Cross Layer Optimal Design with Guaranteed Reliability under Rayleigh block fading channels

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Abstract

Configuring optimization of wireless sensor networks, which can improve the network performance such as utilization efficiency and network lifetime with minimal energy, has received considerable attention in recent years. In this paper, a cross layer optimal approach is proposed for multi-source linear network and grid network under Rayleigh block-fading channels, which not only achieves an optimal utility but also guarantees the end-to-end reliability. Specifically, in this paper, we first strictly present the optimization method for optimal nodal number N*, nodal placement d* and nodal transmission structure p* under constraints of minimum total energy consumption and minimum unit data transmitting energy consumption. Then, based on the facts that nodal energy consumption is higher for those nodes near the sink and those nodes far from the sink may have remaining energy, a cross layer optimal design is proposed to achieve balanced network energy consumption. The design adopts lower reliability requirement and shorter transmission distance for nodes near the sink, and adopts higher reliability requirement and farther transmission distance for nodes far from the sink, the solvability conditions is given as well. In the end, both the theoretical analysis and experimental results for performance evaluation show that the optimal design indeed can improve the network lifetime by 20-50%, network utility by 20% and guarantee desire level of reliability.

Keywords: Wireless sensor networks, utilization, cross layer optimal, reliability, network lifetime

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1. Introduction

Wireless Sensor Networks (WSNs) have captured considerable attention recently due to their enormous potential for environmental monitoring, surveillance operations, and industrial automation [1-3]. Configuring optimization, as it can improve the network performance, has been well discussed in many WSN applications [4-5]. In order to prolong the network lifetime, several optimization measures have been proposed, e.g., Zhang et al. [6] show that the network performance can be significantly improved by optimizing network parameters including the deployed nodal number $N*$, the nodal placement d^* and the nodal transmission structure P^*. In order to achieve the optimal transmission range in physical layer, Chen et al. [7] define the optimal one-hop length for multi-hop communications, which not only minimizes the total energy consumption but also analyzes the influence of channel parameters on the optimal transmission range in a linear network. In [8], a Bit-Meter-per-Joule metric is proposed, which enables us to receive the effects of the network topology, the nodal density and the transceiver characteristics on the overall energy expenditure. Regarding the utility based optimization, Chen et al. [9] introduce a performance measure of utilization efficiency defined as network lifetime per unit deployment cost. In addition, a separate tradeoff between network lifetime and the end-to-end delay is addressed in [10].

Different from the above configuring optimization, in this paper, we would like to propose a cross-layer optimal approach for multi-source linear network and grid network in WSNS, which not only achieves an optimal utility but also guarantees the end-to-end reliability. Specifically, the main contributions of this paper are threefold:

(1) It is proved strictly from mathematics that there exist optimal nodal number $N*$, nodal placement d^* and nodal transmission structure P^* which can meet minimum in not only total energy consumption for data collection but also unit data transmitting energy consumption.

(2) A cross layer optimal design is proposed, which can improve dramatically lifetime without increasing network cost. Based on the network energy consumption feature, i.e., the energy consumption near the sink is high, while energy consumption far from the sink is low, we can know that in regions near the sink, energy consumption is inadequate, while energy consumption is excess in regions far from the sink. Therefore, two measures are adopted to address this problem. Firstly, we decrease nodal transmission distance for nodes near the sink and increase nodal transmission distance for nodes far from the sink in order to balance the nodal energy consumption in the network. Secondly, we adopt the strategy on lower reliability requirement for nodes near the sink while higher reliability requirement for nodes far from the sink to decrease the nodal energy consumption near the sink and improve the nodal energy consumption far from the sink.

(3) Extensive theoretical analyses and simulation evaluations are conducted, and the results demonstrate that the design of both utilization performance and reliability can be achieved simultaneously.

The rest of this paper is organized as follows. In Section 2, the system model and problem statement are described. In Section 3, optimizations for network and channel are presented. In Section 4, we present the analysis and comparison of experimental results. In Section 5, we conclude the paper.

2. The System Model and Problem Statement

2.1 Energy Consumption Model

According to [9], the energy consumption for transmitting one packet E_p is composed of three parts: the energy consumed by transmitter E_t, receiver E_r and the acknowledgment packet exchange E_{ACK}, i.e.:

$$E_p = E_t + E_r + E_{ACK} \tag{1}$$

The energy model for transmitters and receivers are given respectively by:

$$E_t = T_{start} . P_{start} + \frac{N_{head} + N_b}{R_b . R_{code}} \cdot \left(P_{txElec} + \beta_{amp} . P_t \right) \tag{2}$$

$$E_r = T_{start} . P_{start} + \frac{N_{head} + N_b}{R_b . R_{code}} \cdot P_{txElec} \tag{3}$$

where P_t is the transmission power, N_{head} is the number of bit in the overhead of a packet for the synchronization of physical layer, R_{code} is the code rate. The other parameters are described in **Table 1**.

Table 1. Some parameters in the transceiver energy consumption

Symbol	Description	Value
α	Path-loss exponent(≥ 2)	3
β_{amp}	Amplifier proportional offset(>1)	14.0
τ_{ack}	ACK ratio	0.08125
B	Bandwidth of channel	250kHz
f_c	Carrier frequency	2.4GHz
λ	the wavelength	
G_{Tant}	Transmitter antenna gain	1
G_{Rant}	Receiver antenna gain	1
L	Circuitry loss (≥ 1)	1
N_b	Number of bits per packet	2560
N_{head}	Number of bits of overhead in a packet	0
N_0	Noise level	-150dBm/Hz
P_{start}	Startup power	38.7mV

P_{txElec}	Transmitter circuitry power	59.1mV
P_{rxElec}	Receiver circuitry power	59.1mV
R_b	Transmission bit power	250Kbps
T_{start}	Startup time	0 μs
T_{ACK}	ACK duration	1ms

The energy expenditure model of an acknowledgment is given by:

$$E_{ACK}=\tau_{ack}\cdot(E_t+E_r)\ ,\ \tau_{ack}=\frac{N_{ack}+N_{head}}{N_b+N_{head}}$$

(4)

In [6], the energy model for each bit is:

$$E_b=\frac{E_p}{N_b}=E_c+K_1\cdot P_t \tag{5}$$

Put (1)–(4) into (5), we have

$$E_c=(1+\tau_{ack})\left(\frac{2T_{start}\cdot P_{start}}{N_b}+(1+\tau_{head})\frac{P_{txElec}+P_{rxElec}}{R_b R_{code}}\right) \tag{6}$$

and

$$K_1=(1+\tau_{ack})(1+\tau_{head})\frac{\beta_{amp}}{R_b R_{code}}\ \ ,\ \ \tau_{head}=\frac{N_{head}}{N_b}\ . \tag{7}$$

