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A Transportation Problem with Uncertain Truck Times and Unit Costs

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ABSTRACT
Motivated by the emergency scheduling in a transportation network, this paper considers a transportation problem, in which, the truck times and transportation costs are assumed as uncertain variables. To meet the demand in the practical applications, two optimization objectives are considered, one is the total costs and another is the completion times. And then, a multi-objective optimization model is developed according to the situation in applications. Because there are commensurability and conflicting between the two objectives commonly, a solution does not necessarily exist that is best with respective to the two objectives. Therefore, the problem is reduced to a single objective model, which is an uncertain programming with a chance-constrain. After some analysis, its equivalent deterministic form is obtained, which is a nonlinear programming. Based on a stepwise optimization strategy, a solution method is developed to solve the problem. Finally, the computational results are provided to demonstrate the effectiveness of our model and algorithm.

Keywords: Emergency Management, Emergency Scheduling, Transportation Problem, Uncertain Programming

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1. INTRODUCTION

In recent years, the emergency management (crisis management) for an unconventional sudden event has become a hot topic for the governments and enterprises. Emergency management is a kind of conscious comprehensive prevention and control management when the human beings face disasters. The theoretical methods of quantitative analysis such as the mathematical modeling for risk assessment, measurement of information and other factors have become one of the most important core foundational scientific issues in emergency management.

Scheduling is the disposition of orders and quantities for actual requirements and the time-phased allocation of the resulting internal orders to the available resources. It requires reliable data and accurate information. Emergency scheduling is a key component of emergency management. For natural disasters or other special mission requirements such as war, atrocious weather and earthquake, etc., the decision-makers must reschedule the specific systems activities in case a disaster happens such that the loss is minimized. A specific system is often consumed in the process inaccurately measured in advance, which is often manifested in a complex of uncertainty.

When delivering emergency supplies, for example, path transportation time in some paths fraction may be subject to the traffic conditions and the possible impact of natural disasters. Generally, truck time on the path is uncertain, and in some cases may submit to random distribution function, such as normal distribution and uniform distribution.

Since many parameters of the emergency scheduling system are uncertain, sudden changes and diversity of events also cause the lack of data. There are some limitations when the traditional stochastic model deals with such problems. When an unconventional sudden event happens, the stochastic method is unsuitable. The present approaches mentioned above may be called as "conventional scheduling" (Deyi, 2012), which is suitable to the scheduling problem in the known conditions, including the determinative and stochastic situations. In that case, we have to invite some experts to evaluate their degree of belief that each event will occur. However, humans tends to overweight unlikely events (Kahneman and Tversky, 1979), thus the degree of belief may have a much larger range than the real frequency. In this situation, if we insist on dealing with the degree of belief using the probability theory, some counterintuitive results will be obtained (Liu, 2012).

In order to deal with the experts' degree of belief, the uncertainty theory was founded by Liu (2007) and refined by Liu in 2013. Many researchers have contributed to this area. The uncertainty theory has been applied to uncertain programming, uncertain risk analysis, uncertain game, uncertain inference, uncertain logic, uncertain finance, and uncertain optimal control. Nowadays, the uncertainty theory has become a branch of axiomatic mathematics to model human uncertainty (Liu, 2013).

Depending on the analysis as mentioned above, we think that it is necessary to apply the uncertainty theory as a basic approach to model the uncertainty in scheduling issues in emergency management. To differ from the approaches based on probability theory, Deyi (2012) introduced a concept of 'emergency scheduling' (Holmberg, 1995), that is, for an unconventional sudden event, the decision-makers employ the uncertainty theory (including uncertain programming, uncertain statistics, etc.) as a fundamental modeling tool to deal with the uncertainty in scheduling issues in emergency management, such an analysis approach known as emergency scheduling.

As a sort of the scheduling issues, the problem of transporting cargo from a set of supply nodes (factories) to a set of demand nodes (customers) so as to minimize linear transportation costs is well known. The basic transportation problem was originally developed by Hitchcock (Hoffman, 2007). In the conventional transportation problems, it is assumed that a decision maker is sure about the precise values of transportation cost, availability and demand of the product. In real world applications, however, all these parameters may not be known precisely due to uncontrollable factors. Recently, researching under the uncertain environment, stochastic demand has been introduced, and methods are developed (Goossens and Spieksma, 2009; Puri and Puri, 2006). The resulting problem may be called the stochastic transportation problem (STP). However, for the reasons mentioned above, in face of an unconventional sudden event, the decision making method based on uncertain theory should be more suitable to the issues.

