Joint Subcarrier and Bit Allocation for Secondary User with Primary Users' Cooperation

Xiaorong Xu[1,2], Yu-Dong Yao[2], Sanqing Hu[2], and Yingbiao Yao[1]
[1] College of Telecommunication Engineering, Hangzhou Dianzi University
Hangzhou, Zhejiang 310018 - China
[2] Electrical and Computer Engineering Department, Stevens Institute of Technology
Hoboken, NJ 07030 - USA
[e-mail: xuxr@hdu.edu.cn]
*Corresponding author: Xiaorong Xu

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Abstract

Interference between primary user (PU) and secondary user (SU) transceivers should be mitigated in order to implement underlay spectrum sharing in cognitive radio networks (CRN). Considering this scenario, an improved joint subcarrier and bit allocation scheme for cognitive user with primary users' cooperation (PU Coop) in CRN is proposed. In this scheme, the optimization problem is formulated to minimize the average interference power level at the PU receiver via PU Coop, which guarantees a higher primary signal to interference plus noise ratio (SINR) while maintaining the secondary user total rate constraint. The joint optimal scheme is separated into subcarrier allocation and bit assignment in each subcarrier via arith-metric geo-metric (AM-GM) inequality with asymptotical optimization solution. Moreover, the joint subcarrier and bit optimization scheme, which is evaluated by the available SU subcarriers and the allocated bits, is analyzed in the proposed PU Coop model. The performance of cognitive spectral efficiency and the average interference power level are investigated. Numerical analysis indicates that the SU's spectral efficiency increases significantly compared with the PU non-cooperation scenario. Moreover, the interference power level decreases dramatically for the proposed scheme compared with the traditional Hughes-Hartogs bit allocation scheme.

Keywords: Primary users' cooperation (PU Coop), Joint subcarrier and bit allocation, Signal to interference plus noise ratio (SINR), Arith-metric geo-metric (AM-GM) inequality, Interference power level

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1. Introduction

With the emergence of new wireless systems, the demand for radio resources increases dramatically. Meanwhile, the fixed spectrum allocation policy in the authorized networks leads to spectrum underutilization. In order to address the spectrum scarcity issue, cognitive radio (CR) has been proposed as a key technique for the next generation mobile communications to improve the spectrum utilization. In CR, a wireless communication system is able to monitor the changes of its surrounding radio environment and dynamically modify its radio parameters accordingly to make the best use of radio resources [1][2].

CR network (CRN) has been proposed to efficiently exploit the overall spectrum avalability by allowing secondary users (SUs) to opportunistically access the spectrum that has been assigned to primary users (PUs). SUs are allowed to transmit and receive data over portions of the spectrum when PUs are inactive, which is determined by spectrum sensing. Therefore, PUs and SUs could share the spectrum resources without harmful interference to each other in this scenario [3].

Considering that the SU's transmissions may cause harmful interference to PUs for a certain period of time in underlay spectrum sharing scenario [3][4], the SU's transmitting power must be controlled in order to guarantee the PU's normal communications while maintaining the quality-of-service (QoS) for SU's transmissions [5][6]. Therefore, SU must execute "cognition" process to adjust its transmitting power adaptively, namely, both satisfying signal-to-interference-plus-noise ratio (SINR) at the PU receiver and maintaining SU's transmission rate requirements. SU's transmission power control is equivalent to the interference power mitigation in SU dynamic spectrum access, which allows SU to utilize the resources effectively with PUs' cooperation (PU Coop) [4-5,7-8].

Generally, there are more than one PU in the authorized netwok in practical scenarios. PU Coop is proposed as an approach to increase primary system capacity while mitigating mutual interference between primary network and secondary network [9-10]. PU Coop introduces significant performance improvement for the primary network while increasing secondary spectral efficiency. It exploits primary cooperative diversity to improve PU transmission QoS through detecting the secondary signal strength and adjusting the transmit power to meet the average SINR constraint at PU receiver [9-10]. Meanwhile, SUs can use the cognitive network resources with PU Coop [9-10]. Notice that PU Coop requires transmission overhead during the process of power allocation and power adjustment.

In PU Coop research, recent studies are mainly related with power control and channel allocation [9], or efficient spectrum sensing [10]. In contrast to previous research, we consider multi-resource allocation scenario in a secondary network with PU Coop, and propose a joint subcarrier and bit allocation scheme for SU with PU Coop. In our proposed scheme, the PU's transmission QoS is guaranteed and SU's rate requirement is satisfied.

