Stationary Distribution for the Mobilities in Catastrophe Rescue Scenario

Yong Wang[1], Wei Peng[1], Qiang Dou[1] and Zhenghu Gong[1]
[1] School of Computer, National University of Defense Technology,
Changsha, Hunan, China
[e-mail: {wangyong, wpeng, douq, gzh}@nudt.edu.cn]
*Corresponding author: Yong Wang

Received September 28, 2012; revised January 8, 2013; revised February 3, 2013; accepted February 8, 2013; published February 26, 2013

Abstract

Mobility Model has drawn more and more attentions since its critical role in Mobile Wireless Networks performance evaluation. This paper analyzes the mobility patterns in the catastrophe rescue scenario, and proposes the Random Waypoint with Base Point mobility model to model these characteristics. We mathematically analyze the speed and spatial stationary distributions of the nodes and derive explicit expressions for the one dimensional case. In order to keep the stationary distribution through the entire simulation procedure, we provide strategies to initialize the speed, location and destination of the nodes at the beginning of the simulation. The simulation results verify the derivations and the proposed methods in this paper. This work gives a deep understanding of the properties of the Random Waypoint with Base Point mobility model and such understanding is necessary to avoid misinterpretation of the simulation results. The conclusions are of practical value for performance analysis of mobile wireless networks, especially for the catastrophe rescue scenario.

Keywords: stationary distribution; Wireless Network; Network mobility; catastrophe rescue; performance evaluation

http://dx.doi.org/10.3837/tiis.2013.02.008

1. Introduction

Mobility model is an indispensable part in the simulation-based studies of mobile wireless networks, such as MANET [1], DTN [2] and vehicular WLAN [3]. The performance analysis can depend heavily on the mobility model that governs the movement of the nodes [4]. Currently, mobility models could be divided into two categories: traces and synthetic models [5]. Traces are the mobility patterns that are observed in real life scenario. Although traces models could provide accurate information, it is difficult to obtain the real trace data and model it. A common approach is to use synthetic mobility models [6], which aim to resemble the mobility patterns of mobile nodes in the real world.

During the past years, lots of synthetic mobility models [4, 7-18] have been proposed to model the mobility in different scenarios. Unfortunately, these models cannot properly represent the mobile characteristics of the catastrophe rescue scenario, e.g., rescue after an earthquake. In the previous synthetic mobility models, the destination of a node is usually randomly selected and there is only one movement pattern in a model. But in the catastrophe rescue scenario, the destinations are not totally randomly chosen and there is more than one movement pattern. A typical catastrophe rescue scenario mainly contains two types of movement patterns. The rescue group may randomly select the destination most of the time, but it needs to move back to the base point to get some replenishment or have a rest when there is no new task. If there is a new survivor found when they are going back, they need to move to the new destination directly without any pause time. The transport group has to move back to the base point every time when it gets the survivor.

In this paper, we focus on how to model the movement characteristics of the mobile nodes in the catastrophe rescue scenario, and the stationary distributions of speed and location. The main contributions of this work are summarized below:

1) The Random Waypoint with Base Point mobility model is proposed in this paper. This model could perform various movement patterns according to different parameters which will properly model the mobility characteristics of catastrophe rescue scenario.
2) We derive mathematical expressions for the stationary distributions of the node speed and location. These results give a deep understanding of the properties of the Random Waypoint with Base Point mobility model and such understanding is necessary to avoid misinterpretation of the simulation results.
3) This paper proposes two methods to initialize the node speed, position and destination at the beginning of the simulation to keep the stationary distributions all the time. Using these methods, the simulations need not to discard the initial sequence of the observations and obtain more accurate simulation results.

The rest of this paper is organized as follows: We outline the related work in section 2. Section 3 analyzes the catastrophe rescue scenario and introduces the Random Waypoint with Base Point mobility model. In section 4, the explicit expressions of the stationary distribution for speed and position in one dimensional scenario are mathematically derived. Initialization problems of the speed, spatial position and destination are considered in section 5. The experimental results are shown in section 6. Finally, we conclude this paper and discuss the topics for future work in section 7.

2. Related Work

The most widely used model in the simulation is the Random Waypoint (RWP) mobility model [7]. It has been implemented in many networks simulation tools (e.g., NS2 [10], GloMoSim [11], ONE [12]). Bettstetter and Wagner's Random Waypoint with Attraction Area (RWPAA) [13] model was an extension of RWP model, which added attraction points to RWP model in order to generate more realistic non-equally distributed mobility. The probability that a node selected an attraction point or a point in an attraction area as the next waypoint was larger than the choice of other points. Besides this model, there are other extensions such as Bettstetter's Smooth Random Mobility Model [9], Chiang's Gauss-Markov Mobility Model [8], Random Waypoint Movement in n dimension area [14] and Random Waypoint on border [14]. Except for the RWP base models, there are lots of other kinds of models. Random Walk (RW) model [4, 5] and Random Direction (RD) [15] model are also very famous. Hong's Reference Point Group Mobility (RPGM) model [16] represented the group behavior of mobile nodes. Sachez and Manzoni [5] proposed a set of mobility models in which the mobile nodes traveled in a cooperative manner, including Column Mobility Model, Pursue Mobility Model and Nomadic Mobility Model. These models were expected to exhibit strong spatial dependency between nearby nodes [17]. Du et al. proposed a gravitation-based mobility model for hotspot scene in [18] in 2012.

The properties of the RWP mobility model were deeply studied in recent years. Under the assumption that the speed of the node was a constant and there was no pause time, Bettstetter [13] investigated the spatial node distribution of RWP, derived an analytical expression of the distribution in one dimension, and approximated that for the two dimensional case. He also analyzed the stochastic properties of RWP, including the *epoch* length, direction distribution, and cell change rate in [19]. In [6] Bettstetter showed the explicit expression of spatial node distribution in RWP when speed was a constant and pause time was randomly chosen. Hyytia considered an arbitrary convex domain of RWP and performed the derivations without any approximations [14]. Le Boudec [20] had pointed out that under the stationary regime of RWP Model, speed and location are independent. Navidi discussed the stationary distribution problem of RWP in [21]. He showed how to initialize the speed and choose the initial location and destination both with and without pause time in the two dimensional case.

3. The Random Waypoint with Base Point Mobility Model

In this section, we will first describe the typical movements in a catastrophe rescue scenario, and then introduce the Random Waypoint with Base Point mobility model to represent these mobility characteristics.

3.1 The Catastrophe Rescue Scenario

In the catastrophe rescue scenario, there are principally two kinds of groups, the rescue group and the transport group. We assume that, there is a base point where the ground situation is better, and the helicopter, vehicles and other heavy equipment can be there, as well as the command post. When a survivor is found, a rescue group is assigned to the position. After a long time of rescue, the rescue succeeds or fails (the survivor unfortunately dies while rescuing). If the survivor has been rescued successfully, the rescue group will inform the transport group of taking the survivor back to the base point for treatment. Then the rescue group moves to the next rescue position or the base point (to have a rest or get some replenishment). If emergencies (e.g., new survivors are found in some places) happen during its backing way, it will move to the new destination directly without any pause. The transport group moves from the base point to a destination, and after pausing a short time at the destination, the group moves back to the base point carrying the survivor.

3.2 The Mobility Model

As we have described in the previous section, the catastrophe rescue scenario always has a base point and the rescue and transport groups have more probability to move towards the base point. Now we will introduce the ***R**andom **W**ay**P**oint with **B**ase **P**oint* (RWPBP) mobility model which aims to represent these mobility characteristics in this special scenario.