In this paper, we will study a transportation problem in the uncertain environment and employ firstly the uncertain programming as a mathematical modeling tool to deal with the uncertain factors from the view of emergency management. The remainder of the paper is organized as follows. Section 2 introduces some preliminaries on the uncertainty theory for later use. Section 3 gives a more detailed description of the problems that we consider. Then, in Section 4, we present an approach to model these problems. Section 5 describes how we construct a feasible solution and apply this approach in Section 6 to a real life application. Section 7 ends this paper by giving our conclusions and suggestions for further research.

2. PRELIMINARIES

In this section, some basic definitions and arithmetic operations of uncertainty theory needed throughout this paper are presented.

Definition 1 (Liu, 2013). An uncertain variable ξ is a measurable function from an uncertainty space (Γ, L, M) to the set of real numbers, i.e., for any Borel set B of real numbers, the set $\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$ is an event.

For a sequence of uncertain variables $\xi_1, \xi_2, \cdots, \xi_n$ and a measurable function f, Liu (2013) proved that

$$\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$$

defined as $\xi(\gamma) = f(\xi_1(\gamma), \xi_2(\gamma), \cdots, \xi_n(\gamma)), \forall \gamma \in \Gamma$ is also an uncertain variable. In order to describe an uncertain variable, a concept of uncertainty distribution is introduced as follows.

Definition 2 (Liu, 2013). The uncertainty distribution Φ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathrm{M}\{\xi \le x\}$$

for any real number x.

Peng and Iwamura (2010) proved that a function $\Phi : \Re \to [0, 1]$ is an uncertainty distribution if and only if it is a monotone increasing function except for $\Phi(x) \equiv 0$ or $\Phi(x) \equiv 1$. The inverse function Φ^{-1} is called the inverse uncertainty distribution of ξ. Inverse uncertainty distribution is an important tool in the operation of uncertain variables.

Theorem 1 (Liu, 2013). Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent uncertain variables with regular uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively. If $f(x_1, x_2, \cdots, x_n)$ is strictly increasing with respect to $x_1, x_2, \cdots, x_m$ and strictly decreasing with respect to $x_{m+1}, x_{m+2}, \cdots, x_n$, then

$$\xi = f(\xi_1, \xi_2, \cdots, \xi_n)$$

is an uncertain variable with inverse uncertainty distribution

$$\Phi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \cdots, \Phi_m^{-1}(\alpha), \Phi_{m+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha)).$$

Expected value is the average of an uncertain variable in the sense of uncertain measure. It is an important index to rank uncertain variables.

Definition 3 (Liu, 2013). Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty} M\{\xi \ge r\} dr - \int_{-\infty}^{0} M\{\xi \le r\} dr$$

provided that at least one of the two integrals is finite.

In order to calculate the expected value via inverse uncertainty distribution, Liu and Ha (2010) proved that

$$E[\xi]=\int_0^1 f(\Phi_1^{-1}(\alpha),\cdots,\Phi^{-1}{}_m(1-\alpha),\Phi^{-1}{}_{m+1}(1-\alpha),\cdots,\Phi^{-1}{}_n(1-\alpha))d\alpha$$

under the condition described in Theorem 1. Generally, the expected value operator E has no linearity property for arbitrary uncertain variables. But, for independent uncertain variables ξ and η with finite expected values, we have

$$E[a\xi+b\eta]=aE[\xi]+bE[\eta]$$

for any real numbers a and b.

3. PROBLEM DESCRIPTION

Denote S as the set of *supply* nodes, with each supplying $a_i, i\in S$, and denote D as the set of *demand* nodes, with each demanding $d_j, j\in D$. For each pair consisting of supply node $i\in S$ and demand node $j\in D$, a unit cost $c_{ij}\geq 0$ is given. The problem is to send all supply to the demand nodes at a minimum total cost. That is the classical transportation problem. Due to its wide applicability and economic importance, the transportation problem has been extensively studied.

In this paper, the transportation problem will be modeled by uncertain programming in which the transportation time and cost are assumed to be uncertain variables with given uncertainty distributions, and at the same time, there are two optimization objectives to be considered, i.e., total cost and completion time of the emergency scheduling.

4. MODEL DEVELOPMENT

At first, we introduce the following notations to represent the mathematical formulation throughout the remainder of this paper.

- a_i : supply available at the *i*th supply point
- b_j : demand required at the *j*th demand point
- c_{ij} : freight cost involved when one unit of product is transported from the *i*th supply point to the *j*th demand point
- t_{ij} : transportation time from the *i*th supply point to the *j*th demand point
- x_{ij} : total number of units transported from the *i*th supply point to the *j*th demand point
- r_{ij} : capability transported from the *i*th supply point to *j*th demand point
- ξ_{ij} : independent uncertain transportation time from the *i*th supply point to the *j*th demand point
- Φ_{ij} : uncertainty distribution of ξ_{ij}

Next, based on the analysis of the decision making process, we have proposed the following transportation problem model,

$$min\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij},\ \max_{x_{ij}} t_{ij}(x_{ij})\right\} \quad (1)$$

s.t

$$\sum_{j=1}^{n}x_{ij}=a_i,\ (i=1,2,\cdots,m) \quad (2)$$

$$\sum_{i=1}^{m}x_{ij}=b_j,\ (j=1,2,\cdots,n) \quad (3)$$

$$0\leq x_{ij}\leq r_{ij},\ (i=1,2,\cdots,m\quad j=1,2,\cdots,n) \quad (4)$$

$$\sum_{i=1}^{m}a_i=\sum_{j=1}^{n}b_j \quad (5)$$

$$t_{ij}(x_{ij})=\begin{cases}t_{ij}, & if\quad x_{ij}>0\\ 0, & if\quad x_{ij}=0.\end{cases}$$

The problem is now to minimize both of the total cost and the completion time. The constraints are linear with a simple structure. That is a multi-objective optimization problem. In such problem, multiple objective functions need to be optimized simultaneously. Because of incommensurability and conflicting objectives, a solution does not necessarily exist that is best with respective to all objectives. Therefore, for this multi-objective optimization problem, we seek to reduce it into a single objective problem.

For simplicity, we write $\xi=\{\xi_{ij},(i,j)\in A\}$ and $x=\{x_{ij},(i,j)\in A\}$. Let $T(x,\xi)=\max_{x_{ij}>0}\xi_{ij}$ denote the completion time and $\varphi^{-1}(x,\alpha)$ denote the inverse uncertainty of $T(x,\xi)$. Let $C(x,\xi)=\sum_{i=1}^{m}\sum_{j=1}^{n}c_{ij}x_{ij}$ denote the function of the total cost and $\Psi^{-1}(x,\alpha)$ denote the inverse uncertainty distribution of $C(x,\xi)$.

The formulation of the problem is then:

$$\begin{cases}\min_x E[T(x,\xi)] & (6)\\ s.t & \\ M\{C(x,\xi)\leq C_0\}\geq\alpha_0 & (7)\\ Ax=B & (8)\\ 0\leq x\leq R & (9)\end{cases}$$

In this model, the objective is to minimize the expected completion time, another objective is treated as a chance constraint and other constraints remain in the model.

Some additional notations and function expressions are listed as follows:

$$A=\{(i,j)|i\in S,j\in D\}$$

k_{ij} : the original cost transporting from depot i to destination j
h : the coefficient of the transportation cost
$c_{ij} = h_{ij} + h\xi_{ij}$, transportation cost each unit from depot i to destination j
$C(x, \xi) = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$, the function of the total cost
$T(x, \xi) = \max_{x_{ij}>0} \xi_{ij}$, the completion time function
C_0 : budget constraint
R : transportation capability
α_0 : a predetermined confidence level

In order to analyze and solve this model (6–9), firstly, we introduce two corollaries which are from the uncertainty theory (Liu, 2013).