PU Coop aims to meet average SINR at PU receiver while increasing cognitive access ability with interference mitigation. Therefore, it is important to investigate the appropriate joint multi-resource allocation strategy for SUs with PU Coop. We assume that the orthogonal frequency division multiplexing (OFDM) is implemented in primary and secondary network in an underlay spectrum sharing scenario. In this paper, we investigate joint subcarrier and bit optimization in cognitive OFDM-based CR networks. SUs opportunistically utilize the subcarriers through power control. The joint subcarrier and bit allocation scheme is studied with the objective of total interference power minimization under the constraint of SU transmission rate and SINR at PU receiver.

Ref. [11] studies a joint subcarrier and power allocation algorithm for cooperative multiuser

OFDM CR systems. A survey of resource allocation and scheduling schemes in OFDM wireless network is presented in Ref. [12], which discusses different scenarios such as single cell and multicell, cooperative and non-cooperative, optimal and suboptimal, etc. Ref. [13] investigates a fast barrier method and efficient heuristic algorithm for subchannel assignment and power distribution in multiuser OFDM-based CR networks, in order to satisfy SU heterogeneous realtime and non-realtime services in cognitive networks. In contrast to these multi-resource joint optimization schemes in OFDM-based CR networks, the problem considered in this paper is mainly related to the joint allocation of available subcarriers and bits for SU utilization while satisfying average SINR with PU Coop, which is different from the conventional resource allocation schemes [11-13].

To summarize, the contributions of this paper are as follows. PU Coop model is introduced in the proposed joint resource optimization scheme. The subcarriers and bits are jointly optimized with the objective of minimizing the average interference power level under the constraints of SU's transmission rate and PU received SINR. Additionally, the joint optimal problem is solved by an approximate approach via arith-metric geo-metric (AM-GM) inequality to obtain asymptotical optimization results.

The rest of the paper is organized as follows. System model is introduced in Section 2. A joint subcarrier and bit allocation scheme for cognitive user with PU Coop is proposed in Section 3. Simulation results and performance analysis are presented in Section 4. Finally, conclusions are drawn in Section 5.

2. System Model

Referring to Ref. [9], we consider the coexistence of multiple pairs of PU transceivers with PU Coop and a pair of SU transceivers in OFDM-based CRN depicted in **Fig. 1**. Compared with other scenarios such as multiple PUs and multiple SUs, this model is easy to analyze the joint resource allocation for SU with PUs' cooperation without the consideration of central cognitive base station (fusion center) for resource management and scheduling in the secondary network. In this model, the spectrum is divided into K OFDM subcarriers and there are I ($I \le K$) PU links. We assume that each of the primary links occupies at least one subcarrier to perform point-to-point communication. Meanwhile, a SU transmitter (SUT) attempts to communicate with its SU receiver (SUR) by opportunistically using the available K subcarriers. Peer-to-peer communication is implemented between SUT and SUR, and SU can use multiple subcarriers opportunistically at a given time over the detected available subcarriers in underlay spectrum sharing scenario.

In our system model, we define the following notations. h_k^{ss} denotes the cognitive channel coefficient from SUT to SUR over subcarrier k, whereas $h_{i,k}^{pp}$ denotes the primary channel coefficient from the i-th PU transmitter (PUT) to the i-th PU receiver (PUR) over subcarrier k. $g_{i,k}^{sp}$ is the interference channel coefficient over the k-th subcarrier from SUT to the i-th PUR, whereas $g_{i,k}^{ps}$ is the interference channel coefficient over the k-th subcarrier from the i-th PUT to SUR. Channel coefficients are assumed to be independent identical distribution (i.i.d) over Rayleigh flat fading channels. $P_{i,k}^{p}$ and P_k^{s} denote the transmit power of the i-th PUT and SUT over the k-th subcarrier respectively. N_0 is the unilateral power spectral density (PSD) of additive white gaussian noise (AWGN). Furthermore, the bandwidth of OFDM-based CRN is demoted as B.

In CRN, different PU can transmit and receive over different subcarriers. Meanwhile, SU detects the available subcarriers and accesses to the spectrum with power control. Multiple PUs cooperate to exchange control signaling and utilize the subcarriers. They also monitor SU access behavior. SU detects the subcarriers that are utilized by PUs, and it will adjust its transmitting power level to satisfy the average SINR at PUR in order to maintain PU communication QoS in underlay spectrum sharing scenario [9]. We assume that the maximum number of PU Coop is I (all PU Coop transceivers participate in cooperation). The coefficients of cognitive channel, primary channel and interference channel could be obtained via channel estimation. Moreover, SU utilizes the detected orthogonal subcarriers with power control, and it can adjust its transmitting power to satisfy secondary link QoS requirements under SINR constraint at primary PUR [9,11].

