follows that

$$\bar{N}(X,Y) = N(X,Y),$$

for arbitrary vector fields X, Y on M^n. This completes the proof of Theorem 2.3. □

Since an almost Hermitian manifold with $N = 0$ is Hermitian, we have

Corollary 2.4. *Let $M^n = (M^n, g, J)$ be an almost Hermitian manifold with PSRM connection $\bar{\nabla}$. If the Nijenhuis tensor $\bar{N}$ with respect to $\bar{\nabla}$ vanishes, then M^n is Hermitian.*

Since the Nijenhuis tensor N vanishes on the Kähler manifold, we get

Corollary 2.5. *On a Kähler manifold $M^n = (M^n, g, J)$ with PSRM connection $\bar{\nabla}$, the Nijenhuis tensor $\bar{N}$ with respect to $\bar{\nabla}$ vanishes.*

On the other hand, we have the following:

Theorem 2.6. *Let $M^n = (M^n, g, J)$ $(n > 2)$ be an almost Kähler manifold equipped with PSRM connection $\bar{\nabla}$. Then the Kähler form Ω of M^n is closed with respect to $\bar{\nabla}$ if and only if the connections $\bar{\nabla}$ and ∇ coincide.*

Proof. We have

$$\begin{aligned} X(\Omega(Y,Z)) &= (\bar{\nabla}_X \Omega)(Y,Z) + \Omega(\bar{\nabla}_X Y, Z) + \Omega(Y, \bar{\nabla}_X Z) \\ &= (\nabla_X \Omega)(Y,Z) + \Omega(\nabla_X Y, Z) + \Omega(Y, \nabla_X Z). \end{aligned}$$

Then

$$(\bar{\nabla}_X \Omega)(Y,Z) = (\nabla_X \Omega)(Y,Z) - \Omega(\bar{\nabla}_X Y - \nabla_X Y, Z) - \Omega(Y, \bar{\nabla}_X Z - \nabla_X Z).$$

Taking account of (2.1), the last identity becomes

$$\begin{aligned} (\bar{\nabla}_X \Omega)(Y,Z) &= (\nabla_X \Omega)(Y,Z) - \Omega(-u(X)Y - u(JX)JY, Z) \\ &\quad - \Omega(Y, -u(X)Z - u(JX)JZ) \\ &= (\nabla_X \Omega)(Y,Z) + 2u(X)\Omega(Y,Z). \end{aligned} \tag{2.10}$$

Taking cyclic sum of (2.10) in X, Y, Z, we have

$$\begin{aligned} &(\bar{\nabla}_X \Omega)(Y,Z) + (\bar{\nabla}_Y \Omega) + (\bar{\nabla}_Z \Omega)(X,Y) \\ =&(\nabla_X \Omega)(Y,Z) + (\nabla_Y \Omega)(Z,X) + (\nabla_Z \Omega)(X,Y) \\ &+ 2u(X)\Omega(Y,Z) + 2u(Y)\Omega(Z,X) + 2u(Z)\Omega(X,Y). \end{aligned} \tag{2.11}$$

Considering the almost Kähler condition, we have from the last identity

$$\begin{aligned} &(\bar{\nabla}_X \Omega)(Y,Z) + (\bar{\nabla}_Y \Omega)(Z,X) + (\bar{\nabla}_Z \Omega)(X,Y) \\ =&2u(X)\Omega(Y,Z) + 2u(Y)\Omega(Z,X) + 2u(Z)\Omega(X,Y). \end{aligned} \tag{2.12}$$

If we assume that the Kähler form Ω of M^n is closed with respect to $\bar{\nabla}$, then putting $X = e_i$, $Y = e_j$ and $Z = Je_j$ $(e_i \notin span\{e_j, Je_j\})$ in (2.12), we get $u(e_i) = 0$ for each $i = 1, 2, ..., n(= 2m)$. Hence we have $\bar{\nabla} = \nabla$. Conversely, suppose that $\bar{\nabla} = \nabla$, i.e., $u = 0$, then it follows from (2.12) that

$$(\bar{\nabla}_X \Omega)(Y,Z) + (\bar{\nabla}_Y \Omega)(Z,X) + (\bar{\nabla}_Z \Omega)(X,Y) = 0.$$