2.2 Realistic Unreliable Link Model

The realistic unreliable link model is also the same as that in literatures [6, 11]. The unreliable radio link probability (pl) is defined as the packet error rate (PER) [6]:

$$pl(\gamma_{x,x'})=1-PER(\gamma_{x,x'})\quad,\ \gamma_{x,x'}=K_2\cdot P_t\cdot d_{hop}^{-\alpha} \tag{8}$$

where $K_2=\dfrac{G_{Tant}\cdot G_{Rant}\cdot\lambda^2}{(4\pi)^2\cdot N_0\cdot R_s\cdot L}$. d_{hop} is the distance between node x and x', λ is the wavelength, R_s is the symbol rate. Other parameters are the same as those in **Table 1**. Note that $R_b=R_s\cdot b$, where b is the modulation order. The unreliable link models are approximated to Rayleigh block fading channels as follows [9].

$$pl_b(\gamma)=\exp(\frac{-4.25\log_{10}N_b+2.2}{\beta_m\gamma})\qquad\text{when }\alpha_m=1\ . \tag{9}$$

where α_m and β_m rely on the modulation type and order, e.g., for Multiple Quadrature Amplitude Modulation (MQAM) am α_m = $4\left(1-1/\sqrt{M}\right)\Big/\log_2(M)$ and β_m = $3\log_2(M)/(M-1)$. For BPSK, α_m = 1 and β_m = 2.

2.3 Problem Statement

(1) The total energy consumption for each source node transmitting one bit data to the sink is defined as E_{tot}.

(2) Energy consumption rate ξ is defined as transmitting one bit data to the sink with energy consumption E_{tot} divided by the number of nodes (n) participating in transmission, i.e.,

$$\xi = \frac{E_{tot}}{n} \tag{10}$$

(3) Network lifetime ℓ is defined as the average amount of time until any sensor runs out of energy (the first failure) [9. 13]. Utilization efficiency η is defined as network lifetime ℓ divided by the number of deployed sensors (N), i.e.,

$$\eta = \frac{\ell}{N} \tag{11}$$

Utilization efficiency (η) indicates the rate at which network lifetime (ℓ) increases with the number of nodes (N). It causes the tradeoff between network lifetime and deployment cost.

The design goal is to find the optimal node number $N*$, sensor placement d^*, and transmission structure P^* which can minimize E_{tot} and ξ, while maximizing utilization efficiency η:

$$\left\{N*,\ d^*, P^*\right\} = \arg\ \left\{\min_{\mathrm{N,d,P}}(E_{tot}, \xi), \max_{\mathrm{N,d,P}}(\eta)\right\} \tag{12}$$

At the same time, the network has to ensure the end-to-end reliability to meet the minimum requirements of the application, such as C, i.e..

$$\gamma = \max\left(\prod_{i=1}^{k}\gamma_i\right) > C \tag{13}$$

In summary, the optimization goal of this paper is shown as follows:

$$\begin{cases} \left\{N*,\ d^*, P^*\right\} = \arg\ \left\{\min_{\mathrm{N,d,P}}(E_{tot}, \xi), \max_{\mathrm{N,d,P}}(\eta)\right\} \\ s.t. \quad \gamma = \min\left(\prod_{i=1}^{k}\gamma_i\right) > C \end{cases} \tag{14}$$

3. Scheme Design

3.1 Multi-source Linear Network

The multi-source linear network is that each node in the network is deployed to monitor the surrounding environment and generates a sensed data in each cycle. Such linear networks are widely applied into applications such as roads, oil pipelines and border detection. Many routes such as shortest route, HEED route which are widely used in two-dimensional network can also be considered in a linear network. Thus, this research has an important significance and is referred as a multi-source linear network in this paper. As shown in **Fig. 1**, there are n nodes linearly deployed in the network, each node generates one data in a data collection round (cycle), and then transmits it to the sink. For S_1 nearest to the sink, the data load is n data packets, and for S_2, it is n-1 data packets, ... for S_n, the data load is one data packet.

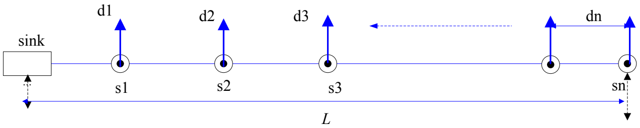

Fig. 1. Illustration of the line network of each node as source

First, we discuss on how to decrease d_{hop} for nodes near the sink and increase d_{hop} for nodes far from the sink. As shown in **Fig. 2**, the nodal data load is much higher for nodes near to the sink, therefore the transmission distance can be decreased in order to reduce the energy consumption for unit data transmission.

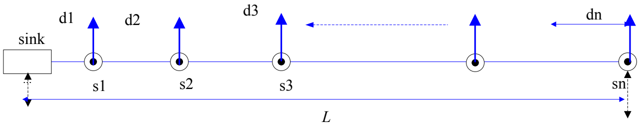

Fig. 2. Illustration of the un-equidistant linear network

Theorem 1: The total energy consumption is minimal when nodes are equidistantly deployed.

Proof: Assuming the number of equidistantly deployed nodes is n, then the energy consumption for node i is:

$$E_i = (n-i+1)N_b\left(E_c + K_1 \cdot P_t\right) \tag{15}$$

First, when nodes are equidistant, the total energy consumption is as follows:

$$\sum_{i=1}^{n} E_i = \sum_{i=1}^{n}\left((E_c + \frac{K_1 n d_{hop}^{\alpha}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C})(n+1-i)N_b \right) \tag{16}$$

subject to $nd_{hop} = D$.

Proof (16): Assuming the distance between any two nodes is d_{hop}, then data is sent to the sink via D/d_{hop} hops. To meet reliability (more than or equal to C), the following should be ensured.

$$pl(\gamma)^{\frac{D}{d_{hop}}} \geq C \tag{17}$$

Put (9) into (17) we can derive :

$$\gamma \geq \frac{n(-4.25\log_{10} N_b + 2.2)}{\beta_m \ln C} \tag{18}$$

Put (18) into (8), we have:

$$P_t \geq \frac{n d_{hop}^{\alpha}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C} \tag{19}$$

Where γ is the S/N (signal-to-noise ratio), P_t is the transmission power, and $pl_g(\gamma)$ is the reliability. Therefore

$$P_t(\min) = \frac{n d_{hop}^{\alpha}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C} \tag{20}$$

Put (20) into (15), we get (21).

While if the nodes are not equidistant, there is:

$$P_{t_i}(\min) = \frac{n d_i^{\alpha}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C}$$

Put the above formula into (15),we get:

$$\sum_{i=1}^{n} E_i = \sum_{i=1}^{n}\left((E_c + \frac{K_1 n d_i^{\alpha}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C})(n+1-i)N_b \right) \tag{21}$$

subject to $d_1 + d_2 + ... + d_n = D$.

The following need to be proved:

$$\sum_{i=1}^{n}\left((E_c + \frac{K_1 n d_i^{\alpha}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C})(n+1-i)N_b \right) \geq \sum_{i=1}^{n}\left((E_c + \frac{K_1 n d_{hop}^{\alpha}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C})(n+1-i)N_b \right) \tag{22}$$

That is, we need to prove:

$$\frac{n(n+1)E_c}{2} + \sum_{i=1}^{n}\left(\frac{K_1 n d_i^{\alpha}(-4.25\log_{10} N_b + 2.2)(n+1-i)}{K_2 \beta_m \ln C} \right) \geq \frac{n(n+1)E_c}{2} + \sum_{i=1}^{n}\left(\frac{K_1 n d_{hop}^{\alpha}(-4.25\log_{10} N_b + 2.2)(n+1-i)}{K_2 \beta_m \ln C} \right)$$

Reorganize the above, we get:

$$\sum_{i=1}^{n} d_i^{\alpha}(n+1-i) \geq \left(\frac{(n+1)n}{2} \right) d_{hop}^{\alpha} \tag{23}$$

Then, we prove (23):

$$\text{Min } E_{tot} = \sum_{i=1}^{n} d_i^{\alpha}(n+1-i),$$

$$\text{Subject to } d_1 + d_2 + ... + d_n = D.$$

Set $F = E_1 + E_2 + ... + E_n + \lambda(d_1 + d_2 + ... + d_n - D)$, where $\lambda \neq 0$ is Lagrange multiplier. According to Lagrange multipliers:

$$\begin{cases} \dfrac{\partial E_1}{\partial d_1} + \lambda = 0 \\ ... \\ \dfrac{\partial E_n}{\partial d_n} + \lambda = 0 \\ S.t.\ d_1 + d_2 + ... + d_n = D \end{cases} \tag{24}$$

(24) shows that when $\partial E_1 / \partial d_1 = ... = \partial E_n / \partial d_n = -\lambda$, the minimum F can be obtained, since nodes are all the same in linear network, so we define $E_i = d_i^{\alpha}(n+1-i)$,therefore

$$\frac{\partial^2 E_i}{\partial d_i^2} = (n+1-i)\cdot\alpha\cdot(\alpha-1)d_i^{\alpha-2} > 0$$

Then it is clear that when $\alpha > 2$, if the above formula is bigger than 0, then $\partial E / \partial d_i$ is a function with d_i which is strictly monotonically increasing, and then (24) is solvable, $d_1 = d_2 = ... = d_n = D/n$. Thus,

$$E_{tot}(\min) = \frac{(n+1)nd_{hop}^{\alpha}}{2} \quad (d_{hop} = D/n)$$

So far, (23) is proved. As can be seen from previous proof, (22) is correct. Thus, the total energy consumption is the minimum when nodes are equidistantly deployed.

■

Theorem 2: For multi-source linear network, there must be a d_{hop} which minimizes E_{tot} in $(0, D]$, while $E_{tot} = \sum_{i=1}^{n} E_i = \sum_{i=1}^{n}(n-i+1)N_b\left(E_c + K_1 \cdot P_t\right)$.

Proof: The total energy consumption is the minimum when nodes are equidistantly deployed, that is:

$$E_{tot} = (E_c + \frac{K_1 D d_{hop}^{\alpha-1}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C}) \cdot \frac{n(n+1)}{2} \tag{25}$$

Obviously, when $d_{hop} \to 0$, since $nd_{hop} = D$,so $n \to +\infty$, $n(n+1)/2 \to +\infty$.While when $d_{hop} \to 0$, $\frac{K_1 D d_{hop}^{\alpha-1}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C} \to 0$,so

$E_c+\dfrac{K_1 D d_{hop}^{\alpha-1}(-4.25\log_{10}N_b+2.2)}{K_2\beta_m\ln C}\to E_c$.Thus we have $E_{tot}\to+\infty$.And when $d_{hop}=D$, $n=1$,since $E_{tot}=E_c+\dfrac{K_1D^{\alpha}(-4.25\log_{10}N_b+2.2)}{K_2\beta_m\ln C}$,so E_{tot} is bounded and E_{tot} is continuously derivative in $(0,D]$, so there must be a $d_{hop}\in(0,D]$ which minimizes E_{tot}.

■

Theorem 1 proves that the network total energy consumption is optimal when n nodes are equidistantly deployed. Theorem 2 shows that there must be an optimal nodal distance (d_{hop}) which can minimize network total energy consumption. According to the definition in Section 2 we can see the second goal is to maximize ξ, which is to minimize unit nodal energy consumption. While the energy consumption for unit node is: $\xi=E_{tot}/n$.

Theorem 3: For multi-source linear network in Rayleigh block fading channels, there is a d_{hop} which can minimize ξ in $(0,D]$.

Proof: Put $n=D/d_{hop}$ into (25) and reorganize it, we can get:

$$\xi=E_c\left(\frac{1}{2}+\frac{D}{2d_{hop}}\right)+\frac{K_1Dd_{hop}^{\alpha-2}(-4.25\log_{10}N_b+2.2)(D+d_{hop})}{2K_2\beta_m\ln C} \tag{26}$$

Obviously, when $d_{hop}\to0$, we have $\xi\to+\infty$,when $d_{hop}=D$, since ξ is bounded, and ξ is continuously derivative in $(0,D]$,then there must be a $d_{hop}\in(0,D]$ which minimizes ξ (The certification process is similar to theorem 2).

■

When n is determined, the network lifetime is determined by the node which has the maximum energy consumption, while in multi-source network, it is the node which is nearest to the sink. Therefore, the network utilization optimization is to minimize the energy consumption of this node, that is, $\min\ \max(E_i)\,|\,i\in\{1..n\}$.

Theorem 4: For multi-source linear network, to solve d_1, d_2, … d_n which achieves $\min\ \max(E_i)\,|\,i\in\{1..n\}$, s.t. $d_1+d_2+\ldots+d_n=D$, $d_0\le d_1\le d_2\le,\ldots\le d_n$ is to solve the following:

$$\begin{cases} nM_1 d_1^{\alpha-1} - (n-1)M_1 d_2^{\alpha-1} + E_c = 0 \\ (n-1)M_1 d_2^{\alpha-1} - (n-2)M_1 d_3^{\alpha-1} + E_c = 0 \\ \cdots \\ 2M_1 d_{n-1}^{\alpha-1} - M_1 d_n^{\alpha-1} + E_c = 0 \\ \sum_{i=1}^{n} d_i = D, M_1 = \dfrac{K_1 D(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C} \end{cases} \tag{27}$$

Proof: The network lifetime is the maximum when all nodal energy consumption equals, the following formula can be obtained:

$$\begin{cases} E_1 = E_2, E_2 = E_3 = \ldots = E_{n-1} = E_n \\ S.t \quad \sum_{i=1}^{n} d_i = D \end{cases}$$

(28)

Then: $$E_i = N_b(n+1-i)\left(E_c + \frac{K_1 D d_i^{\alpha-1}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C}\right)$$

Set $$M_1 = \frac{K_1 D(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C}$$

Put the above formula into the equation set (28), that is (27).

■

For instance, in the network when D=240, C=0.9, n=6, then the following data can be got:

$d_1 = 24.6450, d_2 = 27.8566, d_3 = 32.0767, d_4 = 38.0849, d_5 = 47.8909, d_6 = 69.4462$. Compare these two schemes, the proportion of declined energy is:

$$\phi = \frac{\max(E_i^1) - \max(E_i^2)}{\max(E_i^1)} = 54.07\% .$$

Theorem 4 has proved that η can be improved by decreasing d_{hop} for nodes near the sink and increasing d_{hop} for nodes far from the sink. Similarly, the energy consumption for node i near the sink can be decreased by decreasing nodal reliability c_i, and increase energy consumption for nodes far from the sink by using remaining energy, to ensure the reliability C of the entire routing meet the requirement of applications. Then, Theorem 5 can be derived.

Theorem 5: For multi-source linear network in Rayleigh block fading channels, there must be $c_1 \le c_2 ... \le c_n$ which achieves $\min \max\left(E_i\right) | i \in \{1..n\}$ s.t. $\prod_{i=1}^{n} C_i = C, C < 1, d_i = D/n$.

Proof: If there are n nodes and the energy consumption is balanced, thus we can derive the following :

$$\begin{cases} E_1 = E_2 = ... = E_{n-1} = E_n \\ S.t \quad \prod_{i=1}^{n} C_i = C \end{cases}$$

And
$$E_i = N_b(n+1-i)(E_c + \frac{K_1 D d_i^{\alpha-1}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C_i}) \tag{29}$$

Set
$$M_2 = \frac{K_1 D d_{hop}^{\alpha-1}(-4.25\log_{10} N_b + 2.2)}{\beta_m K_2}$$

Put the formula into (29) ,then put result into the equations which above (29) ,arrange it we can get:

$$C_i = \exp(\frac{(n-i+1)M_2 \ln C_1}{(i-1)E_c \ln C_1 + nM_2}) \quad (i \ge 2)$$

Set
$$H_1(C_1) = C_1 \cdot \exp(\sum_{i=2}^{n} \frac{(n-i+1)M_2 \ln C_1}{(i-1)E_c \ln C_1 + nM_2}) - C$$

Obviously,when $C_1 = 0$, $\lim_{C_1 \to 0} C_1 \cdot \exp(\sum_{i=2}^{n} \frac{(n+1-i)M_2 \ln C_1}{(i-1)E_c \ln C_1 + nM_2}) = 0$,so $H_1(C_1) < 0$; When $C_1 = 1$, $\ln C_1 = \ln 1 = 0$,so $H_1(C_1) = 1 \cdot \exp(0) - C = 1 - C$,thus $H_1(C_1) > 0$, since $H_1(C_1)$ is a continuous function, then there must be a solution in $C_1 \in [0,1]$ which achieves $H_1(C_1) = 0$. ■

3.2 Grid Network

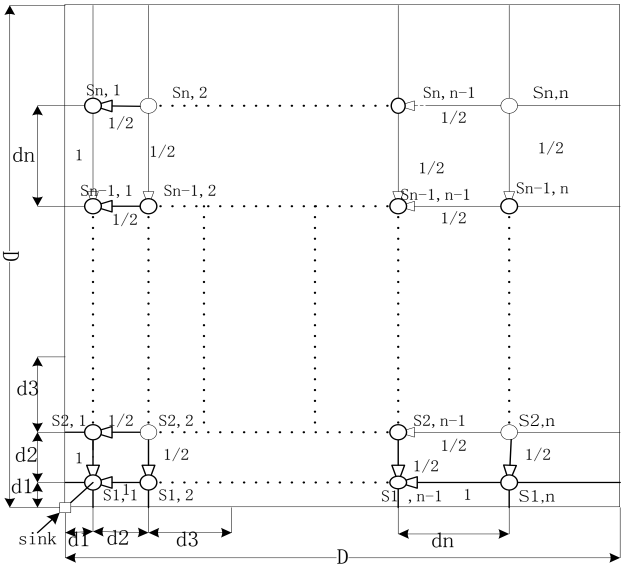

Fig. 3. Grid network

In this section, it is extended to two-dimensional network. In this kind of network, nodes are regularly deployed in intersections of rows and columns. The sink is located in the intersection of bottom left row and column, as shown in **Fig. 3**. In a grid network, each node generates one data and then it is sent to the sink in a cycle, and the transmission direction is restricted in downward or leftward direction with the same probability. To address the problem, this section first calculates the energy consumption for each node in the network and then discusses how to process cross layer optimization for grid network.

Theorem 6: In grid network, the nodal data load is:

$$\begin{cases} B_{n,n} = 1 \\ B_{n,j} = 2 - (\frac{1}{2})^{n-j} \qquad (1 \le j \le n-1) \\ B_{i,1} = 1 + B_{i+1,1} + \frac{1}{2} B_{i,2} \qquad (1 \le i \le n-1) \\ B_{i,j} = 1 + \frac{1}{2}(B_{i+1,j} + B_{i,j+1}) \qquad (1 < i, j < n) \\ B_{i,j} = B_{j,i} \qquad (i \ne j) \end{cases} \tag{30}$$

Proof: Since $B_{n,n}=1$, then $B_{n,n-1}=1+\frac{1}{2}=\frac{3}{2}$, $B_{n,n-2}=1+\frac{1}{2}B_{n,n-1}=\frac{7}{4}$,..., we obtain $B_{n,j}=1+\frac{1}{2}B_{n,j+1}=2-(\frac{1}{2})^{n-j}$, then n^{th} row is determined. The following is $n-1^{th}$ row, through analysis, we get:

$$B_{n-1,j}=1+\frac{1}{2}(B_{n,j}+B_{n-1,j+1}),\ \ 2\le j\le n-1.$$

While $B_{n-1,1}=1+B_{n,1}+\frac{1}{2}B_{n-1,2}$. Thus, $n-1^{th}$ row is determined. And so on, the summarized formula is (30). ■

Theorem 7: In grid network, each node in the first row (column) has bigger data load than other nodes in the same row.

Proof: According to Theorem 6, we get:

$$B_{n-1,n-1}=1+\frac{1}{2}(B_{n,n-1}+B_{n-1,n}) \tag{31}$$

$$B_{n-1,n-2}=1+\frac{1}{2}(B_{n,n-2}+B_{n-1,n-1}) \tag{32}$$

Since $B_{n,n-1}<B_{n,n-2}, B_{n,n-1}<B_{n-1,n-1}$, so $B_{n-1,n-2}>B_{n-1,n-1}$. Similarly, $B_{n-1,1}>B_{n-1,2}$. Then, it follows that $B_{i,1}>B_{i,2}$ ($1\le i\le n$).Therefore, each node in the first row (column) has bigger data load than other nodes in the same row. ■

This section discusses optimization in Rayleigh block fading channels for grid networks. In such networks, the number of nodes is fixed $n*n$, and thus the deployment cost is determined, the optimization goal is how to maximize the network lifetime. Factors that can be optimized are the placement of nodes (d^*) and nodal transmission structure (P^*). In this paper, two optimization methods are proposed, one is to optimize d^*, the other is to optimize P^*. The network lifetime can be maximized through these two approaches. In Theorem 8, we propose an optimal solution of nodal placement.

Theorem 8: In Rayleigh block fading channels grid network, the energy consumption of nodes in maximum consumption row (column) can be balanced if d_i meets:

$$\begin{cases} (B_{1,1}-B_{2,1})E_c+B_{1,1}M_3(\sqrt{2}d_1)^{\alpha-1}-B_{2,1}M_3d_2^{\alpha-1}=0 \\ (B_{2,1}-B_{3,1})E_c+B_{2,1}M_3d_2^{\alpha-1}-B_{3,1}M_3d_3^{\alpha-1}=0 \\ \cdots \\ (B_{n-1,1}-B_{n,1})E_c+B_{n-1,1}M_3d_{n-1}^{\alpha-1}-B_{n,1}M_3d_n^{\alpha-1}=0 \\ \sum_{i=1}^{n}d_i=D \end{cases} \tag{33}$$

Proof: According to (30), the data amount of each node can be calculated. Therefore, the optimal $d_i (1 \le i \le n)$ can be obtained if the first column is optimized. First, we need to solve optimal $d_i (1 \le i \le n)$. It is optimal when the energy consumption is balanced, then:

As can be seen from previous analysis, we get:

$$E_{i,1} = N_b B_{i,1}(E_c + K_1 \cdot \frac{D d_i^{\alpha-1}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C}) \tag{34}$$

$$E_{1,1} = N_b B_{1,1}(E_c + K_1 \cdot \frac{D(\sqrt{2}d_1)^{\alpha-1}(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C})$$

$$\begin{cases} E_{1,1} = E_{2,1}, E_{2,1} = E_{3,1} = \ldots = E_{n-1,1} = E_{n,1} \\ S.t. \sum_{i=1}^{n} d_i = D \end{cases} \tag{35}$$

Set $$M_3 = \frac{K_1 D(-4.25\log_{10} N_b + 2.2)}{K_2 \beta_m \ln C}$$

Put the above formula into (34) yields :

$$E_{i,1} = N_b B_{i,1}(E_c + M_3 d_i^{\alpha-1})$$

Put the above formula into (35), we get (33). Then, compute the front $n-1$ equations of (35), there is:

$$d_i = (\frac{B_{1,1} M_3 (\sqrt{2}d_1)^{\alpha-1} + (B_{1,1} - B_{i,1})E_c}{M_3 B_{i,1}})^{\frac{1}{\alpha-1}} \quad (i \ge 2).$$

Represent all d_i with d_1 ($D \ge d_1 > 0$), and then put them into the n^th^ equation, thus:

$$d_1 + \sum_{i=2}^{n} (\frac{B_{1,1} M_3 (\sqrt{2}d_1)^{\alpha-1} + (B_{1,1} - B_{i,1})E_c}{M_3 B_{i,1}})^{\frac{1}{\alpha-1}} = D$$

Set $$H_2(d_1) = d_1 + \sum_{i=2}^{n} (\frac{B_{1,1} M_3 (\sqrt{2}d_1)^{\alpha-1} + (B_{1,1} - B_{i,1})E_c}{M_3 B_{i,1}})^{\frac{1}{\alpha-1}} - D$$

Through analysis, it is obvious that $H_2(d_1)$ is a monotone increasing function in $d_1 \in (0, D]$ .Obviously, when $d_1 = D$, $H_2(d_1) = H_2(D) = \sum_{i=2}^{n} (\frac{B_{1,1} M_3 (\sqrt{2}D)^{\alpha-1} + (B_{1,1} - B_{i,1})E_c}{M_3 B_{i,1}})^{\frac{1}{\alpha-1}} > 0$,so $H_2(d_1) > 0$;when $d_1 \to 0$, $H_2(d_1) = \varepsilon + \sum_{i=2}^{n} (\frac{B_{1,1} M_3 (\sqrt{2}\varepsilon)^{\alpha-1} + (B_{1,1} - B_{i,1})E_c}{M_3 B_{i,1}})^{\frac{1}{\alpha-1}} - D \quad | \varepsilon \to 0$ .Therefore, if and only if the above is not more than 0, then the original equation has solutions and the solution is obtained.

■

Theorem 10: In Rayleigh block fading channels grid network, the energy consumption of nodes in maximum consumption row(column) can be balanced if reliability c_i meets:

$$\begin{cases} (B_{1,1}-B_{2,1})E_c+\frac{B_{1,1}M_4d_1^{\alpha-1}}{\ln C_1}-\frac{B_{2,1}M_4d_2^{\alpha-1}}{\ln C_2}=0 \\ B_{2,1}-B_{3,1})E_c+\frac{B_{2,1}M_4d_2^{\alpha-1}}{\ln C_2}-\frac{B_{3,1}M_4d_3^{\alpha-1}}{\ln C_3}=0 \\ \cdots \\ (B_{n-1,1}-B_{n,1})E_c+\frac{B_{n-1,1}M_4d_{n-1}^{\alpha-1}}{\ln C_{n-1}}-\frac{B_{n,1}M_4d_n^{\alpha-1}}{\ln C_n}=0,\prod_{i=1}^{n}C_i\geq C \end{cases}$$

(36)

Proof: As can be seen from previous analysis, if $d_i(1\leq i\leq n)$ is obtained, then the problem is converted to solve optimal $C_i(1\leq i\leq n)$. It is optimal when the energy consumption is balanced, then:

$$\begin{cases} E_{1,1}=E_{2,1},E_{2,1}=E_{3,1}=\ldots=E_{n-1,1}=E_{n,1} \\ S.t.\ \prod_{i=1}^{n}C_i\geq C \end{cases}$$

(37)

Then, that is:

$$E_{i,1}=N_b(E_c+K_1\cdot\frac{Dd_i^{\alpha-1}(-4.25\log_{10}N_b+2.2)}{K_2\beta_m\ln C_i})B_{i,1} \tag{38}$$

Set

$$M_4=\frac{K_1D(-4.25\log_{10}N_b+2.2)}{K_2\beta_m}$$

Put the above formula into (38) we yield:

$$E_{i,1}=N_bB_{i,1}(E_c+\frac{M_4d_i^{\alpha-1}}{\ln C_i})$$

Put the above formula into (37), we have (36). Then, compute the front $n-1$ equations of (38), we get:

$$C_i = \frac{(B_{1,1}-B_{i,1})E_c}{B_{i,1}M_4 d_2^{\alpha-1}} + \frac{B_{1,1}}{B_{i,1}\ln C_1} \quad | \, i \geq 2$$

Represent all C_i with C_1, and then substitute them into the n^{th} equation and yield:

$$C_1 \cdot \prod_{i=2}^{n} \frac{(B_{1,1}-B_{i,1})E_c}{B_{i,1}M_4 d_2^{\alpha-1}} + \frac{B_{1,1}}{B_{i,1}\ln C_1} = C$$

Set

$$H_3(C_1) = C_1 \cdot \prod_{i=2}^{n} \frac{(B_{1,1}-B_{i,1})E_c}{B_{i,1}M_4 d_2^{\alpha-1}} + \frac{B_{1,1}}{B_{i,1}\ln C_1} - C \tag{39}$$

Obviously, when $C_1 = 0$, $C_1 \cdot \prod_{i=2}^{n} \frac{(B_{1,1}-B_{i,1})E_c}{B_{i,1}M_4 d_2^{\alpha-1}} = 0$ and

$\lim_{C_1 \to 0} \frac{B_{1,1}}{B_{i,1}\ln C_1} = 0$,so $H_3(C_1) = \lim_{C_1 \to 0} H_3(C_1) = -C < 0$,thus $H_3(C_1) < 0$; when $C_1 = 1$,

$\lim_{C_1 \to 1} \frac{B_{1,1}}{B_{i,1}\ln C_1} \to +\infty$,and $H_3(C_1) = H_3(1) = \prod_{i=2}^{n} \frac{(B_{1,1}-B_{i,1})E_c}{B_{i,1}M_4 d_2^{\alpha-1}} + \frac{B_{1,1}}{B_{i,1}\ln C_1} - C$,thus $H_3(C_1) > 0$, since $H_3(C_1)$ is a continuous function, there must be $C_1 \in [0,1]$ which achieves $H_3(C_1) = 0$.

■

4. Performance Analysis and Experimental Results

4.1 Multi-source Linear Network

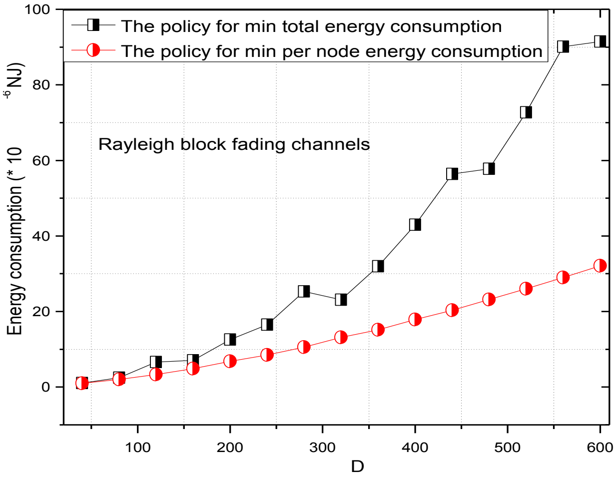

Fig. 4. The energy consumption of MTC VS MPNC

In this section, we provide some simulation examples to verify the cross layer optimal design proposed in this paper. We define *M*inimum *T*otal energy *C*onsumption for transmitting unit bit data to the sink as MTC, and *M*inimum *P*er *N*ode *C*onsumption for transmitting unit bit data to the sink as MPNC. **Fig. 4** shows the energy consumption of MTC VS MPNC in Rayleigh block fading channels. As can be seen from **Fig. 4** and **Fig. 5**, the energy consumption of MPNC is only 10% to 67% of the energy consumption of MTC. Obviously, utilization based on design can improve network lifetime (decrease energy consumption).

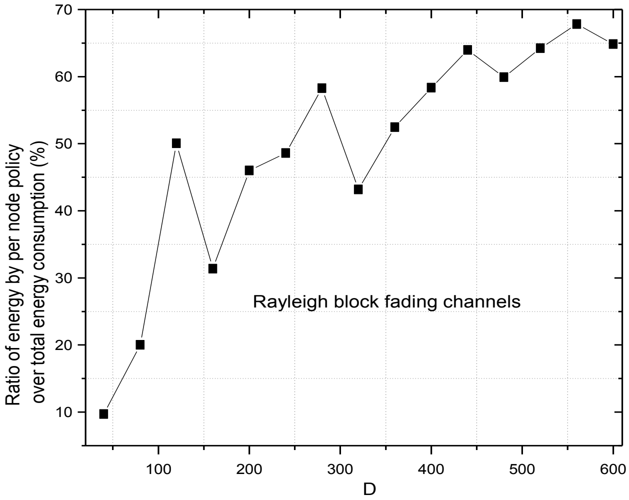

Fig. 5. Ratio of energy consumption by MPNC VS MTC

In this paper, balancing energy consumption is obtained by reducing transmission distance near the sink and increasing transmission distance far from the sink, which is called Unequal Distance of Nods Policy (UDNP). While nodes have equal distance in previous research is called Equal Distance of Nods Policy (EDNP). **Fig. 6** shows the energy consumption under UDNP and EDNP in Rayleigh block fading channels. Combined with **Fig. 7**, the energy consumption of UDNP is decrease by 2.36 to 2.46 times compared with EDNP, that is, the network lifetime is improved by more than 2 times. **Table 2** shows the nodal deployment distance with UDNP in Rayleigh block fading channels.

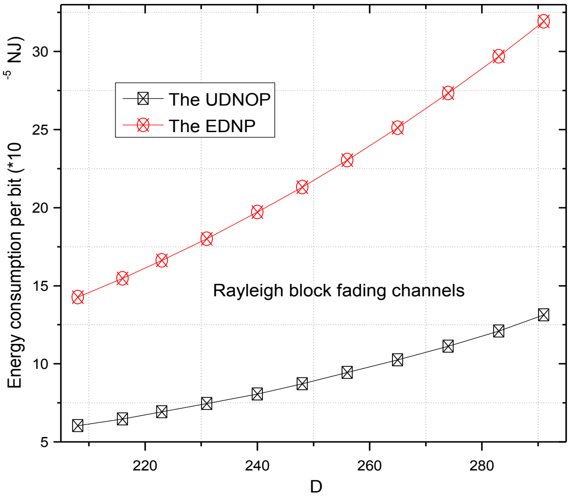

Fig. 6. The energy consumption under UDNP and EDNP

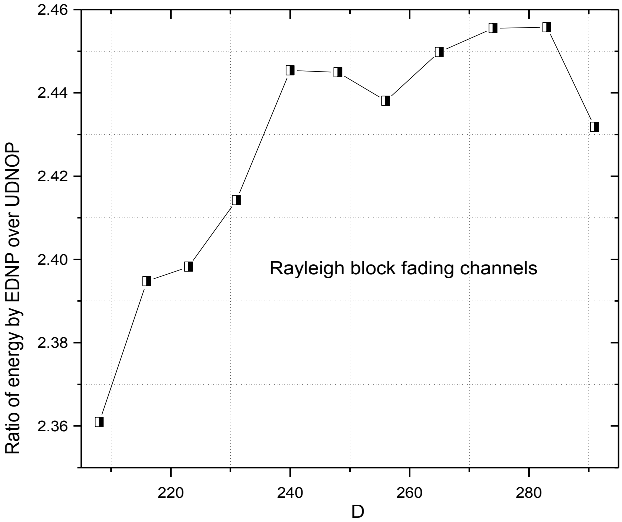

Fig. 7. The ratio of energy consumption by UDNP and EDNP

Table 2. The unequal distance of nodes

d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	D
20	23	25	27	30	34	39	50	248
21	24	26	28	31	35	40	51	256
22	25	27	29	32	36	41	53	265
23	26	28	30	33	37	43	54	274
24	27	29	31	34	38	44	56	283

25	28	30	32	35	39	45	57	291

In addition, the network lifetime can be improved by decreasing nodal reliability near the sink and increasing nodal reliability far from the sink, under the premise of total reliability meets the requirement of applications, which is called URNP (Unequal reliability of nods policy) in this paper. The policy each node adopts equal reliability is denoted as ERNP (equal reliability of nods policy).

Fig. 8 gives the energy consumption under these two schemes (URNP vs ERNP) and **Fig. 9** gives the ratio of energy consumption. As can be seen, URNP has better performance, which can improve network lifetime by more than 20%.

Table 3 shows nodal reliability with URNP. If the total reliability requirement is 0.579, then the nodal reliability for each node in ERNP is 0.933, however, in URNP scheme, nearly Sink node reliability is 0.8 when it can meet the requirements of 0.579. Thus in URNP policy, energy consumption of the maximum energy consumption node is less than that in ERNP strategy. So the network lifetime can be improved in URNP scheme.

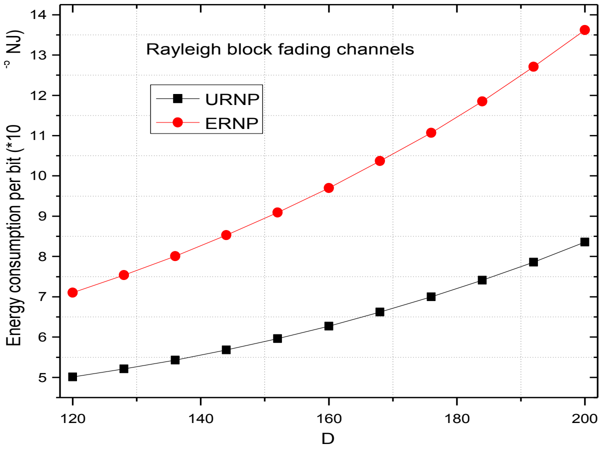

Fig. 8 The energy consumption under URNP and ERNP

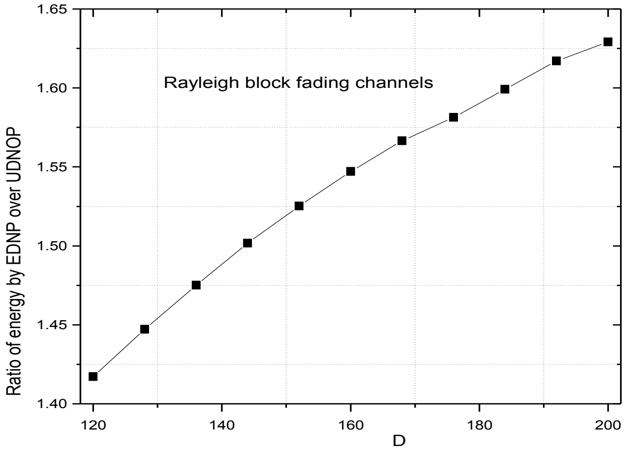

Fig. 9 The ratio of energy consumption of ERNP VS URNP

Table 3. The reliability of node (Rayleigh block fading channels)

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	C
0.8	0.88	0.92	0.95	0.96	0.98	0.98	0.996	0.579
0.8	0.87	0.91	0.94	0.96	0.97	0.98	0.995	0.561

4.2 Grid Network

In this section, we present the verification for grid network optimization design in this paper, which is in a 5*5 grid network. **Table 4** shows the nodal deployment distance with UDNP under Rayleigh block fading channels for grid network.

Fig. 10 shows the energy consumption under UDNOP and EDNP in Rayleigh block fading channels, combined with **Fig. 11**, it can be known easily that when compared with EDNP the energy consumption is decreased by more than 3.7 times with UDNP.

Table 4. The unequal distance of nodes (Rayleigh block fading channels)

d_1	d_2	d_3	d_4	d_5	d_6	Total nodes
20	32	38	47	63	200	5*5
21	33	39	48	65	206	5*5
22	34	40	49	66	211	5*5
23	35	41	50	68	217	5*5
24	36	42	51	69	222	5*5
25	37	43	52	71	228	5*5

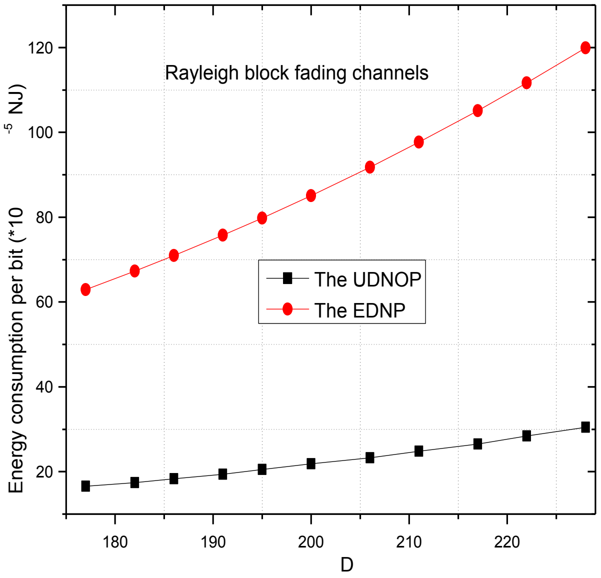

Fig. 10. The energy consumption under UDNOP and EDNP (Rayleigh block fading channels)

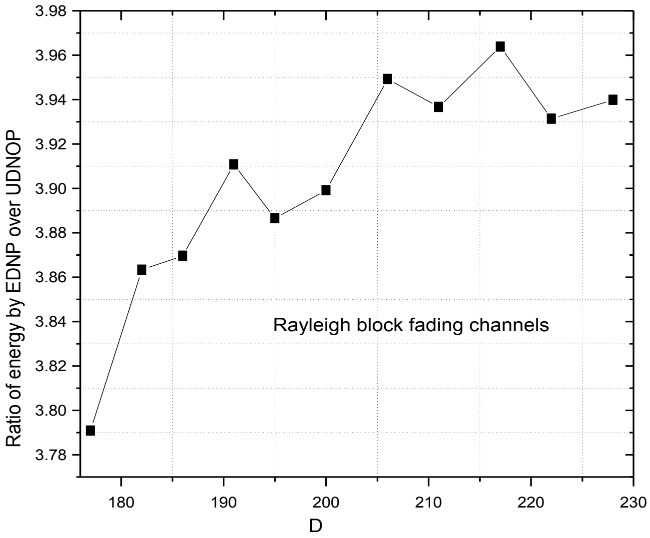

Fig. 11. Ratio of energy by EDNP over UDNOP (Rayleigh block fading channels)

Table 5. The reliability of node (Rayleigh block fading channels)

c_1	c_2	c_3	c_4	c_5	C	c_i
0.8	0.97	0.989	0.995	0.999	0.764	0.948
0.8	0.97	0.987	0.994	0.999	0.764	0.948

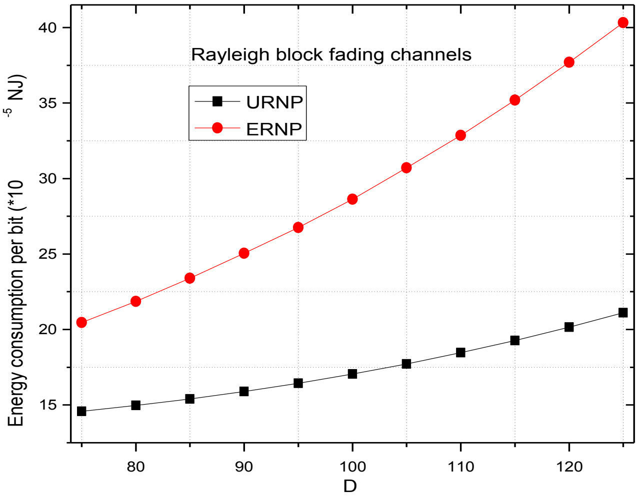

Fig. 12. The energy consumption under URNP and ERNP

Table 5 shows the nodal reliability with URNP in Rayleigh block fading channels. When the required total reliability is 0.764, the nodal reliability for each node with ERNP is 0.948. In URNP, the reliability of maximum energy consumption node is only 0.8, and thus the energy consumption is less than that in ERNP. **Fig.12** shows the energy consumption under URNP and ERNP, as can be seen, URNP can improve network lifetime by more than 30%.

5. Conclude and Discussion

In this paper, the optimization for multi-source linear network and grid network under Rayleigh block-fading channels have been proposed. The main conclusion can be drawn as follows: (1) The optimal nodal number N*, nodal placement d* and nodal transmission structure p* are given which can meet minimum total energy consumption and minimum unit data transmitting energy by the strict mathematics method. When the network total energy consumption for unit data is minimum, the unit data energy consumption ξ is not necessarily the minimum; (2) The minimum ξ does not necessarily maximize utilization efficiency η . A cross layer optimal design is proposed which adopts lower nodal reliability and shorter transmission distance for nodes near the sink. Through those, the network energy consumption can be balanced, and the network utilization is improved. Besides, we find that the network lifetime can be improved by several times (2-5 times) with UDNP, compared with EDNP. Meanwhile, the network lifetime can be improved by more than 30% with URNP, compared with ERNP; (3) Different from previous research, this paper gives the optimization equations and their solvable conditions by the strict mathematics, which have good theoretical significance.

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