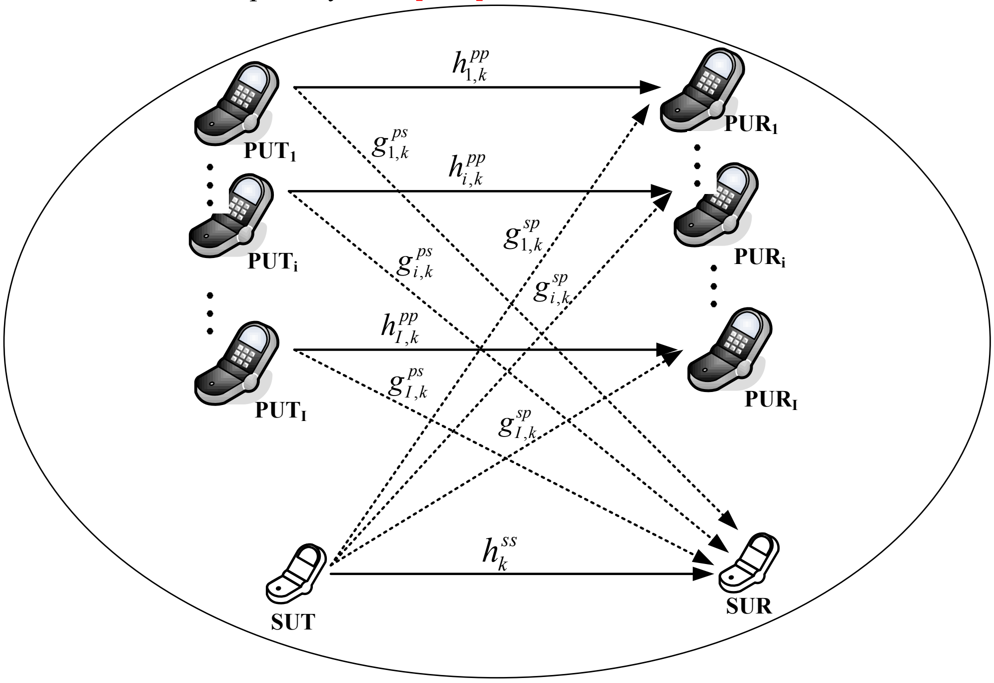
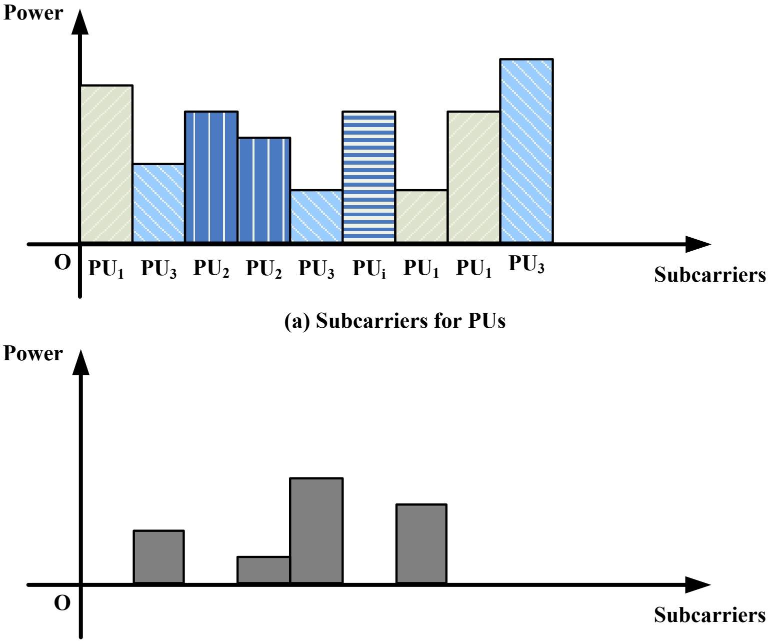


Fig. 1. System model of coexistence of PU Coop and SU

The subcarrier utilization diagram is shown in **Fig. 2**. In the figure, we assume that full K OFDM subcarriers are licensed to I primary users, and each of the primary transceivers occupies at least one subcarrier in a given time [9,12]. For simplicity, inter-subcarrier interference is not considered. SU could utilize multiple subcarriers opportunistically in an underlay spectrum sharing scenario. Based on this model, we propose a joint subcarrier and bit optimization scheme which is implemented for SU to meet secondary transmission rate requirements with proper power control while satisfying average SINR constraint at PUR, in order to implement spectrum sharing.



(a) Subcarriers for PUs

(b) Subcarriers for SU

Fig. 2. Subcarrier utilization diagram for PUs and SU

Next, we consider SINR at the i -th PUR and SUR respectively. If the k -th subcarrier is only occupied by the i -th PUT, SINR at the corresponding PUR can be expressed as

$$\mathrm{SINR}_{i,k}^{\mathrm{PUR}} = \frac{P_{i,k}^{p}\left|h_{i,k}^{pp}\right|^{2}}{N_0 B}, \quad i=1,2,\cdots \qquad \cdots \tag{1}$$

If the k -th subcarrier is occupied by SUT and the i -th PUT simultaneously, SINR at the i -th PUR is shown as follows

$$\mathrm{SINR}_{i,k}^{\mathrm{PUR}} = \frac{P_{i,k}^{p}\left|h_{i,k}^{pp}\right|^{2}}{P_{k}^{s}\left|g_{i,k}^{sp}\right|^{2} + N_0 B}, \quad i=1,2,\cdots \qquad \cdots \tag{2}$$

where $P_{k}^{s}\left|g_{i,k}^{sp}\right|^{2}$ is the interference power from SUT to the i -th PUR over the k -th subcarrier.

Assume that the k -th subcarrier is occupied by SUT and the i -th PUT simultaneously, then SINR at the corresponding SUR can be given as

$$\mathrm{SINR}_{k}^{\mathrm{SUR}} = \frac{P_{k}^{s}\left|h_{k}^{ss}\right|^{2}}{\sum_{i=1}^{I}\alpha_i P_{i,k}^{p}\left|g_{i,k}^{ps}\right|^{2} + N_0 B}, \quad k=1,2,\cdots \tag{3}$$

where $\sum_{i=1}^{I}\alpha_i P_{i,k}^{p}\left|g_{i,k}^{ps}\right|^{2}$ is the interference power from the i -th PUT to SUR over the k -th subcarrier, and α_i is a binary variable that denotes whether the subcarrier is occupied by the i-th PUT or not.

It is assumed that the average received SINR at each PUR should be above a threshold $\overline{\text{SINR}}^{p}$, in order to guarantee PUs' transmissions. Moreover, PU Coop can detect the potential SU signal strength and adjust PUs' transmitting power, in order to tolerate a certain interference level from cognitive transmission and satisfy average SINR at PUR [9].

Therefore, the total interfernce power from SUT to PUR must be minimized under the constraints of cognitive transmission rate and average SINR at PUR. Cognitive transmission rate varies in accordance with SUR's SINR. However, the maximum cognitive transmission rate is fixed to satisfy SU QoS requirements for a certain utilized subcarriers. Therefore, margin adaptation (MA) criterion is applied to establish mathematical model of joint subcarrier and bit allocation for SU with PU Coop, which is often referred as MA criterion in multi-user multi-resource joint optimization problem [14][15][16]. Combining Eq. (1) and Eq. (2), we can deduce that the objective function is the total interference power from SUT to PUR, while satisfying cognitive transmission rate and PU SINR constraint. Therefore, we have

$$\arg\min_{P_k^s,\rho_k} \sum_{i=1}^{I}\sum_{k=1}^{K} \rho_k \left(\left| g_{i,k}^{sp} \right|^2 P_k^s \right) \tag{4}$$

where ρ_k is a binary variable that indicates whether the k-th subcarrier is utilized by SUT or not. The MA criterion based optimization problem is subjected to the following conditions

$$R_{\text{total}}^{s} = \sum_{k=1}^{K} \rho_k B \log_2 \left[1 + \text{SINR}_k^{\text{SUR}} \right] \tag{5}$$

$$\sum_{k=1}^{K} \rho_k = 1,\ \ \rho_k = \{0,1\},\ k = 1,2,\cdots \tag{6}$$

$$\text{SINR}_{i,k}^{\text{PUR}} \geq \overline{\text{SINR}}^{p} \tag{7}$$

$$0 < P_{i,k}^{p} \leq \overline{P}^{p},\ i = 1,2,\cdots \qquad \cdots \tag{8}$$

$$0 < P_k^s \leq \overline{P}^{s},\quad k = 1,2,\cdots \tag{9}$$

Eq. (5) is the cognitive link transmission rate constraint, which is fixed to satisfy SU transmission QoS. Eq. (6) denotes that each subcarrier is used by one user at a time. Eq. (7) is the SINR constraint for PU links. Eq. (8) and Eq. (9) represent the maximum transmit power at the k-th subcarrier for each PU and SU respectively. The maximum powers can be regarded as the intrinsic limits for PU and SU, which are normally set out by spectrum regulators [9][14][15].

It is remarkable that, the minimization of interference power shown in Eq. (4) can be seen as a joint subcarrier and bit allocation optimization for SU. Bits are assigned at each cognitive subcarrier with the objective of SU power control, under cognitive rate constraint and PUR average SINR constraint simultaneously, as Eq. (5) and Eq. (7) indicated.

Due to the fact that the joint optimization problem shown in Eqs. (4)-(9) is non-convex, the global optimal solution for subcarrier and bit assignment could not be achieved properly. We evaluate joint subcarrier and bit allocation scheme with PU Coop into two separate phases. As indicated in Refs. [14][15], the first one is optimal subcarrier allocation based on PU Coop, and the second one is bit assignment at the determined SU's available subcarriers. Therefore, the approximate solution is divided into two single parameter optimization problems related to subcarriers and bits in two dependent phases. Specifically, in the first phase, multiple PUs cooperate to adjust their transmit power level, so that available subcarriers are allocated to SU properly under SU power control and SINR constraint, thus interference mitigation can be achieved between SU and PUs. In the second phase, given the minimum interference power level at PUR, bit allocation in terms of AM-GM inequality is performed at the determined available subcarriers, in order to satisfy cognitive transmission rate requirements.

3. Joint Subcarrier and Bit Allocation with PU Coop

In this section, we present and evaluate the joint subcarrier and bit allocation scheme for SU with PU Coop.

Assume the point-to-point communication is implemented in multiple primary links and secondary link respectively. SU utilizes the available OFDM subcarriers opportunistically by optimum allocation of subcarriers and bits. It transmits data over these subcarriers in underlay spectrum sharing scenario. Therefore, the joint multi-resource optimization problem shown in Eqs. (4)-(9) can be divided into two steps. The specific solution for the joint optimization problem can be described as follows.

The optimization problem shown in Eqs. (4)-(9) is a multi-parameter optimization problem, which is non-convex NP-hard problem. An approximate optimal solution can be obtained by the transformation of the optimization objectives [14,17]. Our proposed approximate optimization method can be divided into two single-parameter optimization problems related with subcarriers and bits in two dependent phases. Specifically, one is subcarrier allocation with PU Coop, and the other one is bit assignment at the determined subcarriers via AM-GM inequality. Specific steps of the proposed scheme are described in the following subsections.

3.1 Subcarrier Allocation Based on PU Coop

In order to satisfy primary transmission QoS requirements and secondary reliable communication with power control, mutual interference that exists between primary links and secondary link must be mitigated in underlay spectrum sharing scenario [9,14,18-19].

PU Coop is implemented to monitor SU access behavior via exchanging control signaling. If some subcarriers are utilized by SU, PUs will negotiate to increase their transmitting power level in order to satisfy average SINR at PUR. Meanwhile, SU will allocate subcarriers and bits to minimize interference power while satisfying SU rate requirements. PU Coop can enhance primary wireless capacity and maintain primary transmission QoS [19].

First, we consider subcarrier allocation for SU with PU Coop to realize interference mitigation, which satisfies average SINR constraint at PUR shown in Eq. (7). Suppose that original bits are equally distributed within each subcarrier. If SUT detects PU subcarriers with the lowest power level, SUT has the priority to obtain those subcarriers for its usage in underlay spectrum sharing scenario [18]. The detailed subcarrier assignment procedure can be illustrated as follows.

Assume that K_i denotes the set of available subcarriers from the i-th PUT for SU to utilize, and A represents the set of available cognitive OFDM subcarriers allocated to SU. We have

$\sum_{i=1}^{I} K_i = K$.

- Initialization
  Set $K_i = \phi$, $R_{\text{total}}^s = 0$, $A = \{1,2,...,K\}$.
- For $i = 1,2,...,I$,
  Find k that satisfies $\left|\text{SINR}_{i,k}^{\text{PUR}}\right| \geq \left|\text{SINR}_{i,j}^{\text{PUR}}\right|$ for all $j \in A$;
  Let $K_i = K_i \cup \{k\}$, $A = A - \{k\}$ and update $R_{K_i}^s = B\log_2\left[1+\text{SINR}_{K_i}^{\text{SUR}}\right]$.
- While $A \neq \Phi$,

(a) