한국학교수학회논문집 제 16 권, 제 1 호
Journal of the Korean School Mathematics Society
Volume 16, Number 1, 219-240, March 2013

The Construction of an Abstract Schema in the Similar Mathematical Problem Solving Process[1)]

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It is the aim of this paper to suggest the method constructing abstract schema in similar mathematical problem solving processes. We analyzed closely the existing studies about the similar problem solving. We suggested the process designing a method for helping students construct an abstract schema. We designed the teaching method constructing abstract schema by appling this process to a group of similar problems chosen by researchers. We applied the designed method to a student. And we could check the possibility and practice of designed teaching method by observing the student's reaction closely.

Key Words : Construction of an Abstract Schema, Similar Mathematical Problem Solving Process, Structural Similarity, Essential Elements, Non-Essential Elements.

Ⅰ. INTRODUCTION

Analogy is a sort of similarity. Similar objects agree with each other in some respect, analogous objects agree in certain relations of their respective parts(Polya, 1973). Analogy plays a significant role in problem solving, decision making, perception, memory, creativity, emotion, explanation and communication(Wikipedia, 2012). Analogy is a subject studied energetically in psychology and pedagogy. Many researchers have been interested in problem solving by analogy(Gick & Holyoak, 1980, 1983; Gick & paterson,

1) To help readers understand, this is a modified paper adding contents of which help understanding to a thesis presenting at the conference of MCG 7.

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The third author is supported by the research fund in Chinju National University of Education, 2012.

1992; Holyoak, 1985; Novick, 1988, Perfetto, Bransford & Franks, 1983).

Analogical mapping is the core process in analogy. In a typical instance of analogical mapping, a familiar situation – the base or source description – is matched with a less familiar situation – the target description. Analogical mapping requires aligning the two situations – that is, finding the correspondences between the two representations – and projecting inferences from the base to the target(Gentner, 1983).

Most theories of analogy agree that analogical processing involves finding a correspondence between the conceptual structures of the two domains compared. As Kokinov and French(2003) put it, "Experimental work has demonstrated that finding this type of structural isomorphism between base and target domains is crucial for mapping." In the same vein, Holyoak and Thagard(1989) said that at the core of analogical thinking lies the process of mapping: the construction of elderly correspondences between the elements of a source analogy and those of a target. Almost all current theories of analogy agree on the importance of structural consistency, although models vary in whether it is implemented as a hard constraint or as a soft constraint(Gentner & Markman, 2005). That is, the structural similarity is crucial in analogical processing.

Meanwhile, the term schema is used to refer to an internal model or representation that contains knowledge about a specific category. Problem solvers get his own schema when he has an experience solving a specific problem. For example, students have his own schema as he learn about the expansion of polynomial $(a+b)^2$. An analogical transfer would depend on the schema he construct. If he have a schema taking the focus on the specificity of a, b of a polynomial $(a+b)^2$, it will be difficult to expand the polynomial $(a+b+c)^2$. That is, if he consider a, b as a object limited to number, he will think that the expansion of $(a+b+c)^2$ is separate matter with the expansion of $(a+b)^2$. On the other hand, if he have a schema not taking the focus on the specificity on a, b of a polynomial $(a+b)^2$, it will be easy to cognize that the expansion of $(a+b+c)^2$ is of the same type with the expansion of $(a+b)^2$. Thus it is very important which schema problem solvers construct. Especially, it is required to get an abstract schema eliminated the surface properties to expect the increase of analogical transfer.

In the same vein, Gick and Holyoak(1983) insisted that problem solvers had to get an abstract schema. They called the process accumulated this schema as the schematic learning in particular. This abstract schema is formed by eliminating the surface properties and preserving the structural properties between base problem and target problem[5)].

Many studies are focused on finding the learning condition of base problem to help students have an insight into the structural similarity between the base problem and the target problem(Bassok, 1997; English, 1997; Gick & Holyoak, 1983; Holyoak & Koh, 1987). One of the research's results is that students who construct the high level of schema for the principle solution of the base problem can solve the target problem because they are less affected by the surface difference between the base problem and the target problem. Thus students have to construct the high level of schema for the base problem to transfer effectively from the base problem to the target problem. And so the researches for concrete methods to help problem solvers construct the high level of schema(i.e. the abstract schema) are needed. There was, however, less study on how to have students build the high level of schema.

Specifically there was little study on the teaching methods helping students build an abstract schema in mathematics. Thus it was difficult to know how to help students construct an abstract schema and to apply this method in math classes. And so we thought that it was needed to give body to the method helping students construct an abstract schema. That is, we thought that it was required to present the concrete methods to realize actually the construction of an abstract schema in math classes. We now are attempting to look for the teaching methods helping the construction of an abstract schema. First of all, we try to suggest the process designing a method for helping students construct an abstract schema. Next, we try to design the teaching method constructing abstract schema by appling this process to a group of similar problems. Third, we will check the possibility and practice of teaching method by appling a student.

Ⅱ. THEORETICAL BACKGROUND

To argue the similar problem solving, it is necessary to treat the structural similarity. At first, we tried to argue the structural similarity. Gentner and Markman(2005) classified the meaning of a structural similarity into two categories – the relational similarity view and the pure graph-isomorphism view. The structural similarity is defined differently in different theories. On the relational similarity view, structural similarity requires conceptual similarity between corresponding relations. On the pure graph-isomorphism view, structural similarity requires only graph isomorphism; although

5) The base problem is the familiar problem of which problem solvers have an experience to solve previously. The target problem is the new problem similar to prior familiar problem(i.e. base problem). Our study will use these as terms with the such a meaning.

conceptual similarity between relations can be helpful it is not central to the algorithm. Gentner and Markman(2005) explain two views through the following example.

(a) Mary hugged John because she loves him.
(b) Rover nuzzled Sarah because he loves her.
(c) Fred heated the sandwich before he ate it.

Both views would agree that the relationally similar pair [(a) and (b)] is processed as an analogy. However, the views diverge on the graph-isomorphic pair - [(a) and (c)]. On the relational similarity view, such nonconceptual matches are not processed as analogies.

According to Gentner(1983), structural similarity involves conceptual similarity between corresponding relations. His opinion about structural similarity is the relational similarity view. On the other hand, structural similarity need only entail graph matches in Holyoak and Thagard(1989)'s view. We agree with Gentner's opinion on the meaning of structural similarity, because we think that it will be more proper in mathematics. For example, we judged it is necessary to take the relational similarity into account to cognize the structural similarity between $(a+b)^2$ and $(a+b+c)^2$. On the relational similarity view, b in $(a+b)^2$ can be matched $b+c$ in $(a+b+c)^2$. Thus our study agree with Gentner's opinion.

The basic assumption of Gentner's structure-mapping theory is that in comparing two objects, people are influenced not merely by sets of separate features, but also by how the features are related to one another, by what structure they form(Mussweiler & Gentner, 2007). Gentner and Toupin(1986) checked how people match. In this study, they checked that people typically prefer the correspondence preserving the relational match. For example, if participants compare two cross-mapped figures such as (d) and (f), they are not only guided by the individual objects (the O's, X's and V's), but also by how these individual objects relate to one another. Many of them match X in (d) to O in (f) and match O in (d) to V in (f). This is considerable evidence that people's sense of similarity is influenced by common relational structure.

(d) XOOXXOOX
(f) OVVOOVVO

When solving a problem, we form a schema to this problem. Next, this schema should have a decisive effect on solving similar problems. For this reason, we need to build a good schema, where this is the schema increasing the analogical transfer. It is necessary to approach problem solving from the viewpoint of the schema rather than from the viewpoint of the problem.

Gick and Holyoak(1983) claimed schematic learning is the process accumulated to an abstract schema. An abstract schema is formed by eliminating the surface properties and preserving the structural properties between base problems and target problems. Surface properties are inessential in respect to the nature of the problem and its solution. Structural properties are essential in respect to the nature of the problem and its solution. For example, the specific letter a, b, c is a surface property in the expansion of $(a+b)^2$ and $(a+b+c)^2$. The form of polynomial is a structural property in the expansion of $(a+b)^2$ and $(a+b+c)^2$. Gick and Holyoak(1983) call the shared essential structural features between the base problem and the target problem the convergence schema. They said that the process of schema induction involves deleting the differences between the analogs while preserving their commonalities. Our study agree with Gick and Holyoak's opinion. That is, an abstract schema is the schema which is preserved essential, structural features and eliminated the surface features between base problem and target problem in our study. And the teaching method we try to look for will be one which preserve essential, structural features and eliminate the surface features. Thus we will judge based on the quantity of unnecessary elements recognized that a student construct a better abstract schema than before.

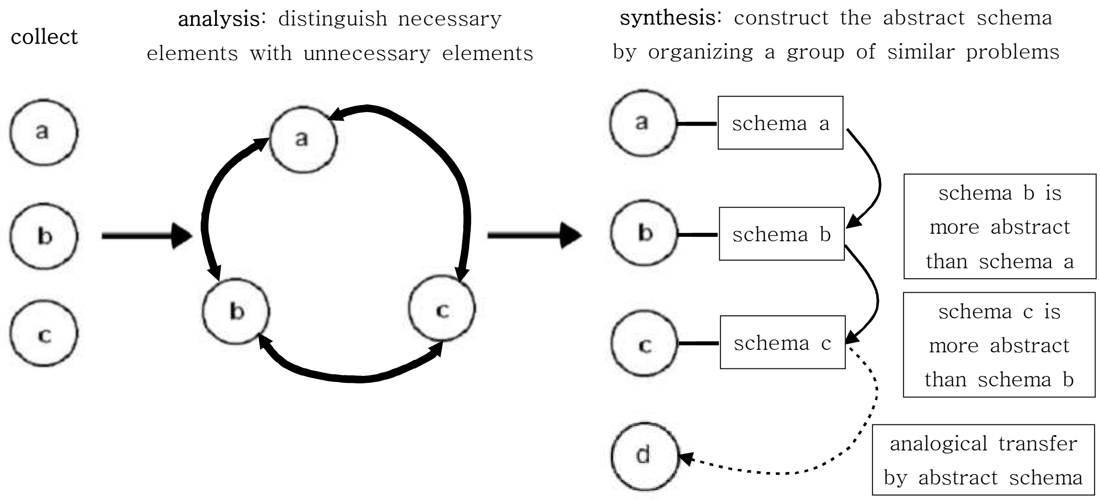
We tried to approach similar problem solving from the viewpoint of the schema. If a problem solver doesn't have a full insight into the similarity of the similar problem, he may construct the schema to each problem individually. The schema to each problem is not connected to each other. For example, if a problem solver doesn't have an insight into the similarity with the expansion of $(a+b)^2$ and $(a+b+c)^2$, he can't connect to each problem but form a separate schema to each problem. That is, he will think that two problems are different. He doesn't have an abstract schema. Thus we tried to help students connect with each schema and construct an abstract schema as below. An abstract schema is not a simple combination of each schema but a meaningful connection of each schema. The construction of abstract schema is the work to integrate each separate schema in a statement of disconnection.

We refer to the schema getting rid of the surface properties and preserving the structural properties between base problem and target problem as an abstract schema. We wish for students to have this abstract schema, deleting the differences between the analogs while preserving their commonalities. We attempt to look for the teaching methods to help build an abstract schema preserving the relational, essential structure. We try to suggest the process designing a method for helping students construct an abstract schema. And we try to design a concrete method to help build an abstract schema in mathematics by appling this process to a group of similar problems. Furthermore, we will check the possibility and practice of the method by appling a student.

Ⅲ. Designing the method to help construct an abstract schema

In this chapter, we will show the process of designing the method to construct an abstract schema at first. Next, we will design the method constructing abstract schema by appling this process to a group of similar problems. As stated above, an abstract schema is the schema which is preserved essential, structural features and eliminated the surface features between base problem and target problem in our study. We thought that it is a major principle to eliminate the surface features.

At first, we should collect a group of similar problems to help students form an abstract schema. It doesn′t matter whether we perceive the structural similarity between them or not in this situation. We may only collect similar problems by superficial similarity. After that, we have to analyse a group of similar problems. We should classify elements in problems by structural similarity. That is, we will distinguish necessary elements with unnecessary elements between similar problems. If a problem is completely unrelated in the other problems in analysis process, we may remove the problem. In this case, we may add to another problem. Finally, we can organize the teaching method to construct an abstract schema in the similar problems. we should synthesize the analyzed situation. That is, we have to organize a group of similar problems. We will deliberate over how to help students to construct an abstract schema by the analysis. We will determine the order of each problem. The problem which was first proposed will naturally be the base problem. The problem which next proposed will naturally be the target problem for the prior. We should decide each schema we intend between the base problem and the target problem. We will intend to construct more abstract schema than before. Ultimately, we will try to lead to the increase of analogical transfer by constructing the abstract schema. That is, we want to make students solve the other similar problem d by constructing the abstract schema c in below. The upper contents are displayed in the following diagram form. This is a process of designing the method to construct an abstract schema in the similar problems.



[Figure 1] The process of designing the method to construct an abstract schema

Let's take a close look at this process through an example. First, we can collect a group of similar problems as the following polynomial expression; $(a+b)^2$, $(2+3)^2$, $(x+y)^2$, $(a+b+c)^2$. Next, we will try to analysis four similar problems. We could distinguish necessary elements with unnecessary elements by focusing on the structural similarity. Every problem is related to the expansion of polynomial. $(a+b)^2$ will be exactly the same with $(2+3)^2$ if we substitute 2 for a and 3 for b. That is, $(a+b)^2$ is equivalent to $(2+3)^2$ and this implies that the characteristic as the letter or number is unnecessary in the expansion of polynomial. Similarly, $(a+b)^2$ will be exactly the same with $(x+y)^2$ if we substitute x for a and y for b. That is, $(a+b)^2$ is equivalent to $(x+y)^2$ and this implies that it is not important what kind of letter it is in the expansion of polynomial. Furthermore $(a+b)^2$ will be exactly the same with $(a+b+c)^2$ if we substitute $a+b$ for a and c for b. That is, $(a+b)^2$ is equivalent to $(a+b+c)^2$ and this implies that the complexity of expression is not important in the expansion of polynomial. We can keep on analyzing like this.

Finally, we can organize the teaching method to construct an abstract schema in the similar problems. We will synthesize the analyzed situation. That is, we will organize a group of similar problems. At first, we have to determine the order of each problem. We organized a group of similar problems in following order: $(2+3)^2$ → $(a+b)^2$ → $(x+y)^2$ → $(a+b+c)^2$. Next, we should decide each schema we intend between the base problem and the target problem. A schema we intend between $(2+3)^2$ and $(a+b)^2$ is a thing got rid of the characteristic as the letter or number. A schema we intend between $(a+b)^2$ and $(x+y)^2$ is a thing not being important what kind of letter it is. And finally a schema we intend between $(x+y)^2$ and $(a+b+c)^2$ is a thing that the complexity of expression is not important. After going through the upper process, we thought it is possible that students can integrate four problems. That is, we thought they could consider four problems as the equivalence.

Ⅳ. Applying the method to construct an abstract schema

1. Method

1) Subject

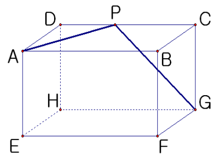
The subject for the study was a third-grade student in Namsan middle school located in Changwon, Gyeong-Sang-Nam-do of South Korea. A student in the sample had volunteered to participate in a day long experiment. He had a good school record in mathematics and possessed a good communication ability. Since we tried to focus on a schema after solving the problem, we thought that it is appropriate to select a student with a good school record in mathematics. That is, we judged that a participant must be a student who didn't have trouble in solving the problem. If a student have trouble in solving the problem, we thought that it would be long to connect similar problems. Because the integration of similar problems can occur after solving similar problems. To observe the student's reaction focusing the integration of similar problems, we judged that a student with a good record is appropriate. He had already known of the Pythagorean theorem, so we didn't need to be concerned about a shortage of content knowledge in the experiment.

2) Organizing similar problems

The followings are the similar problems suggested a student. We tried to collect problems in related to a shortest path in the geometry. In particular, we tried to find similar problems in terms of the solution. We could find similar problems related to Pythagorean theorem as follows. We tried to analysis four similar problems. We could distinguish necessary elements with unnecessary elements by focusing on the structural similarity. Every problem is related to the shortest path. Furthermore the faces are containing in the path of each problem. But the condition of each face differs from each other. some faces are plane, other faces are curved. We could distinguish necessary elements with unnecessary elements. This problems are equivalent in the point of view of abstract schema. We could organize a group of similar problems by analysis. We try to suggest a group of similar problems in following order: problem 1 → problem 2 → problem 3 → problem 4.

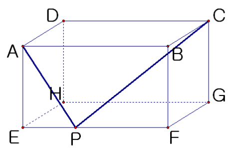
Problem 1

There is a rectangular parallelepiped ABCD − EFGH with $\overline{AB} = 5\,cm$, $\overline{BC} = 3cm$, $\overline{BF} = 4\,cm$ as below. The point P is on the line $\overline{DC}$. What is the length of a shortest path starting at a point A, passing through a point P and reaching a point G?



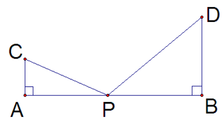
Problem 2

There is a rectangular parallelepiped ABCD − EFGH with $\overline{AB} = 5\,cm$, $\overline{BC} = 3cm$, $\overline{BF} = 4\,cm$ as below. The point P is on the line $\overline{EF}$. What is the length of a shortest path starting at a point A, passing through a point P and reaching a point G?



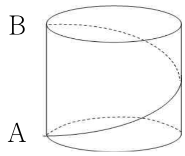
Problem 3

There are points A, B, C, D, P with $\overline{CA} = 2\,cm$, $\overline{DB} = 5\,cm$, $\overline{AB} = 15\,cm$ as below. A point P is on the line $\overline{AB}$. What is the length of a shortest path starting at a point C, passing through a point P and reaching a point D?



Problem 4

There is a cylinder with a radius $3cm$ and a height $8cm$ as below. What is the length of a shortest path starting at a point A in a base surface, going round the cylinder and reaching a point B in an upper surface?



3) Date collection and analysis

After school, an experiment was performed in the classroom at the student's school for about one hour. We suggested similar problems already organized and asked that he solve them. We were not interested in the success or failure of problem solving. We tried to know which schema a participant constructed after he solved similar problems. To find out his schema, we made him compare similar problems and look into the similarity he constructed at first. We judged the degree of construction of abstract schema on the basis of how many unnecessary elements he cognize. We decided that if he cognize more unnecessary elements than ever before, he construct more abstract schema than before. We applied the designed method to him. The teaching method is a reflective activity performed after the problem solving. It is the activity connecting each schema formed from each problem. To know his schema in each situation, we asked him to divide into two classes - structural properties with the surface properties. We recorded the process in the video and wrote student's words and behavior in the field note. Our goal is to check the possibility and practice of the method by appling a student.

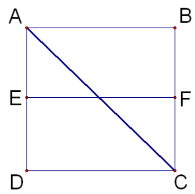
2. Results of observation[6)]

6) S refers to an anonymous student and R refers to a researcher.

At first, the student had solved the first problem easily. He had some experience solving this in the past, but he had a difficulty in solving the second problem. For a long time, he could not solve the problem. He spoke his mind as follows.

S : $\overline{PC}$ is not located in a plane, but in a space. And so I think that this problem is different from the previous problem.

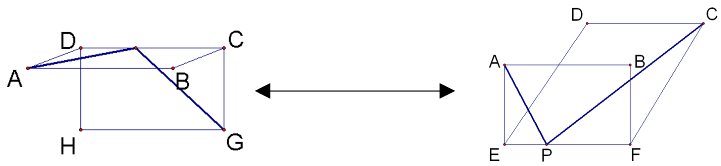
He couldn't move forward for a while. We tried to help him solve the problem. The purpose of this study is to verify the applicability of designed method to construct the abstract schema after problem solving. To achieve this purpose, we tried to apply the method we designed and check the possibility of this method. We judged that it would be fine to give hints for solving the problem. Thus we explained that the line $\overline{PC}$ in second problem is on the plane DCFE. Despite any researcher's aid, he could not still solve the problem. He struggled for a long time and finally solved the problem. He had cognized the structural similarity between first problem and second problem. He drew the picture unfolding the face AEFB and the face DCFE in the process of problem solving as below.



[Figure 2] student's drawing to solve the second problem

1) Construction of the schema eliminated of unnecessary components

We could map elements in first problem into elements in second problem in the respect of structural similarity as follows: $\overline{AP} \leftrightarrow \overline{AP}$, $\overline{PG} \leftrightarrow \overline{PC}$, □ABCD↔□ABFE, □DHGC↔□DCFE. We wished that students could connect first problem with second problem as follows.

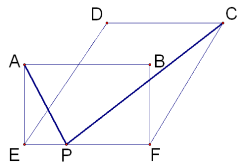


[Figure 3] The connection of first problem and second problem

We intended the following schema between first problem and second problem: it is a schema eliminated of unnecessary components such as the faces not containing the path. For example, □ABFE in first problem is not containing the path.

The following shows the process which a student constructed the schema eliminated of unnecessary components in second problem. That is, this shows that he constructed the better schema than before.

R : Let's get rid of any unnecessary elements. What are unnecessary elements in this problem?
S : Um…. I think that the face ABCD is an unnecessary element
R : Another thing?
S : Um…. To answer your inquiry, we can draw the following(Figure 4).



[Figure 4] Removing unnecessary components form second problem.

We judged that he built schema being god rid of unnecessary components through dialog, and we tried to go on to the next problem.

2) Construction of the schema eliminated the angle between two faces

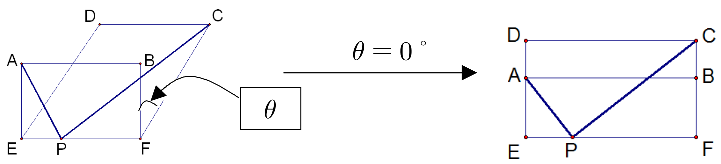
After that, the student was given the third problem which was a "shortest distance" problem solved by Heron of Alexandria, who lived about 10-75 A.D. He had tried to work out this problem for a long time. Finally he succeeded in solving the problem. The following shows how the student cognize similar problems.

R : Do you think that this problem is similar with the prior two problems?
S : Yes, I think that this problem is similar with the priors.

R : What is the similarity?
S : I think that unfolding is similar.
R : Another thing?
S : I think that there is not anything else.

From this interview, we could know that a student is cognizing the similarity undefinedly. This problem is similar to the priors in respect of the shortest path. The solution of the third problem is similar to the solution of the priors in respect of unfolding the path, but we thought that these similarities were superficial. The third problem could clearly be connected with the priors if the student was focusing on $\angle BFC$ in the [Figure 4].

We could map the second problem into the third problem in the respect of structural similarity. We intended the following schema between the second problem and the third problem: it is a schema got rid of the angle between two faces. We hoped that students could connect the second problem with the third problem as follows.



[Figure 5] The connection of second problem and third problem

That is, if $\angle BFC = 0^\circ$, Figure 4 could become the third problem. This insight could help to form a deeper abstract schema. We tried to help an obvious insight into the similarity as follows.

R : If the angle between the face ABFE and the face DCFE is zero in this figure(pointing Figure 4), how become this figure?
S : Um…. Ah, then this figure is equivalent to the third problem.
R : What are unnecessary elements in this problems?
S : An angle between two faces is not necessary.

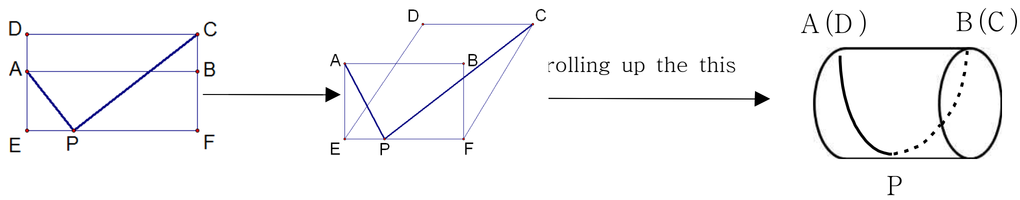
We judged that he built a good schema eliminated the angle between two faces through dialog. And we tried to go on to next problem.

3) Construction of the schema with removal of the condition of the face

After that, the student was given the fourth problem. He could easily solve the problem. The following shows how the student cognize similar problems.

R : Do you think that this problem is similar with the prior two problems?
S : Yes, I think that this problem is similar with the priors.
R : What is the similarity?
S : I think that unfolding is similar.
R : Another thing?
S : I don't think that there is anything else.

From this interview, we could know that a student is cognizing the similarity a little ambiguously. This problem is similar to the priors in respect of shortest path. The solution of fourth problem is similar to the solution of the priors in respect of unfolding the path. We also thought as before that the similarities were superficial. The fourth problem could clearly be connected with the priors if a student was focusing on how to make a cylinder out of [Figure 4]. That is, if [Figure 4] is rolled, the [Figure 4] could become the fourth problem. We intended the following schema: it is a schema removed of the condition of the face. We hoped that students could connect third problem with fourth problem as follows.



[Figure 6] The connection of third problem and fourth problem

This insight could help to form a deeper abstract schema. And so we tried to help an obvious insight into the similarity as follows.

R : Can you transform this figure(pointing [Figure 4]) to be equivalent to the forth problem?

He considered the method carefully from this question for a long time. And he said the following.

S : Rolling up this figure, this figure becomes this problem.
R : What are any unnecessary elements in this problems?
S : Um…. I think that… I don't know.
R : The condition of the face is an nonessential element in this similar problems. It is not important whether the face is flat or curved.
S : At first, I thought that these problems were similar but different. I realize now that these problems become similar if unnecessary elements are removed.

From the above interview, we judged that he built a good schema with removal of the condition of the face, and the experiment was closed.

3. Analysis

We would discuss the results. At first, the student didn't solve the second problem. Specifically, he could not find the plane containing the line $\overline{PC}$ in the second problem. That is, His difficulty came from not finding the plane in second problem corresponding to the plane DCGH containing the line $\overline{PG}$ in the first problem. He could not get an insight to the structural similarity. To put it delicately, he could not find the mapping elements in target problem corresponding to elements in base problem. And so he could not apply a solution of first problem to second problem. He even said that first problem is different from second problem. He could not connect first problem with second problem at all. That would seem to imply that the insight into structural similarity is required to get a good schema in the similar problem. Not getting an insight into structural similarity, it is difficult to have a good schema.

After solving second problem with a researcher's help, the student could build a good schema. Figure 4 shows that he had formed an abstract schema. There are a lot of elements in second problem. For example, there are points A, B, C, …, H, P and there are six faces, but not all of the elements are used in problem solving. There are elements being used in problem solving and elements not being used in problem solving. For example, the face ABCD is an element being used in problem solving. The face ABFE is an element being used in problem solving. Figure 2 drawing to solve second problem didn't have unnecessary elements but necessary elements in problem solving. This shows that he had formed an abstract schema by getting rid of unnecessary components.

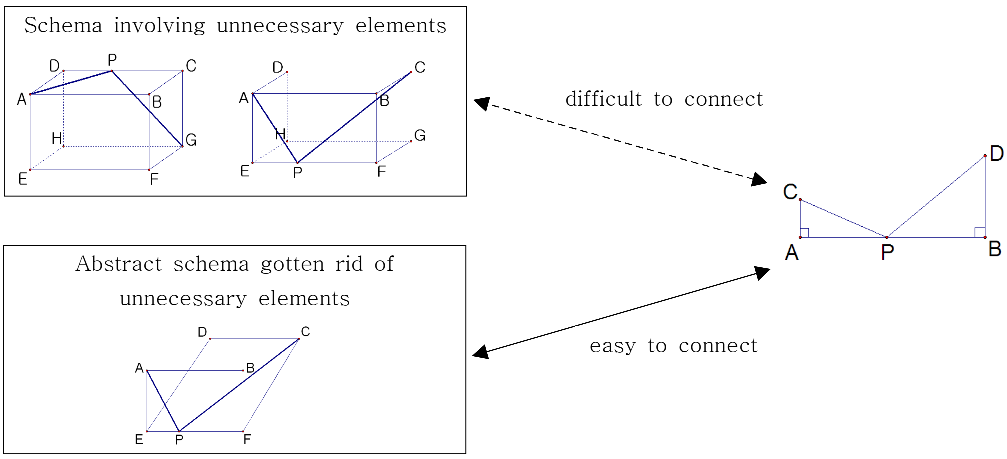
[Figure 4] is a figure that unnecessary components are removed in second problem.

This shows that a student gets to be able to regard second problem as Figure 4. This schema would also help him connect this problem with next problems. That is, he was prepared to embrace the other problems.

At first, the student could not build a good schema after solving third problem. He cognized the similarity between the priors and third problem superficially. He could not find the other similarity except the folding the path in respect of the solution. He could not connect to third problem with the priors. We then tried to help the student build an abstract schema. It is the process of getting rid of the surface properties and preserving the structural properties. From the assistance of the researcher, the student was able to connect the priors with third problem.

The student could have a schema getting rid of the angle between two faces. Whatever the angle between two faces is, three problems(first problem, second problem, third problem) are equivalent. He cognized that the angle between two faces is an nonessential element in the similar problem. That is, he got to know that angle between two faces is not important through this activity.

We think that the abstract schema being already formed helps construct a schema on a higher level. If a student does not have an abstract schema gotten rid of unnecessary elements, we think that he could not connect the prior problems to third problem, despite researcher's help. He would be interfered with the construction of another abstract schema due to unnecessary elements. This could be described as follows.



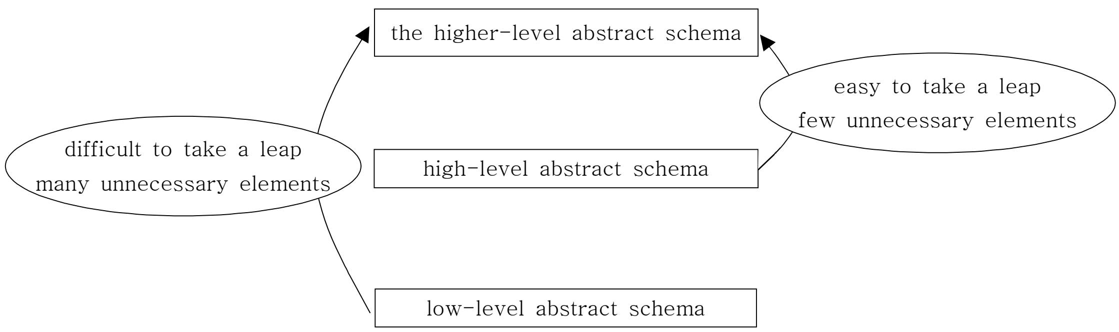
[Figure 7] The difference of connection with third problem in each schema

At first, the student could not build a good schema after solving fourth problem. He cognized the similarity between the priors and fourth problem superficially. As before, he could not find the other similarity except the folding of the path in respect of the solution. He could not connect fourth problem with the priors. We then tried to help a student build an abstract schema. From the assistance of the researcher, the student was able to connect the [figure 4] with the fourth problem.

The student could connect the prior problems with fourth problem. He constructed an abstract schema. He had a schema of getting rid of the condition of the face. Whatever the condition of face is, the four problems(first problem, second problem, third problem, fourth problem) are equivalent. The faces in first problem, second problem and third problem are a flat surface and the face in fourth problem is a curved surface. He cognized that the condition of the face is an nonessential element in the similar problem. That is, he got to know that the condition of the face is not important through this activity.

The student said "I realize now that these problems become similar if unnecessary elements are removed." This means that the student constructed an abstract schema. Thus the method we designed is an applicable thing to help the student construct an abstract schema. However everyone will not receive a help from this method. To expand the applicability, we need to develop teacher question.

Like the preceding, we think that the abstract schema being already formed helps construct a schema on a higher level. If a student does not have an abstract schema gotten rid of unnecessary elements, he would be interfered with the construction of another abstract schema due to a lot of unnecessary elements. The more unnecessary elements there are in the existing schema, the more difficult it is to build a good schema. We displayed this fact in a diagram form as follows. In this figure 8, the standard for distinction of the level is the relative number of unnecessary elements eliminated. That is, the more unnecessary elements there are in the schema, the more high-level schema it is to build.



[Figure 8] Degree of difficulty of taking a leap depending on the level of existing schema

We applied to the student the methods we designed to help build an abstract schema, and discussed for the methods from the result. The method we designed is based on the theory of Gick and Holyoak(1983). It is the process getting rid of the surface properties and preserving the structural properties. It is the process connecting the previous problems with the present problem. We think that if the method is applied for students using a transparent plastic paper, it helps effective construction of abstract schema. The following is the process helping the abstract schema using a transparent plastic paper.

[Figure 9] The activity helping the abstract schema using a transparent plastic paper

One must be careful to abstract the schema to the highest degree by Kintsch & Dijk(1978). They said that if abstracting the schema to the highest degree, all problems seem to be similar. That is, all problems become a problem of which find the answer. To overcome this difficulty, Gick & Holyoak(1983) insisted that there is an optimal level of abstraction. They said that the optimal level of representation will be that which maximizes the degree of correspondence between causally relevant features of the analogs. Many researchers insist that since mapped elements are typically similar but not identical, they must be decomposable into identities and differences(Gick & Holyoak, 1983; Hesse, 1966; Tversky, 1977). This means that it is necessary to cognize the identities and differences in order to construct a good schema. And so it is not good to remove even identities. To form a good schema, it is necessary to preserve identities. Thus it ought to be careful to abstract the schema to the highest degree.

There are many products in mathematics through the construction of abstract schema. For example, the concept of isomorphism in abstract algebra is a typical example. We think that the concept of isomorphism was generated by constructing an abstract schema. That is, the concept of isomorphism could be required by getting rid of unnecessary elements and preserving a algebraic structure. $C_n \cong Z/Z_n$ doesn't mean that the cyclic group C_n is equal to the quotient group Z/Z_n. $C_n \cong Z/Z_n$ means that the cyclic group C_n is equivalent to the quotient group Z/Z_n. That is, C_n is identical to Z/Z_n in respect of structure. This implies that the concept of isomorphism is a product which could be required through construction of abstract schema. Ultimately, one must get rid of unnecessary elements and to preserve necessary elements in order to construct an abstract schema.

Ⅴ. CONCLUSION

We attempted to look for the teaching methods in helping to build an abstract schema in the similar mathematical problem solving. First of all, we suggested the process designing a method for helping students construct an abstract schema. We thought that it is a major principle to eliminate the surface features. First, one must collect a group of similar problems. It doesn't matter whether we perceive the structural similarity between them or not in this situation. And one has to only collect similar problems by superficial similarity. Second, one must analysis a group of similar problems. One should distinguish necessary elements with unnecessary elements between similar problems. Finally, we can organize the teaching method to construct an abstract schema in the similar problems. We have to synthesize the analyzed situation. We will deliberate over how to help students to construct an abstract schema by the analysis. We will determine the order for each problem. We should decide each schema we intend between the base problem and the target problem. We will intend to construct more abstract schema than before. The order of each problem proposed in the synthesis is not fixed. It can be changed. The order is proposed for convenience by researchers and doesn't have a special meaning. If it is proposed in a different order, we think that the learners can construct different types of abstract schema. We think that this will be a subject of follow-up research.

Next, we tried to design the teaching method constructing abstract schema by appling this process to a group of similar problems. We tried to organize a group of similar problems. We would deliberate over how to help students to construct an abstract schema. We decided unnecessary elements between the base problem and the target problem. Furthermore, we determined the order of each problem.

Third, we checked the possibility and practice of teaching method by appling a student. We applied similar problems to a student in turn, and gave time to solve the problems. After that, we checked the student's schema by standard of our study. He didn't have an abstract schema. We applied the designed method to help him construct an abstract schema.

First, we could check that the insight into structural similarity is required to construct an abstract schema. Not getting an insight into structural similarity, it is difficult to have a good schema. Second, we could find that the abstract schema being already formed helps construct higher-level abstract schema than ever before. If a student does not have an abstract schema gotten rid of unnecessary elements, he would be interfered with the construction of another abstract schema due to many unnecessary elements. Third, we could check the possibility for the method we designed. However everyone will not receive help from this method. To expand the applicability, we need to develop teacher question.

The construction of an abstract schema helps develope a student's ability to look at the similar problems. We hope that a teaching method developed from our study help form a good abstract schema about other similar problems. And it is needed to study the construction of abstract schema considering the level of students.

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유사 문제 해결 과정에서 추상적 스키마 구성하기

강정기[7] · 전영배[8] · 노은환[9]

초 록

각 문제를 해결하는 과정에서 우리는 필연적으로 문제에 대한 스키마를 갖게 된다. 그런데 유사 문제를 해결하다 보면 각 문제를 관련 짓는 추상적 스키마를 발견하게 된다. 이러한 추상적 스키마는 문제 해결자에게 통합적 시각을 갖게 함으로써 문제를 바라보는 안목을 높이며, 아울러 유추 전이의 상승에 기여한다는 점에서 매우 중요한 것이라고 생각된다. 유사 문제에서 추상적 스키마를 구성하기 위해서는 등장하는 어떤 요소를 제외하고도 문제의 본질을 훼손하지 않는 것과 훼손하는 것을 찾아야 한다고 생각하였다. 이와 같은 관점에서 본 연구는 유사 문제에서 추상적 스키마의 구성을 돕는 방법을 설계하였다. 또한 그것을 한 학생에게 적용하여 그 방법의 가능성을 살펴보았다. 본 연구를 통해 학생들이 갖는 개별적 수학적 지식이라고 생각되는 요소들을 통합하는 것이 가능함을 확인하였다. 이는 기존 학자들(Gick and Holyoak, 1983; Kintsch & Dijk, 1978)이 언급한 추상적 스키마의 구성을 구체화하는 방법을 제시했다는 의의를 갖는다. 이런 결과는 향후 교수·학습 방법의 개선에 도움을 줄 것으로 기대된다.

주요용어 : 추상적 스키마 구성, 유사 문제 해결 과정, 구조적 유사성, 본질적 요소, 비본질적 요소

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