Corollary 1. Assume the objective function $f(x, \xi_1, \xi_2, \cdots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \cdots, \xi_m$ and strictly decreasing with respect to $\xi_{m+1}, \xi_2, \cdots, \xi_n$. If $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with uncertainty distribution $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively, then the expected objective function $E[f(x, \xi_1, \xi_2, \cdots, \xi_n)]$ is equal to

$$\int_0^1 f(x, \Phi_1^{-1}, \cdots, \Phi_m^{-1}, \Phi_{m+1}^{-1}, \cdots, \Phi_n^{-1})d\alpha.$$

Corollary 2. Assume the constraint function $g(x, \xi_1, \xi_2, \cdots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \cdots, \xi_k$ and strictly decreasing with respect to $\xi_{k+1}, \xi_2, \cdots, \xi_n$. If $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively, then the chance constrain

$$M\{g(x, \xi_1, \xi_2, \cdots, \xi_n) \le 0\} \ge \alpha$$

holds if and only if

$$g(x, \Phi_1^{-1}(\alpha), \cdots, \Phi_k^{-1}(\alpha), \Phi_{k+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha)) \le 0.$$

Secondly, we give the following theorems.

Theorem 2. The expected objective function $E[T(x, \xi_{ij})]$ in the model (6–9) is equal to

$$\int_0^1 T(x, \Phi_{ij}^{-1})d\alpha (i = 1, 2, 3, \cdots, m \quad j = 1, 2, 3, \cdots, n)$$

proved that $E[T(x, \xi_{ij})]$ *exists.*

Proof. Owing to the assumption before, the function Φ_{ij} is strictly increasing with respect to ξ_{ij} and ξ_{ij} are independent uncertain variables with distribution Φ_{ij} respectively. By using Corollary 1, we obtain the theorem. □

Theorem 3. The chance constraint

$$M\{C(x, \xi_{ij}) \le C_0\} \ge \alpha$$

in the model (6–9) is equivalent to

$$C(x, \Phi_{ij}^{-1})(i = 1, 2, \cdots, m \quad j = 1, 2, \cdots, n).$$

Proof. Owing the assumption before, the function $C(x, \xi_{ij})(i = 1, 2, \cdots, m, \ j = 1, 2, \cdots, n)$ is strictly increasing with respect to ξ_{ij}. And ξ_{ij} are independent uncertain variables with uncertainty distributions Φ_{ij}, respectively. Since the inverse uncertainty distribution of $C(x, \xi_{ij})$ is $\Psi^{-1}(x, \alpha) = C(x, \Phi^{-1}(\alpha))$.

$$M\{C(x, \xi_{ij}) \le C_0\} = M\{C(x, \xi_{ij}) - C_0 \le 0\}.$$

Let $g(x, \xi_{ij}) = C(x, \xi_{ij}) - C_0 (i = 1, 2, \cdots, m, \ j = 1, 2, \cdots, n)$. Since that $M\{C(x, \xi_{ij}) \le C_0\} \ge \alpha$ is equivalent to $M\{g(x, \xi_{ij}) \le 0\} \ge \alpha$. It follows from Corollary 2 that we have $g(x, \Phi_{ij}^{-1}) \le 0$.

It is easy to prove that $g(x, \Phi_{ij}^{-1}) = C(x, \Phi_{ij}^{-1}) - C_0$, so, $C(x, \Phi_{ij}^{-1}) - C_0 \le 0$, that is, $C(x, \Phi_{ij}^{-1}) \le C_0$.

The theorem is proved. □

Theorem 4. The uncertain programming

$$\begin{cases} \min_x E[T(x, \xi_{ij})] \\ s.t \\ M\{C(x, \xi_{ij}) \le C_0\} \ge \alpha \end{cases}$$

in the model (6–9) is equivalent to the crisp mathematical programming

$$\begin{cases} \min_x \int_0^1 \Phi_{ij}^{-1}(x, \alpha)d\alpha \\ s.t \\ \Psi^{-1}(x, \alpha) \le C_0 \end{cases}.$$

Proof. It follows from Theorems 2 and 3 immediately. □

The model is changed to deterministic form by the Theorems 4, then, this model is equivalent to

$$\begin{cases} \min_x \int_0^1 \Phi_{ij}^{-1}(x, \alpha)d\alpha \\ s.t \\ \Psi^{-1}(x, \alpha) \le C_0 \\ Ax = B \\ 0 \le x \le R \end{cases}$$

which is a deterministic nonlinear programming.

5. SOLUTION METHOD

To solve the model, based on stepwise optimization strategy, we develop a solution algorithm. The heuristic algorithm is described as follows:

Step 1: According to the known values of α, h and the $\Phi_{ij}(\alpha)$ of uncertainty distributions of the ξ_{ij}, we can obtain the inverse uncertainty distribution $\Psi^{-1}(x, \alpha)$ of the cost function $C(x, \xi)$, and since the total cost $\boldsymbol{C}_0$ is known, we can gain inequality constraint $\Psi^{-1}(x, \alpha) \le C_0$.

Step 2: Using the constraint of the balance of supply and demand, we can obtain the equality constraints of $Ax = B$.

Step 3: Basing on the capacity constraint $0 \le x \le R$, and step 1 and step 2, we can immediately get the feasible set of the objective function: the n groups solutions of decision variables, $X_1, X_2, \cdots, X_n$.

Step 4: Return each group of values of X_i to the completion time function $T(x,\xi) = \max_{x_{ij}>0} \xi_{ij}$, then we can obtain a set of $E[T(x, \xi)] = \int_0^1 \Phi^{-1}(x, \alpha) d\alpha$ which contains n elements, and then we can select the minimum one from the set. The minimum value is just the least completion time.

Step 5: The X_i is corresponding to the minimum $E[T(x, \xi)] = \int_0^1 \Phi^{-1}(x, \alpha) d\alpha$ which is calculated from the step 4, its value is the volume that corresponding route transports cargo. Then calculate the real cost based on X_i.

Form the step 1, the constraints of the model are changed to deterministic form, step 2 and step 3 give the feasible solutions, and its calculating complexity is $O((mn)^2)$. Then, the complexity of selecting the minimum time of completed task is $O(mn^2)$. In all, it is a nonlinear programming, and the complexity is $O((mn)^2) + O(mn^2) = O(m^2n^2)$. The complexity of the algorithm is a polynomial.

6. ILLUSTRATIVE EXAMPLE

To test how well the model and the solution algorithms may be applied in the real world, we performed numerical tests based on the domestic operation of a department with reasonable assumptions.

Consider a transportation problem with respect to emergency scheduling in which there are two supply nodes and three demand nodes. Assume that the truck times are linear uncertain variables,

$$\xi_{ij} \sim \mathrm{L}(\underline{t}_{ij}, \overline{t}_{ij}),$$

and their values are following,

i	j	$\underline{t}_{ij}$	$\overline{t}_{ij}$
1	1	2	3
	2	3	5
	3	4	7
2	1	3	4
	2	4	6
	3	5	8

We assume that $h = 0.1$, $h_{ij} = 0$ ($i = 1, 2\ j = 1, 2, 3$), $r_{ij} = 200$ ($i = 1, 2, j = 1, 2, 3$), $C_0 = 400$, and the confidence level $\alpha_0 = 0.95$. Then we can get the optimal solution is $x_{11} = 101.9$, $x_{12} = 143.3$, $x_{13} = 54.8$, $x_{21} = 73.1$, $x_{22} = 81.7$, $x_{23} = 45.2$, $C = 250.375$, and $T = 6.5$.

That is to say we should transport 101.9 from A_1 to B_1, 143.3 from A_1 to B_2, 54.8 from A_1 to B_3, 73.1 from A_2 to B_1, 81.7 from A_2 to B_2, and 45.2 from A_2 to B_3. The total cost is 250.375, and the completion time is 6.5 days.

7. CONCLUSION

To consider a cargo transportation problem under conditions of emergency scheduling that occurs in actual operations, we established an uncertain programming model. Based on the stepwise optimization strategy, a heuristic algorithm was developed to solve the model. Numerical test was performed to evaluate the model and the solution algorithm. The test results show that the model and its heuristic solution are all effective.

There are some suggestions for future research: 1) more uncertain parameters in a system. Such as, demands and supplies, etc.; 2) more complex transportation networks, to be more suitable to real situations in emergency management; 3) integrating multi-decision to a model, such as, production and transportation, transportation and stock, etc.; and 4) integrated uncertain and stochastic model.

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