This completes the proof of Theorem 2.6. □

Since a Kähler manifold is almost Kähler, we immediately have

Corollary 2.7. *On a Kähler manifold $M^n = (M^n, g, J)(n > 2)$ with PSRM connection $\bar{\nabla}$, the Kähler form Ω of M^n is closed with respect to $\bar{\nabla}$ if and only if the connections $\bar{\nabla}$ and ∇ coincide.*

3. Concurrent vector field and PSRM connection

A vector field V on M^n is said to be concurrent if it satisfies

$$\nabla_X V = fX, \tag{3.1}$$

where f is a function on M^n. Then we have the following:

Theorem 3.1. *Let $M^n = (M^n, g, J)$ be an almost Hermitian manifold equipped with PSRM connection $\bar{\nabla}$. Then we have*

$$\bar{R}(X, Y, Z, W) = -\bar{R}(Y, X, Z, W).$$

If the vector field U associated with the 1-form u by $u(X) = g(U, X)$ is concurrent, then we obtain

$$\bar{R}(X, Y, Z, W) = -\bar{R}(X, Y, W, Z).$$

If we further assume that M^n is Kähler, then we have the following results:
(a) $\bar{R}(X, Y, Z, W) = \bar{R}(Z, W, X, Y)$,
(b) $\bar{R}(X, Y, Z, W) + \bar{R}(Y, Z, X, W) + \bar{R}(Z, X, Y, W) = 0$ if and only if the function f of a concurrent vector field U vanishes, that is, the vector field U is parallel,
(c) $\bar{r}(Y, Z) = \bar{r}(Z, Y)$,
(d) M^n is Einstein with respect to $\bar{\nabla}$ if and only if M^n is Einstein.

Proof. It follows from (2.4) that

$$\bar{R}(X, Y, Z, W) = -\bar{R}(Y, X, Z, W)$$

holds true. Suppose that the vector field U associated with the 1-form u is concurrent. Then we have from (2.4)

$$\begin{aligned} \bar{R}(X, Y, Z, W) &= R(X, Y, Z, W) + 2fg(JX, Y)g(JZ, W) \\ &\quad - g(U, (\nabla_X J)Y)g(JZ, W) + g(U, (\nabla_Y J)X)g(JZ, W) \\ &\quad + g(U, JX)g((\nabla_Y J)Z, W) - g(U, JY)g((\nabla_X J)Z, W). \end{aligned} \tag{3.2}$$

It follows from (2.6) and (3.2) that

$$\bar{R}(X, Y, Z, W) = -\bar{R}(X, Y, W, Z).$$

holds true. If we further suppose that M^n is Kähler, then we have from (3.2)

$$\bar{R}(X,Y,Z,W) = R(X,Y,Z,W) + 2fg(JX,Y)g(JZ,W), \tag{3.3}$$

from which follows that

$$\bar{r}(Y,Z) = r(Y,Z) - 2fg(Y,Z) \tag{3.4}$$

and

$$\bar{s} = s - 2fn.$$

With the help of (3.3), it is obvious that $\bar{R}(X,Y,Z,W) = \bar{R}(Z,W,X,Y)$ holds true. On the other hand, from (3.3) and the first Bianch identity, it follows that $\bar{R}(X,Y,Z,W)+\bar{R}(Y,Z,X,W)+\bar{R}(Z,X,Y,W) = 0$ implies that the function f vanishes. Conversely, according to (3.3) and the first Bianch identity, it is clear that $f = 0$ implies $\bar{R}(X,Y,Z,W) + \bar{R}(Y,Z,X,W) + \bar{R}(Z,X,Y,W) = 0$. On the other hand, by the aid of (3.4), it is easy to see that $\bar{r}(Y,Z) = \bar{r}(Z,Y)$ holds. And by virtue of (3.4), we also conclude that if M^n is Einstein with respect to $\bar{\nabla}$, then M^n is Einstein, and vice versa. This completes the proof of Theorem 3.1. □

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