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Maximal United Utility Degree Model for Fund Distributing in Higher School

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ABSTRACT

The paper discusses the problem of how to allocate the fund to a large number of individuals in a higher school so as to bring a higher utility return based on the theory of uncertain set. Suppose that experts can assign each invested individual a corresponding nondecreasing membership function on a close interval I according to its actual level and developmental foreground. The membership degree at the fund $x \in I$ is called utility degree from fund x, and product (minimum) of utility degrees of distributed funds for all invested individuals is called united utility degree from the fund. Based on the above concepts, we present an uncertain optimization model, called Maximal United Utility Degree (or Maximal Membership Degree) model for fund distribution. Furthermore, we use nondecreasing polygonal functions defined on close intervals to structure a mathematical maximal united utility degree model. Finally, we design a genetic algorithm to solve these models.

Keywords: Uncertain Programming, Membership Function, Utility Degree, Higher School

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1. INTRODUCTION

The portfolio selection method was considered to allocate the fund to a large number of securities to bring a more profitable return. Since Markowitz published his path-breaking work in the early 1950s, the model has become a rather popular subject in both theory and practice (Abiyev and Menekay, 2007; Crama and Schyns, 2003; Deng *et al.*, 2005; Hirschberger *et al.*, 2007; Leung *et al.*, 2001; Li and Yang, 2004; Li *et al.*, 2000; Liu *et al.*, 2003; Xia *et al.*, 2000), and several algorithms for solving this problem have also been presented (Abiyev and Menekay, 2007; Lin and Liu, 2008; Perold, 1984; Qin *et al.*, 2009). In these models, their objective functions are the sum of random or fuzzy variables. And the basic idea of these models is to measure the return by expected value. In this paper, we study the problem of how to allocate the fund to a large number of individuals in a higher school so that the investment can bring a bigger utility return. We consider a different objective function of the problem with portfolio selection since it cannot be measured by the amount of incomes.

Recently, the uncertainty theory, as a branch of axiomatic mathematics satisfying normality, self-duality, countable subadditivity and product measure axioms, was proposed by Liu (2007) and refined by Liu (2010). Nowadays, it has been applied to uncertain programming (Liu, 2009a; Gao, 2011, 2012; Meng and Zhang, 2013; Peng and Yao, 2011; Rong, 2011; Sheng and Yao, 2012; Zhang and Chen, 2012; Zhang and Meng, 2013), uncertain risk analysis (Huang, 2011), uncertain logic (Chen *et al.*, 2012; Li and Liu, 2009), uncertain process (Zhang *et al.*, 2013), and others (Chen and Ralescu, 2011; Dai and Chen, 2012; Gao, 2009; Wang *et al.*, 2012; Zhang *et al.*, 2013; Zhu, 2010). Uncertain set can be applied to solve this problem. Therefore, the paper will present an uncertain optimization model about fund distribution for using membership functions of uncertain sets.

The remainder of this paper is organized as follows. Section 2 recalls some basic concepts and results about the uncertainty theory and uncertain set. In Section 3, we introduce the concepts of utility degree and united utility degree from the fund. From them we present an uncertain optimization model, called Maximal United Utility Degree (or Maximal Membership Degree) model (MUUDM) about fund distribution using product and minimum operators. In Section 4, we use nondecreasing polygonal functions defined on a close intervals to structure a material MUUDM, and design an arithmetic using genetic algorithm. Finally, a brief summary is given.

2. PRELIMINARIES

In this section, we will introduce some basic concepts and results about uncertainty theory and uncertain set theory.

Definition 1 (Liu, 2007). Let Γ be a nonempty set, and L a σ-algebra over Γ. Each element $\Lambda \in L$ is called an event. A set function $M: L \to [0, 1]$ is called an uncertain measure if it satisfies the following three axioms:

Axiom 1 (Normality axiom). $M\{\Gamma\} = 1$.

Axiom 2 (Duality axiom). $M\{\Lambda\} + M\{\Lambda^c\} = 1$ for any event Λ.

Axiom 3 (Subadditivity axiom). For every countable sequence of events $\{\Lambda_i\}$, we have

$$M\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty} M\{\Lambda_i\}.$$

The triplet (Γ, L, M) is called an uncertainty space. In order to obtain an uncertain measure of compound event, a product uncertain measure was defined by Liu (2009b), thus producing the fourth axiom of uncertainty theory:

Axiom 4 (Product axiom). Let (Γ_k, L_k, M_k) be uncertainty space for $k = 1, 2, \cdots, n$. Then the product uncertain measure on Γ is an uncertain measure on the product σ-algebra $Ł = Ł_1 \times Ł_2 \times \cdots \times Ł_n$ satisfying

$$M\left\{\prod_{k=1}^{n}\Lambda_k\right\} = \min_{1\le k\le n} M_k\{\Lambda_k\}.$$

Definition 2 (Liu, 2013). An uncertain set ξ is a measurable function from an uncertainty space $(\Gamma, Ł, M)$ to a collection of sets of real numbers i.e., for any Borel set B of real numbers, both of

$$\{B \subset \xi\} = \{\gamma \in \Gamma \mid B \subset \xi(\gamma)\}$$

and

$$\{\xi \subset B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \subset B\}$$

are events.

Definition 3 (Liu, 2013). An uncertain set is said to have a membership function μ if for any Borel set B of real numbers, we have

$$M\{B \subset \xi\} = \inf_{x\in B}\mu(x),$$
$$M\{\xi \subset B\} = 1 - \sup_{x\in B^c}\mu(x).$$

3. MAXIMAL UNITED UTILITY DEGREE MODEL

3.1 Problem Statement

In this section, we will study the problem of how to allocate the fund to a large number of individuals in a higher school so that the fund can bring a higher utility return based on the theory of uncertain set.

Suppose that fund a is distributed to n individuals in a higher school. In order to gain a higher utility return, we invite several experts to evaluate their utility return in the end of s years from developmental foreground, respectively. Uncertain sets ξ_j, $j = 1, 2, \cdots, n$ with nondecreasing membership functions $\mu_j(x)$, $j = 1, 2, \cdots, n$, are defined on interval $[b, c]$, respectively. The $\mu_j(x)$ is called the utility degree from fund x for jth invested individual, $j = 1, 2, \cdots, n$. From them we present the following uncertain optimization model, called MUUDM, about fund distributing for a higher school using product and minimum operators:

$$\begin{cases} \max & \prod_{1\le i\le n} u_i(x_i) \\ s.t. & \sum_{i=1}^{n} x_i = a, \\ & x_{i_1} = e_{i_1}, x_{i_2} = e_{i_2}, \cdots, x_{i_m} = e_{i_m} \in [b, c], \\ & 0 \le b \le x_i \le c \le a, \\ & \min_{1\le i\le n, i\ne i_j, j=1,2,\cdots,m} u_i(x_i) \ge d. \end{cases} \quad (1)$$

where $m, n, a, b, c, d, e_{i_j}$, $j = 1, 2, \cdots, m$ are constants, $\prod_{1\le j\le n}\mu_j(x_j)$ ($\wedge_{1\le i\le n} u_i(x_i)$) is called product (minimum) united utility degrees from the fund a, respectively.

We can see that the model is not complicated from external form. It is similar to models about avail function in economy. Note that the avail functions in economy take values in (0, +∞), and differentiable functions. However, in the model, the objective function takes values in [0, 1] and is a continuous function. The membership functions are chosen as nondecreasing polygonal functions.

4. A NUMERICAL EXAMPLE

For simplification, we first introduce an expressive method of polygonal functions as follows:

$$\mu(x)=\begin{cases}\alpha_1+\dfrac{(\alpha_2-\alpha_1)(x-x_1)}{x_2-x_1}, & if\ \ x_1\le x<x_2\\ \alpha_i+\dfrac{(\alpha_{i+1}-\alpha_i)(x-x_i)}{x_{i+1}-x_i}, & if\ \ x_i\le x<x_{i+1},\ 2\le i\le n-2\\ \alpha_{n-1}+\dfrac{(\alpha_n-\alpha_{n-1})(x-x_{n-1})}{x_n-x_{n-1}}, & if\ \ x_{n-1}\le x\le x_n\end{cases} \quad (2)$$

And it is noted by $\mu\{x_1, \alpha_1; x_2, \alpha_2; \cdots; x_n, \alpha_n\}$.

Now suppose that the fund 1000 dollar is distributed to 10 individuals in a higher school, and $50\le x_i\le 190$, $i=1, 2, \cdots, 10$, $x_9=50$, $x_{10}=80$, $x_5=110$, $x_3=120$, $x_4=130$, and $\underset{1\le i\le 10, i\ne 3,4,5,9,10}{\wedge} u_i(x_i)\ge 0.5$ are satisfied. In order to gain a higher united utility degree, we invite several experts to forecast their utility return in the end of 5 years, which are uncertain sets ξ_j, $j=1, 2, \cdots, 10$ with the following nondecreasing polygonal membership functions, respectively:

$d_1\{50, 0.2; 70, 0.4; 90, 0.5; 110, 0.7; 130, 0.89; 150, 1; 170, 1; 190, 1\}$
$d_2\{50, 0.2; 70, 0.4; 90, 0.5; 110, 0.7; 130, 0.8; 150, 1; 170, 1; 190, 1\}$
$d_3\{50, 0.4; 70, 0.75; 90, 0.85; 110, 0.95; 130, 0.97; 150, 1; 170, 1; 190, 1\}$
$d_4\{50, 0.3; 70, 0.6; 90, 0.88; 110, 0.95; 130, 1; 150, 1; 170, 1; 190, 1\}$
$d_5\{50, 0.2; 70, 0.4; 90, 0.75; 110, 0.85; 130, 0.94; 1; 150, 1; 170, 1; 190, 1\}$
$d_6\{50, 0.2; 70, 0.4; 90, 0.6; 110, 0.7; 130, 1; 150, 1; 170, 1; 190, 1\}$
$d_7\{50, 0.2; 70, 0.4; 90, 0.75; 110, 0.89; 130, 1; 150, 1; 170, 1; 190, 1\}$
$d_8\{50, 0.2; 70, 0.4; 90, 0.75; 110, 0.85; 130, 0.95; 150, 1; 170, 1; 190, 1\}$
$d_9\{50, 0.15; 70, 0.25; 90, 0.40; 110, 0.50; 130, 0.65; 150, 0.67; 170, 0.68; 190, 0.70\}$
$d_{10}\{50, 0.15; 70, 0.60; 90, 0.65; 110, 0.70; 130, 0.75; 150, 0.80; 170, 0.85; 190, 0.90\}$

Then we have the following MUUDM:

$$\begin{cases}\max & \prod_{1\le i\le 10} u_i(x_i)\\ s.t. & \sum_{i=1}^{10} x_i=1000,\\ & x_9=50,\ x_{10}=80,\ x_5=110,\ x_3=120,\ x_4=130\\ & 50\le x_i\le 190\le 1000,\\ & \min_{1\le i\le 10, i\ne 3,4,5,9,10} u_i(x_i)\ge 0.5.\end{cases} \quad (3)$$

i.e.,

$$\begin{cases}\max & \prod_{1\le i\le 10, i\ne 3,4,6,9,10,} u_i(x_i)\\ s.t. & x_1+x_2+x_6+x_7+x_8=510,\\ & x_9=50,\ x_3=120, x_{10}=80,\ x_5=110,\ x_4=130,\\ & 50\le x_i\le 190,\\ & \min_{1\le i\le 10, i\ne 3,4,5,9,10} u_i(x_i)\ge 0.5.\end{cases} \quad (4)$$

Since the objective functions in these models are not differentiable, we cannot use the Lagrange multiplier rule to solve these models. Therefore, we use a genetic algorithm to solve the uncertain programming model. The steps are listed as follows:

Step 1. Import array

$d[5][8]=\{\{0.2, 0.4, 0.5, 0.7, 0.89, 1, 1, 1\}, \{0.2, 0.4, 0.5, 0.7, 0.8, 1, 1, 1\}$
$\{0.2, 0.:90, 0.6, 0.7, 1, 1, 1, 1\}, \{0.2, 0.4, 0.75, 0.89, 1, 1, 1, 1\}$
$\{0.2, 0.4, 0.75, 0.85, 0.95, 1, 1, 1\}.\}$

Step 2. Initialize chromosomes: $x[5]=\{90, 90, 90, 90, 90\}$, $f0=0, e=0.$ and

$v[30][5]=\{\{0, 0, 0, 0, 0\}, \{0.1, -0.1, 0, 0, 0\},$
$\{-0.1, 0.1, 0, 0, 0\}, \{0, 0.1, -0.1, 0, 0\},$
$\{0, -0.1, 0.1, 0, 0\}, \{0, 0, -0.1, 0.1, 0\},$
$\{0, 0, 0.1, -0.1, 0\}, \{0, 0, 0, 0.1, -0.1\},$
$\{0, 0, 0, -0.1, 0.1\}, \{0.1, -0.1, 0.1, -0.1, 0\},$
$\{-0.1, 0.1, -0.1, 0.1, 0\}, \{-0.1, -0.1, 0.1, 0.1, 0\},$
$\{0.1, 0.1, -0.1, -0.1, 0\}, \{0.1, -0.1, 0.1, -0.1, 0\},$
$\{0, 0.1, -0.1, 0.1, -0.1\}, \{0, -0.1, 0.1, 0.1, -0.1\},$
$\{0, 0.1, -0.1, -0.1, 0.1\}, \{-0.1, 0, 0.1, -0.1, 0.1\},$
$\{0.1, 0, -0.1, 0.1, -0.1\}, \{-0.1, 0, 0.1, 0.1, -0.1\},$
$\{0.1, 0, -0.1, -0.1, 0.1\}, \{0, 0.1, -0.1, -0.1, 0.1\},$
$\{-0.1, 0.1, 0, -0.1, 0.1\}, \{0.1, -0.1, 0, 0.1, -0.1\},$
$\{-0.1, 0.1, 0, 0.1, -0.1\}, \{0.1, -0.1, 0, -0.1, 0.1\},$
$\{-0.1, 0.1, -0.1, 0, 0.1\}, \{0.1, -0.1, 0.1, 0, -0.1\},$
$\{-0.1, 0.1, 0.1, 0, -0.1\}, \{0.1, -0.1, -0.1, 0, 0.1\}.\}$

Step 3. Calculate the objective values: for each $k\in\{1, 2, \cdots, 30\}$, $n\in\{1, 2, \cdots, 9, 10\}$, $x[i]=90+10n\times v[k][i]$, $i=1, 2, 3, 4, 5$ if $(50+i\times 20)\le x[j]\le(50+(i+1)\times 20)$, $i=1, 2, 3, 4, 5, 6, 7, 8$, then we have

$$u_j(x[j])=d[j][i]+(v[j][i+1]-v[j][i])(x[j]-50-(i\times 20))/20,$$

thus

$$fn=\prod_{1\le j\le 5} u_j(x[j]).$$

If $fn>e$, then $e=fn$.

Step 4. Crossover operation for initialize chromosomes: for each $k\in\{1, 2, \cdots, 30\}$, we produce two random numbers, such as $1 \le c, d \le 5,$ then we change *c*th line and *d*th line in $v[k][5]$.

Step 5. Repeat the third to the fourth steps for a given cycles time 100.

Step 6. Report the best fund distribution is $x = \{100, 100, 120, 130, 110, 100, 105, 105, 50, 80\}$ with product united utility degree $0.705375,$ and it is maximal approximately, and $d1(100) \wedge d2(100) \wedge d6(100) \wedge d7(105) \wedge d8(105) = 5.15 > 0.5$ is satisfied.

5. CONCLUSION

This paper introduced a new concept of utility degree and united utility degree of the fund, and presented an uncertain optimization model for fund distributing in a higher school. In particular, we developed a mathematical model by using nondecreasing polygonal functions defined on close intervals. Finally, a genetic algorithm for solving the model was designed.

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