Definition 1 (***epoch***): The entire procedure that a node chooses a destination, moves to it and stays there for a certain time until it chooses the next destination.

Obviously, the start point (x_s, y_s), the destination (x_d, y_d), the moving speed v and the pause time t_{pause} could determine an *epoch*.

There are two important parameters in the RWPBP model.

Definition 2 (***go-back-probability***): The probability that a node makes a decision to go back to the base point after an *epoch*.

Definition 3 (***back-to-base-point-probability***): The probability that a node will reach the base point when a node decides to go back to the base point and moves to it.

We use p_g and p_b to denote the *go-back-probability* and *back-to-base-point-probability* respectively. The destination (x_d, y_d) is usually randomly chosen in other mobility models [4, 5, 7]. But in RWPBP model, how to choose a new destination is determined by p_g and p_b. When a node does not make a decision to go back to the base point, it chooses the new destination randomly. But if a node decides to go back to the base point, it will choose the base point or randomly chooses a point on the line between the current point and the base point as the new destination according to p_b. Obviously, a node has a probability of $p_g p_b$ in total to reach the base point.

We define the *epoch* that the node decides to go back to the base point as the *go back epoch*. It is worth noting that after a *go back epoch*, no matter whether the node reaches the base point or not, the next destination will be randomly chosen. The speed v is randomly chosen from the range (v_{min}, v_{max}) for each *epoch*. The pause time t_{pause} will be uniformly chosen from the range (t_{min}, t_{max}) if the node randomly chooses a destination or selects the base point as the destination. But if the node cannot reach the base point in a *go back epoch*, the pause time should be set to 0. The reason is that the *go back epoch* is interrupted and there is no pause time (e.g., the transport group receives a new emergency mission).

When p_g is 0, the RWPBP model performs the same mobility pattern with the RWP model in which the node always randomly chooses a new destination. When p_g and p_b are both 1.0, RWPBP can represent the transport group which will access the base point after a random destination every time. Although RWPBP is a little similar to RWPAA [13] where the base point could be seen as a hot spot, there exists intrinsically difference between these two models. In RWPAA model, if a node chooses the attraction point (or area) as the destination, it will surely reach the point (or the area). However, in the RWPBP model, when a node goes back to the base point, it could only reach the base point with the probability p_b.

The algorithm of RWPBP mobility model is shown in **Algorithm 1**. In this algorithm, A and B represent the length and width of the area respectively. (x_b, y_b) are the coordinates of the base point. Usually, $x_b = y_b = 0$. N is the total number of the waypoints that a node accesses. We use the variable *go_back* to record the state of the current *epoch*. When the node is in *go_back* state, it will randomly select the next destination. Otherwise, it will choose the next destination according to p_g and p_b.

Algorithm 1 *RWPBP()*

Input: $p_g, p_b, A, B, v_{min}, v_{max}, t_{min}, t_{max}, go\_back$
Output: $NextPoint$;

$v \leftarrow v_{min} + (v_{max} - v_{min}) * rand()$;
$t_{pause} \leftarrow t_{min} + (t_{max} - t_{min}) * rand()$;
if go_back or $rand() > p_g$ **then**
  $(x, y) \leftarrow (A * rand(), B * rand())$;
  $go\_back \leftarrow false$;
else
  if $rand() < p_b$ **then**
    $(x, y) \leftarrow (x_b, y_b)$;
  else
    $(x, y) \leftarrow (x, y) * rand()$;
    $t_{pause} \leftarrow 0$;
  end if
  $go\_back \leftarrow true$;
end if
$NextPoint \leftarrow (x, y, v, t_{pause})$;
return $NextPoint$;

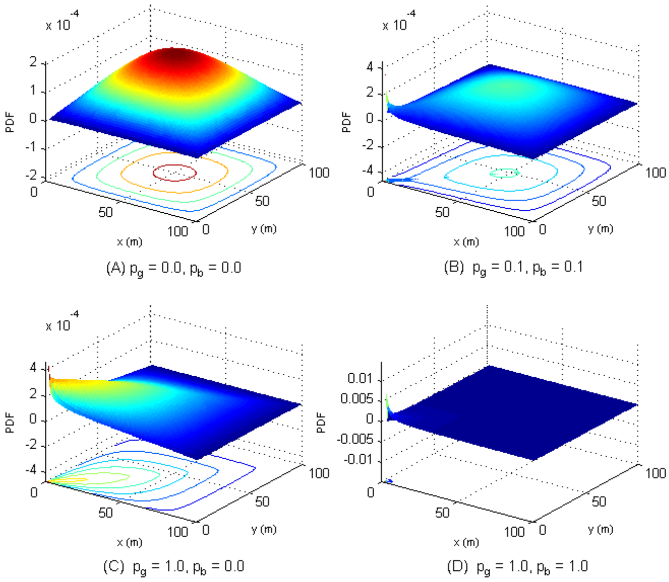

Fig. 1. The spatial distribution of RWPBP model without pause time under simulation

3.3 Study of Node Distribution under Simulation

We will study the node distribution of the RWPBP model using the Monte Carlo Methods [17] here. We simulate the moving patterns in a 100x100m^2 area. We divide the entire area into 100x100 equal square cells, and the size of each cell is 1x1m^2. The node distribution is evaluated by the probability that the node appears in the cells. The value of how often a node will appear in a cell is represented by a histogram. As the time going, the node moves across the cells or stays in a cell. We sample the coordinates of the node using a constant cycle. We add 1 to the histogram of the cell the coordinates belongs to. When the sampling time is long enough, the histogram of each cell is normalized with the total sampling cycles. For the sake of simplicity, we set the speed to a constant value and the pause time is set to 0.

Fig. 1 shows the node distribution after a long time sampling. From this figure we can see that, when $p_g = 0$, there is a peak in the center of the simulation area which means that the node is most likely to access the center. With the growing of p_g, the peak is moving to the base point. Especially when p_g and p_b are large enough, there is a sharp peak at the base point. That is because the node moves to the base point or nearby frequently.

4. Analytical Derivation of the Distribution in One Dimension

We have shown the spatial node distribution of the RWPBP model in a square area through simulation. Now we consider how to calculate the distribution when the size of the area, the speed range, the pause time range, the *go-back-probability* and the *back-to-base-point-probability* are given, i.e., we will derive a mathematical expression of the distribution with these given parameters.

We will consider the one dimensional case here. The one dimensional RWPBP model could represent the scenario that there is only one available road through the entire catastrophe area. The rescue points are the houses locating at the two sides of the road. And the groups only move along the road.

As described above, the node will pause at the destination of each *epoch* for a certain time. The probability that the node appears at these pausing locations (the destinations of the *epoch*s) will be increased, and this will surely affect the stationary distribution of the node. Under such condition, the distribution will contain two parts: the moving distribution and the pausing distribution. To simply the problem, we assume that there is no pause time of each *epoch* and we only consider the moving distribution in this paper. Without loss of generality, we assume that the coordinate of the base point is 0 and the border of the movement is A, which means the node can only move on a finite line segment $(0, A)$. The movement procedure is the same as the two dimensional RWPBP model while ignoring the y-dimension. So we can easily get the one dimensional RWPBP algorithm by substituting 0 for B of **Algorithm 1**. **Fig. 2** illustrates the mobility patterns in one dimensional RWPBP model.

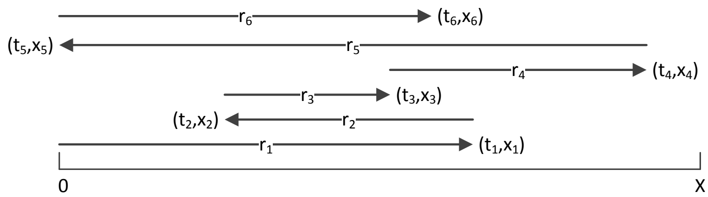

Fig. 2. The Illustration of 1D RWPBP

4.1 Speed Distribution

We use v to denote the speed of the node, and $v \in (v_{min}, v_{max})$. Since the probability that a speed v is sampled is proportional to the time the node travels using this speed, i.e., $f_V(v) \propto \frac{1}{v}$, we get

$$f_V(v) = \begin{cases} \frac{\lambda_V}{v} & \text{if } v_{min} < v < v_{max} \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where λ_V is the normalization term to keep

$$\int_{-\infty}^{\infty} f_V(v)dv = 1. \tag{2}$$

Putting Equation (1) and (2) together yields$\lambda_V = \frac{1}{\ln(v_{max}/v_{min})}$. Therefore

$$f_V(v) = \begin{cases} \dfrac{1}{v \ln(v_{max}/v_{min})} & \text{if } v_{min} < v < v_{max} \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The expect value of v is

$$E(v) = \int_{-\infty}^{\infty} v f_V(v)dv = \frac{v_{max} - v_{min}}{\ln(v_{max}/v_{min})}. \tag{4}$$

4.2 Spatial Distribution

When a point is randomly chosen from a line segment (x_1, x_2), the point is usually taken from a uniform distribution, i.e., the probability density function (PDF) of the point is

$$f(x) = \begin{cases} \dfrac{1}{x_2 - x_1} & \text{if } x_1 < x < x_2 \\ 0 & \text{otherwise.} \end{cases} \tag{5}$$

The destination of last *epoch* is also the start point of the new *epoch*, and we denote the start point as x_s. The node will choose a new destination after each *epoch* and the new destination is represented by x_d. So when a node randomly chooses the destinations, the PDFs of x_s and x_d are

$$f_{X_s}(x_s) = \begin{cases} \dfrac{1}{A} & \text{if } 0 < x_s < A \\ 0 & \text{otherwise,} \end{cases} \tag{6}$$

and

$$f_{X_d}(x_d) = \begin{cases} \dfrac{1}{A} & \text{if } 0 < x_d < A \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

$\forall x \in (0, A)$, if an *epoch* will access it, there are two conditions must be satisfied: $x_s < x$ and $x_d > x$ (or $x_d < x$ and $x_s > x$). We use $p(x)$ to denote the probability that an *epoch* will access x, then $p(x) = P(x_s < x, x_d > x) + P(x_d < x, x_s > x)$.

When an *epoch* starts with a randomly chosen position x_s, $\forall x \in (0, A)$,

$$P(x_s < x) = \int_0^x f(x_s)d(x_s) = \frac{x}{A}, \tag{8}$$

and

$$P(x_s > x) = \int_x^A f(x_s)d(x_s) = \frac{A - x}{A}. \tag{9}$$

According to the movement characteristics described in section 3, we divide the entire movement procedure of RWPBP into three mobility patterns.

Definition 4 (*Random Pattern*): After reaching a randomly selected destination, the node randomly selects a new destination with a probability of $1 - p_g$, and we call this mobility pattern the *Random Pattern*.

Definition 5 (*Base Point Pattern*): The node moves back to the base point and randomly selects a new destination after reaching the base point with a probability of $p_g p_b$. We call this pattern the *Base Point Pattern*. It is worth noting that, the *Base Point Pattern* contains two *epoch*s, i.e., the *epoch* that the node goes back to the base point and the *epoch* that the node moves to a randomly selected new destination.

Definition 6 (*Interrupted Base Point Pattern*): The node may be interrupted during the time it moves back to the base point. It will randomly choose a new destination and move to it with a probability of $p_g(1-p_b)$. We call this pattern the *Interrupted Base Point Pattern*. Note that, this pattern also contains two *epoch*s, i.e., the *epoch* that the node reaches the interrupted point and the *epoch* that the node moves to a randomly chosen new destination.

Let $T(x)$ denote the expected accessed times of x by the node, and let $T_r(x)$, $T_b(x)$ and $T_i(x)$ represent the expected accessed times of x by the three mobility patterns defined above respectively. Then $T(x) = (1-p_g)T_r(x) + p_g\big(p_b T_b(x) + (1-p_b)T_i(x)$. Obviously, the expected accessed times of x is the sum of the probabilities that x is accessed by the *epoch*(s) in each pattern. The PDF of x is proportional to the expected accessed times by the node, i.e., $f_X(x) \propto T(x)$. Then we have

$$f_X(x) \propto (1-p_g)T_r(x) + p_g\big(p_b T_b(x) + (1-p_b)T_i(x). \tag{10}$$

Let λ_X denote the normalization term to keep

$$\int_{-\infty}^{\infty} f_X(x)dx = 1. \tag{11}$$

And we obtain the PDF of the spatial distribution in Equation (12).

$$f_X(x) = \begin{cases} \lambda_X\Big((1-p_g)T_r(x) + p_g\big(p_b T_b(x) + (1-p_b)T_i(x)\big)\Big) & \text{if } 0 < x < A \\ 0 & \text{otherwise.} \end{cases} \tag{12}$$

Now we will analyze the three patterns separately.

4.2.1 Random Pattern

In this mobility pattern, x_s and x_d are both uniformly chosen from $(0, A)$, then

$$P(x_s < x) = \frac{x}{A}, \tag{13}$$

$$P(x_s > x) = \frac{A-x}{A}, \tag{14}$$

$$P(x_d < x) = \frac{x}{A}, \tag{15}$$

and

$$P(x_d > x) = \frac{A-x}{A}. \tag{16}$$

Because the choices of x_s and x_d are independent,

$$P(x_s < x, x_d > x) = P(x_s < x)P(x_d > x) = \frac{x(A-x)}{A^2}$$

and

$$P(x_s > x, x_d < x) = P(x_s > x)P(x_d < x) = \frac{x(A-x)}{A^2}.$$

Thus we have

$$T_r(x) = \frac{x(A-x)}{A^2} + \frac{x(A-x)}{A^2} = \frac{2x(A-x)}{A^2} \; . \tag{17}$$

4.2.2 Base Point Pattern

According to the definition, there are two *epoch*s in this pattern. We use $P_b^1(x)$ and $P_b^2(x)$ to denote the probability that the two *epoch*s in the base point pattern will access x respectively. And we use (x_{s1}, x_{d1}) and (x_{s2}, x_{d2}) to denote the start points and destinations of the two *epoch*s respectively. Obviously, $x_{s2} = x_{d1} = 0$.

We will first discuss the first *epoch*. x_{s1} is randomly selected from $(0, A)$ and $x_{d1} = 0$, then

$$P(x_{s1} < x) = \frac{x}{A}, \tag{18}$$

$$P(x_{s1} > x) = \frac{A - x}{A}, \tag{19}$$

$$P(x_{d1} < x) = 1, \tag{20}$$

and

$$P(x_{d1} > x) = 0. \tag{21}$$

Because the selections of x_s and x_d are independent,

$$P(x_{s1} < x, x_{d1} > x) = P(x_{s1} < x)P(x_{d1} > x) = 0$$

and

$$P(x_{s1} > x, x_{d1} < x) = P(x_{s1} > x)P(x_{d1} < x) = \frac{A - x}{A}.$$

Consequently

$$P_b^1(x) = 0 + \frac{A - x}{A} = \frac{A - x}{A}. \tag{22}$$

Now we will consider the second *epoch*. As $x_{s2} = 0$ and x_{d2} is randomly chosen, we have

$$P(x_{s2} < x) = 1, \tag{23}$$

$$P(x_{s2} > x) = 0, \tag{24}$$

$$P(x_{d2} < x) = \frac{x}{A}, \tag{25}$$

and

$$P(x_{d2} > x) = \frac{A - x}{A}. \tag{26}$$

The derivation is the same as that of $P_b^1(x)$, which yields

$$P_b^2(x) = \frac{A - x}{A}. \tag{27}$$

Therefore,

$$T_b(x) = P_b^1(x) + P_b^2(x) = \frac{2(A - x)}{A}. \tag{28}$$

4.2.3 Interrupted Base Point Pattern

In this pattern, we use $P_i^1(x)$ and $P_i^2(x)$ to denote the probability that the two *epoch*s will access x respectively. And we also use (x_{s1}, x_{d1}) and (x_{s2}, x_{d2}) to denote the start points and destinations of the two *epoch*s respectively. Obviously, $x_{s2} = x_{d1}$.

Now we analyze the first *epoch* of this pattern.

As x_{s1} is randomly chosen, we get

$$P(x_{s1} < x) = \frac{x}{A}, \tag{29}$$

and

$$P(x_{s1} > x) = \frac{A-x}{A}. \tag{30}$$

Because $x_{d1} < x_{s1}$, $P(x_{s1} < x, x_{d1} > x) = 0$. Then $\forall x_{s1} \in (0, A)$,

$$f_{X_{d1}|X_{s1}}(x_{d1}|x_{s1}) = \begin{cases} \frac{1}{x_{s1}} & \text{if } 0 < x_{d1} < x_{s1} \\ 0 & \text{otherwise.} \end{cases} \tag{31}$$

Accordingly,

$$f_{X_{s1}X_{d1}}(x_{s1}, x_{d1}) = f_{X_{d1}|X_{s1}}(x_{d1}|x_{s1}) f_{X_{s1}}(x_{s1}) = \begin{cases} \frac{1}{Ax_{s1}} & \text{if } 0 < x_{d1} < x_{s1} < A \\ 0 & \text{otherwise.} \end{cases} \tag{32}$$

So we get

$$\begin{aligned} P(x_{s1} > x, x_{d1} < x) &= \int_0^x \int_x^A f(x_{s1}, x_{d1}) dx_{s1} dx_{d1} \\ &= \int_0^x \int_x^A \frac{1}{Ax_{s1}} dx_{s1} dx_{d1} \\ &= \int_0^x \frac{1}{A} \ln \frac{A}{x} dx_{d1} \\ &= \frac{x}{A} \ln \frac{A}{x}. \end{aligned} \tag{33}$$

Hence

$$P_i^1(x) = P(x_{s1} < x, x_{d1} > x) + P(x_{s1} > x, x_{d1} < x) = 0 + \frac{x}{A} \ln \frac{A}{x} = \frac{x}{A} \ln \frac{A}{x}. \tag{34}$$

The second *epoch* is different from the first one. Since x_{s2} is randomly chosen from $(0, x_{s1})$, $\forall x_{s1} \in (0, A)$,

$$f_{X_{s2}|X_{s1}}(x_{s2}|x_{s1}) = \begin{cases} \frac{1}{x_{s1}} & \text{if } 0 < x_{s2} < x_{s1} \\ 0 & \text{otherwise.} \end{cases} \tag{35}$$

$$P(x_{s2} < x) = P(x_{s2} < x | x_{s1} < x) P(x_{s1} < x) + P(x_{s2} < x | x_{s1} > x) P(x_{s1} > x) \tag{36}$$

and we have mentioned above that $x_{s2} = x_{d1}$, then

$$P(x_{s2} < x) = P(x_{d1} < x | x_{s1} < x) P(x_{s1} < x) + P(x_{d1} < x | x_{s1} > x) P(x_{s1} > x). \tag{37}$$

x_{d1} is randomly chosen from $(0, x_{s1})$, so $x_{d1} < x_{s1}$. $\forall x \in (0, A)$, if $x_{s1} < x$, then $x_{d1} < x$ must be satisfied, i.e., $P(x_{d1} < x | x_{s1} < x) = 1$. As $P(x_{s1} < x) = \frac{x}{A}$,

$$P(x_{d1} < x | x_{s1} < x) P(x_{s1} < x) = \frac{x}{A}, \tag{38}$$

$$P(x_{d1} < x | x_{s1} > x) P(x_{s1} > x) = P(x_{d1} < x, x_{s1} > x) = \frac{x}{A} \ln \frac{A}{x}. \tag{39}$$

Then we have

$$P(x_{s2} < x) = \frac{x}{A}(1 + \ln \frac{A}{x}) \tag{40}$$

and

$$P(x_{s2} > x) = 1 - P(x_{s2} < x) = 1 - \frac{x}{A}(1 + \ln \frac{A}{x}). \tag{41}$$

x_{d2} is randomly chosen from $(0, A)$, so

$$P(x_{d2} < x) = \frac{x}{A}, \tag{42}$$

and

$$P(x_{d2} > x) = \frac{A - x}{A}. \tag{43}$$

The selections of x_{s2} and x_{d2} are independent, so

$$P(x_{s2} < x, x_{d2} > x) = P(x_{s2} < x)P(x_{d2} > x) = \frac{x}{A}(1 + \ln\frac{A}{x})\frac{A - x}{A}, \tag{44}$$

and

$$P(x_{s2} > x, x_{d2} < x) = P(x_{s2} > x)P(x_{d2} < x) = (1 - \frac{x}{A}(1 + \ln\frac{A}{x}))\frac{x}{A}. \tag{45}$$

Therefore,

$$\begin{aligned} P_i^2(x) &= P(x_{s2} < x, x_{d2} > x) + P(x_{s2} > x, x_{d2} < x) \\ &= \frac{x}{A}(1 + \ln\frac{A}{x})\frac{A - x}{A} + \big(1 - \frac{x}{A}(1 + \ln\frac{A}{x})\big)\frac{x}{A} \\ &= (\frac{x}{A})^2(\frac{A}{x}\ln\frac{A}{x} + 2\frac{A}{x} - 2\ln\frac{A}{x} - 2). \end{aligned} \tag{46}$$

And thus

$$T_i(x) = P_i^1(x) + P_i^2(x) = \frac{2x(A - x)}{A^2}(\ln\frac{A}{x} + 1). \tag{47}$$

Putting Equation (11) (12) (17) (28) and (48) together yields

$$\lambda_X = \frac{18}{A(5p_g + 7p_g p_b + 6)}. \tag{48}$$

We plug Equation (48) into the Equation (12), and finally obtain the expression of the PDF of the spatial node distribution in Equation (49), which is the ***main conclusion*** of this paper.

$$f_X(x) = \begin{cases} \frac{36\Big((1 - p_g)x(A - x) + p_g\big(p_b A(A - x) + (1 - p_b)x(A - x)(\ln\frac{A}{x} + 1)\big)\Big)}{A^3(5p_g + 7p_g p_b + 6)} & \text{if } 0 < x < A \\ 0 & \text{otherwise.} \end{cases} \tag{49}$$

The expected value of x under a certain p_g and p_b is

$$E(x) = \int_{-\infty}^{\infty} x f_X(x)dx = \frac{A(7p_g + 5p_g p_b + 12)}{4(5p_g + 7p_g p_b + 6)}. \tag{50}$$

We have mentioned in the previous section that, when $p_g = 0$, the RWPBP model could perform as the classic RWP. We substitute 0 for p_g in Equation (49) and get

$$f_X(x) = \begin{cases} -\frac{6(Ax - x^2)}{A^3} & \text{if } 0 < x < A \\ 0 & \text{otherwise.} \end{cases} \tag{51}$$

Equation (51) matches the conclusion in [13].

4.3 Discussion on the Two Dimensional Case

For the two dimensional case, it is difficult to mathematically derive the explicit expression of the distribution, since the movement procedure of the two dimensional case is more complex than the one dimensional case. We can roughly estimate the two dimension distribution by assuming x and y are independent, i.e., $f_{XY}(x, y) = f_X(x)f_Y(y)$. To obtain a better approximate or accurate expression of the two dimensional case, we can also divide the total movement into the three basic mobility patterns defined in this paper, and analyze them separately.

5. Stationary Distribution Initialization

In the simulation of the mobile scenario using mobility models, the distribution of the nodes' location typically varies over time, and converges to a steady-state after a long time running [21]. This steady-state is the stationary distribution we analyzed in the preceding section. The network performance evaluated by the simulation is heavily depended on the mobility model [4]. The distributions of speed and location vary during the entire simulation procedure, and Yoon had pointed out that network performance at the beginning may differ substantially from that at the end of the simulation [23]. The previous primary method dealing with this problem is to discard some initial sequence of the observations [24]. Such method is not only inefficient, but also we do not know the convergence time to decide how long time to be discarded [21], and this will surely lead to the inaccuracy of the simulation results. It is a better way to generate the initial speeds and positions of the nodes that obey their stationary distributions. Now we will show how to implement them.

As we have obtained the PDF of the speed distribution, we could use the *inverse transformation technique* to generate the initial speed. We first calculate the *cumulative distribution function (CDF)*

$$F(v) = \int_{-\infty}^{v} f_V(v)dv = \frac{\ln(v/v_{min})}{\ln(v_{max}/v_{min})}, \tag{52}$$

and then we get the inverse function of $F(v)$,

$$F^{-1}(u) = \frac{v_{max}^{u}}{v_{min}^{u-1}}. \tag{53}$$

When we uniformly select a value of u from $(0,1)$, $v = F^{-1}(u)$ will obey the distribution $f_V(v)$.

We will use *rejection sampling method* to initialize the location and destination. *Rejection sampling* is a sampling technique used to generate observations from a distribution in mathematics. The basic *rejection sampling* works as follows: Firstly, sample a point x from the proposal distribution. Secondly, draw a vertical line at x, up to the curve of the proposal distribution $f_X(x)$. Thirdly, sample uniformly along this line and reject points outside of the desired distribution. As we have derived the PDF of the spatial node distribution function $f_X(x)$, we can easily use the basic *rejection sampling* to generate the initial location of the node which obeys the distribution function $f_X(x)$. But only choosing the initial location and randomly selecting the destination will not keep the stationary distribution. They should be both properly initialized. The method used here is that, we first choose an *epoch* using rejection sampling, and then choose a position from the *epoch* as the initial location. The destination of the *epoch* is chosen for the initial destination.

The detail of the method is presented in **Algorithm 2**. In this algorithm, we first randomly select a start point of the *epoch* (line 4). Then we use the destination selection method described in **Algorithm 1** to generate a destination (lines 5-15) of the *epoch*. Afterward we use the *rejection sampling* (lines 16-24) to decide whether this *epoch* will be accepted or not. If accepted, we randomly choose a position from this *epoch* as the initial location and the destination of this *epoch* is selected as the initial destination. If rejected, the foregoing process will be repeated until an *epoch* is accepted. As each pattern contains two *epoch*s, we compute the length of the *epoch*, and divide it by the length of the longest possible path, which is $2A$ (line 16).

Algorithm 2 *Initialization()*

Input: $A, p_g, p_b, v_{min}, v_{max}$
Output: $initial\_info$
1: $u = rand();$
2: $v_{init} = v_{max}^{u}/v_{min}^{u-1};$
3: **while** $true$ **do**
4: $x_{s1} \leftarrow A * rand();$
5: **if** $rand() > p_g$ **then**
6: $x_{d1} \leftarrow A * rand();$
$x_{d2} \leftarrow x_{d1};$
$go\_back \leftarrow false;$
7: **else**
8: **if** $rand() < p_b$ **then**
9: $x_{d1} \leftarrow x_b;$
10: **else**
11: $x_{d1} \leftarrow x_{s1} * rand();$
12: **end if**
13: $x_{d2} \leftarrow A * rand(); go\_back \leftarrow true;$
14: **end if**
15: $x_{s2} \leftarrow x_{d1};$
16: **if** $rand() < (|x_{d1} - x_{s1}| + |x_{d2} - x_{s2}|)/(2 * A)$ **then**
17: $x' \leftarrow (|x_{d1} - x_{s1}| + |x_{d2} - x_{s2}|) * rand();$
18: **if** $x' < |x_{s1} - x_{d1}|$ **then**
19: $x_s \leftarrow x_{s1} + x' * sign(x_{d1} - x_{s1})$;
$x_d \leftarrow x_{d1};$
20: **else**
21: $x_s \leftarrow x_{d1} + (x' - |x_{s1} - x_{d1}|) * sign(x_{d2} - x_{s2});$
$x_d \leftarrow x_{d2};$
$go\_back \leftarrow false;$
22: **end if**
23: **break**;
24: **end if**
25: **end while**
26: $initial\_info \leftarrow (x_s, x_d, go\_back, v_{init});$
27: **return** $initial\_info$

6. Experiment and Results

In this section, we will verify Equations (3) and (49) derived in section 4, as well as the initialization method (i.e., **Algorithm 2**) proposed in section 5.

The experiment method used here is comparable with the method described in section 3.3 for the two dimensional spatial distribution. We simulate the moving patterns in a $100m$ line segment. We divide the entire line segment into 100 equal subsegments, and the length of each subsegment is $1m$. The node distribution is evaluated by the probability that the node appears in the subsegment. The value of how often a node will appear in a subsegment is represented by a histogram. As the time going, the node moves across the subsegments. We sample the location of the node at a constant cycle of 1 second and add 1 to the histogram of the subsegment that the sampled location belongs to. For the sake of more precise comparison, we integrate $f(x)$ and calculate the probability that the node is sampled in the subsegment, and not just calculate $f(x)$ for an approximation as the previous works [13, 21] did. We compare the normalized histogram with $P(x-1<t<x) = F(x) - F(x-1) = \int_0^x f(t)dt - \int_0^{x-1} f(t)dt$, where $f(t)$ could be the *PDF* of speed (Equation (3)) or spatial (Equation (49)) distribution we derived in the previous section.

6.1 Stationary Distribution Results

The speed is uniformly selected from $(0.1, 3)(m/s)$. The destination is chosen using **Algorithm 1** by substituting 0 for B. The pause time is set to 0. We simulate the node movement for 10^7 *epochs*, and the histogram of each subsegment is normalized with the total sampling cycles. We only simulate the speed distribution with $p_g = 0.5, p_b = 0.5$, because the simulation results under other parameters are similar to the result and they are not shown in the figure. But for the spatial node stationary distribution, we simulate the movement of the node under different p_g and p_b.

Fig. 3 and **Fig. 4** show the normalized histograms from simulation and the analytical PDFs from Equation (3) and Equation (49) respectively. In the following figures of this paper, the red curves '*Analytical*' superimposed on the histograms are the analytical results calculated by $\int_0^x f(t)dt - \int_0^{x-1} f(t)dt$, which is mentioned in the last section. From these two figures we can see that, the analytical PDFs exactly match the normalized histograms. Sub-figures **(A)**, **(C)** and **(D)** in **Fig. 4** represent the stationary distributions of the three mobility patterns we defined in section 4.

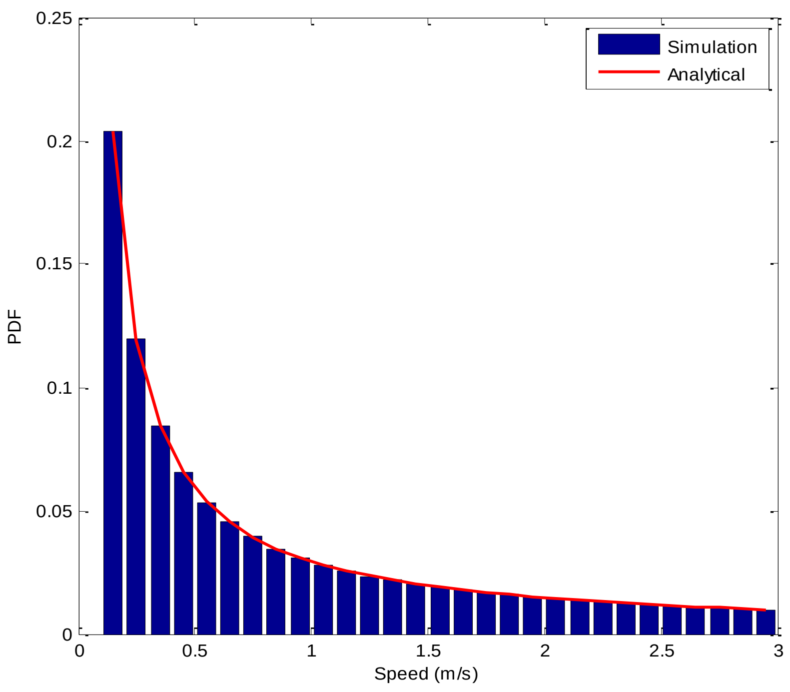

Fig. 3. Stationary speed distribution with $p_g = 0.5, p_b = 0.5$

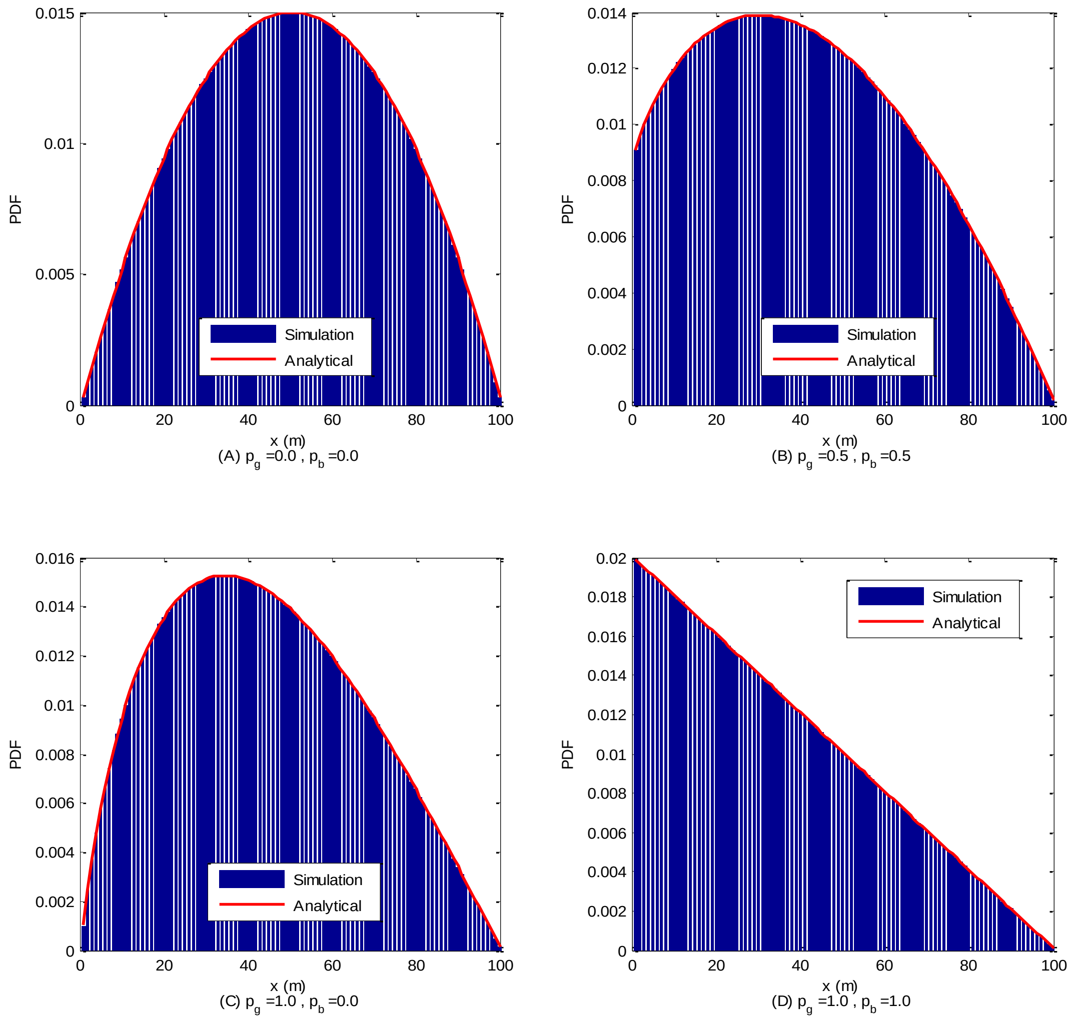

Fig. 4. Stationary spatial distribution under different parameters

6.2 Stationary Initialization Results

The speed range is set to $(0.1, 20)(m/s)$ and the pause time is also set to 0. We simulate that the node travels for 1000 seconds and the sampling cycle is 1 second. We repeat this for 10^7 times. We show the distributions during the first 1 second, the first 100 seconds and the last 100 seconds. It is worth noting that, after initialization, the following speeds are uniformly chosen from the speed range, and the following destinations are generated by **Algorithm 1**.

Fig. 5 presents the normalized node speed histograms. The initial location and destination are chosen using **Algorithm 2**. **Fig. 5. (A1)-(A3)** of this figure are the results of the method in which initial speed is randomly chosen. The initial speed of **Fig. 5. (B1)-(B3)** is selected using **Algorithm 2**. From **Fig. 5** we can see that, at the beginning of the simulation, the distribution of randomly selected initial speed is nearly uniform (see **Fig. 5. (A1)**), which reflects the distribution of the initial value. As the time growing, the distribution is more and more closer to the stationary distribution. During the last 100 seconds of simulation, the distribution is very close to its stationary value. Nevertheless, under the initialization algorithm, the stationary speed distribution keeps all the time.

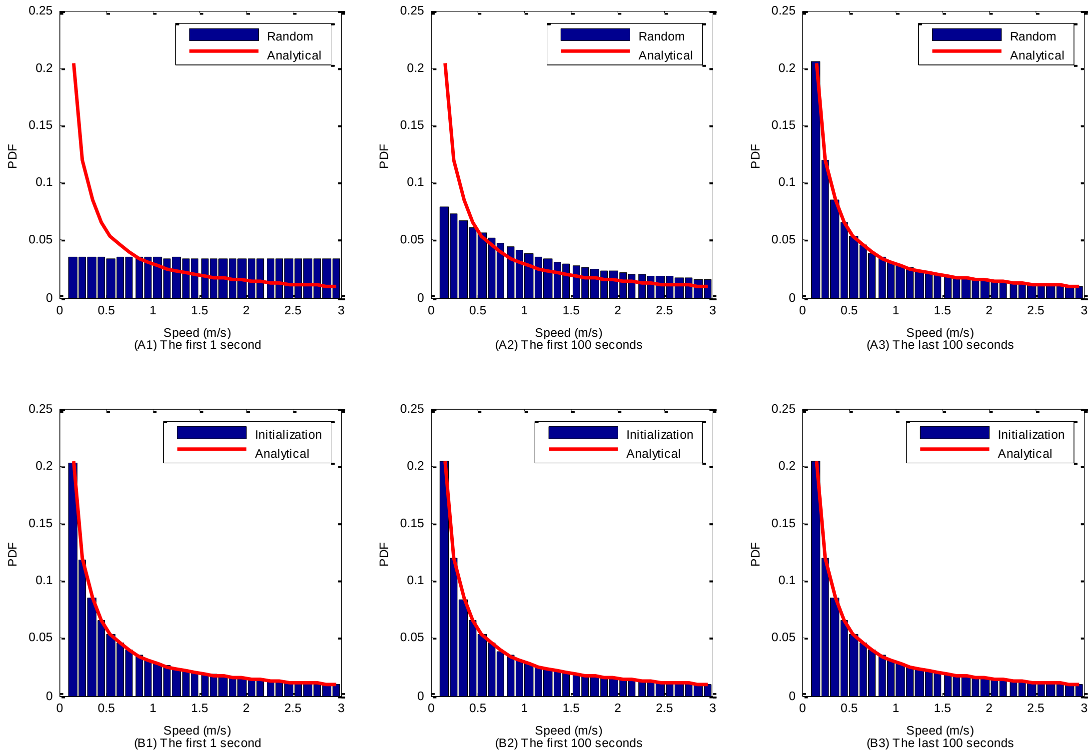

Fig. 5. The speed distribution during different simulation time periods with $p_g = 0.5, p_b = 0.5$

Fig. 6 shows the normalized spatial histograms for the different strategies that initialize the location and the first destination of the node. In this figure, all the initial speeds are chosen using **Algorithm 2**. **Fig. 6. (A1)-(A3)** are the results that the initial location is chosen as the base point and the initial destination is randomly selected. **Fig. 6. (B1)-(B3)** present the results that the initial location and destination are both randomly chosen. In **Fig. 6. (C1)-(C3)**, the initial location and destination are selected using **Algorithm 2**.

There is a sharp peak near the base point in **Fig. 6. (A1)**. That is because the node locates at the base point at the beginning of the simulation and moves away from the base point when simulation starts. When the simulation time is not long enough, the longest point from the base point it can reach is limited by its probable maximum speed. The results of **Fig. 6. (B1)-(B3)** are similar to that of randomly choosing the initial speed which has been interpreted above. The stationary distribution keeps all the time when we use the initialization algorithm in **Fig. 6. (C1)-(C3)**.

7. Conclusion and Future Work

In this paper, we introduced the RWPBP mobility model to describe the movement in the catastrophe rescue scenario. The RWPBP model can generate many different mobility patterns according to different parameters, including the classic Random Waypoint model. After giving the simulation based node distribution, we mathematically derived the stationary distribution of speed and location of RWPBP mobility model for the one dimensional scenario. We also proposed methods on how to initialize the speed, location and destination. Simulation results verified the derived expressions and methods we proposed.

Due to the page limit, in this paper, we only discussed the one dimensional case that the node moves on a straight line segment and there is no pause time. In the future work, we will consider the impact of pause time on the stationary distribution and the two dimensional case should also be studied.

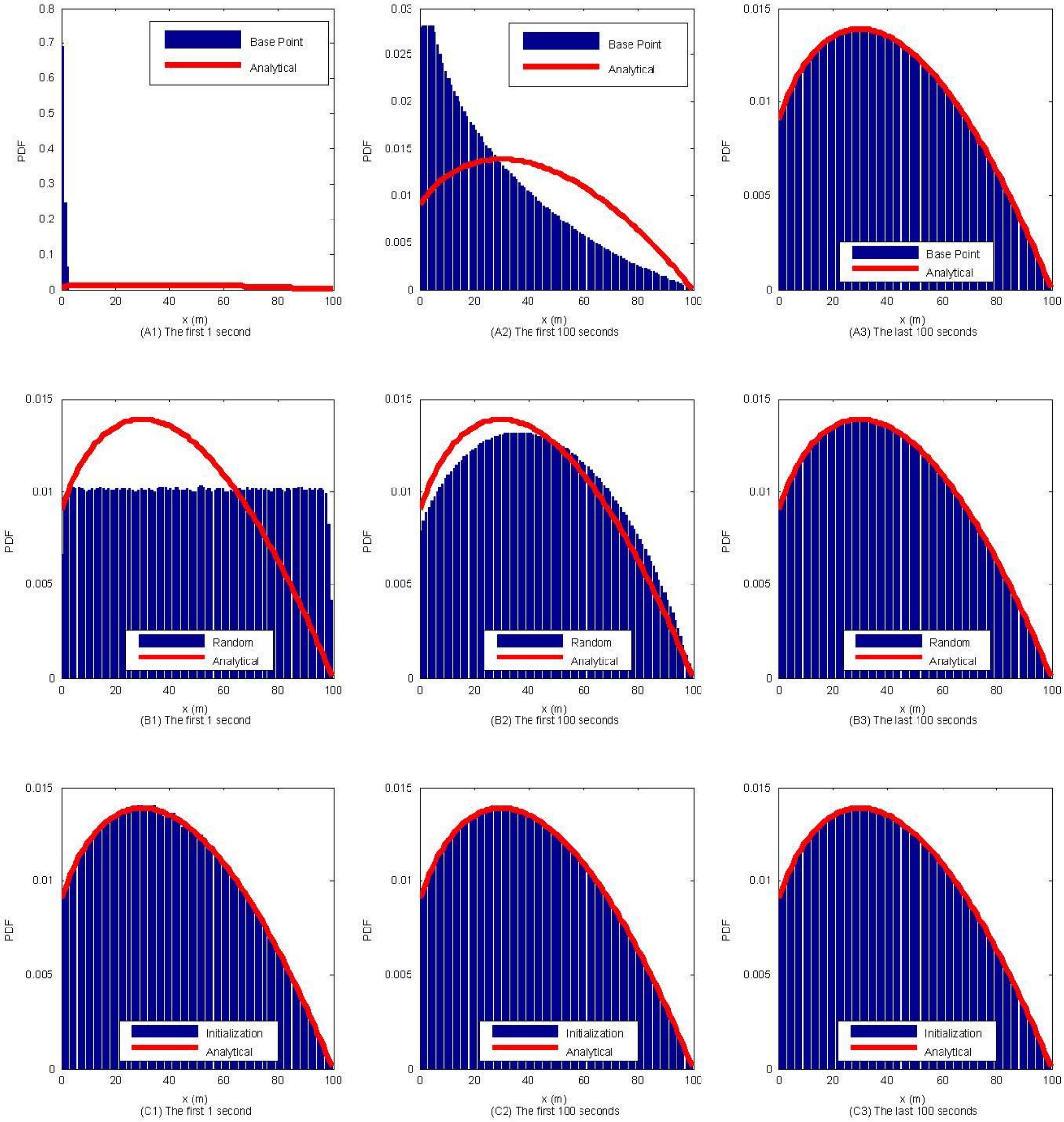

Fig. 6 The spatial distribution during different simulation time periods with $p_g = 0.5, p_b = 0.5$

Acknowledgment

This work has been partially supported by the National High-Tech Research and Development Plan of China under Grant No. 2011AA01A103, the Hunan Provincial Natural Science Foundation of China under Grant No. 11JJ7003, the National Natural Science Foundation of China under Grant No. 61070199, 61070201 and 61103182.

References

[1] M. Zuhairi, H. Zafar, and D. Harle, “Dynamic reverse route for on-demand routing protocol in MANET,” *KSII TRANSACTIONS ON INTERNET AND INFORMATION SYSTEMS*, vol. 6, no. 5, pp. 1354–1372, 2012. Article (CrossRef Link).

[2] K. Fall, “A delay-tolerant network architecture for challenged internets,” in *Proc. of the 2003 conference on Applications, technologies, architectures, and protocols for computer communications*, (Karlsruhe, Germany), pp. 27–34, ACM, 2003. Article (CrossRef Link).

[3] L. Park, W. Na, G. Lee, C. H. Lee, C. Y. Park, Y. S. Cho, and S. Cho, “Mobility-aware interference avoidance scheme for vehicular WLANs,” *KSII TRANSACTIONS ON INTERNET AND INFORMATION SYSTEMS*, vol. 5, no. 12, pp. 2272–2293, 2011. Article (CrossRef Link).

[4] T. Camp, J. Boleng, and V. Davies, “A survey of mobility models for ad hoc network research,” *Wireless Communications & Mobile Computing (WCMC): Special issue on Mobile Ad Hoc Networking: Research*, Trends and Applications, vol. 2, pp. 483–502, 2002. Article (CrossRef Link).

[5] M. Sachez and P. Manzoni, “ANEJOS: a java based simulator for ad hoc networks,” *Future Generation Computer Systems*, vol. 17, no. 5, pp. 573 – 583, 2001. Article (CrossRef Link).

[6] C. Bettstetter, G. Resta, and P. Santi, “The node distribution of the random waypoint mobility model for wireless ad hoc networks,” *Mobile Computing, IEEE Transactions on*, vol. 2, no. 3, pp. 257 – 269, 2003. Article (CrossRef Link).

[7] D. B. Johnson and D. A. Maltz, “Dynamic source routing in ad hoc wireless networks,” *Mobile Computing*, pp. 153–181, 1996. Article (CrossRef Link).

[8] C.-C. Chiang, Wireless network multicasting. *PHD Thesis*, University of California, 1998. Chair-Gerla, Mario. Article (CrossRef Link).

[9] C. Bettstetter, “Smooth is better than sharp: a random mobility model for simulation of wireless networks,” in *Proc. of the 4th ACM international workshop on Modeling, analysis and simulation of wireless and mobile systems*, (New York, NY, USA), pp. 19–27, ACM, 2001. Article (CrossRef Link).

[10] NS2, http://www.isi.edu/nsnam/ns/, 2012. Article (CrossRef Link).

[11] GloMoSim, http://pcl.cs.ucla.edu/projects/glomosim/, 2012. Article (CrossRef Link).

[12] A. Keranen, J. Ott, and T. Karkkainen, “The ONE simulator for DTN protocol evaluation,” in *Proc. of the 2nd International Conference on Simulation Tools and Techniques*, (New York, NY, USA), pp. 55:1–55:10, ICST, 2009. Article (CrossRef Link).

[13] C. Bettstetter and C. Wagner, “The spatial node distribution of the random waypoint mobility model,” in *Proc. of 1st German Workshop Mobile Ad-Hoc Network. WMAN.02*, (Ulm, Germany), pp. 41–58, GI, 2002. Article (CrossRef Link).

[14] E. Hyytia, P. Lassila, and J. Virtamo, “Spatial node distribution of the random waypoint mobility model with applications,” *IEEE Transactions on Mobile Computing*, vol. 5, no. 6, pp. 680–694, 2006. Article (CrossRef Link).

[15] E. M. Royer, P. M. Melliar-smith, and L. E. Moser, “An analysis of the optimum node density for ad hoc mobile networks,” in *Proc. of the IEEE International Conference on Communications*, pp. 857–861,2001. Article (CrossRef Link).

[16] X. Hong, M. Gerla, G. Pei, and C.-C. Chiang, “A group mobility model for ad hoc wireless networks,” in *Proc. of the 2nd ACM international workshop on Modeling, analysis and simulation of wireless and mobile systems*, (New York, NY, USA), pp. 53–60, ACM, 1999. Article (CrossRef Link).

[17] F. Bai and A. Helmy, “A SURVEY OF MOBILITY MODELS,” in *Wireless Adhoc Networks*. Kluwer Academic Publishers. 2008. Article (CrossRef Link).

[18] Z. Du, P. Hong, and K. Xue, “Hotspot gravitation mobility model,” *Communications Letters*, IEEE, vol. 16, no. 2, pp. 193 –195, 2012. Article (CrossRef Link).

[19] C. Bettstetter, H. Hartenstein, and X. P E Rez-Costa, “Stochastic properties of the random waypoint mobility model: epoch length, direction distribution, and cell change rate,” in *Proc. of the 5th ACM international workshop on Modeling analysis and simulation of wireless and mobile*

systems, (New York, NY, USA), pp. 7–14, ACM, 2002. Article (CrossRef Link).
[20] W. Navidi and T. Camp, "Stationary distributions for the random waypoint mobility model," *IEEE Transactions on Mobile Computing,* vol. 3, no. 1, pp. 99 – 108, 2004. Article (CrossRef Link).
[21] D. J. C. MacKay, "Introduction to monte carlo methods," *Learning in Graphical Models*, pp. 175–204, Kluwer Academic Press, 1998. Article (CrossRef Link).
[22] J.-Y. Le Boudec, "On the stationary distribution of speed and location of random waypoint," *IEEE Transactions on Mobile Computing,* vol. 4, pp. 404 – 405. 2005. Article (CrossRef Link).
[23] J. Yoon, M. Liu, and B. Noble, "Random waypoint considered harmful," in *Proc. of INFOCOM'03*, vol. 2, pp. 1312 – 1321, 2003. Article (CrossRef Link).
[24] T. Camp, J. Boleng, B. Williams, W. Navidi, and L. Wilcox, "Performance comparison of two location based routing protocols for ad hoc networks," in *Proc. of INFOCOM'02*, pp. 1678 – 1687, 2002. Article (CrossRef Link).

Yong Wang received the B.Sc. and M.Sc. degrees from National University of Defense Technology, China, in 2006 and 2008 respectively. He is currently a Ph.D. student in the School of Computer, National University of Defense Technology under the supervision of Professor Zhenghu Gong. He was an exchange student at the Department of Computing, The Hong Kong Polytechnic University from May 2011 to April 2012. His current research interests include delay tolerant network, and mobile wireless network.

Wei Peng received the M.Sc. and the Ph.D. degrees in Computer Science from the National University of Defense Technology, China, in 1997 and 2000 respectively. From 2001 to 2002, he was a research assistant in School of Computer, National University of Defense Technology, China, then a research fellow from 2003. He was a visiting scholar of University of Essex, UK, from May 2007 to April 2008. His major interests are internet routing, network science, and mobile wireless networks.

Qiang Dou received the B.Sc. and the Ph.D. degrees in Computer Science from the National University of Defense Technology, China, in 1995 and 2001 respectively. Now he is a research fellow in School of Computer, National University of Defense Technology, China. His major interests are high performance computer architecture, high performance microprocessor design and verification, and computer networks.

Zhenghu Gong is a professor in National University of Defense Technology. His research interests include wireless network, and network communication and protocol.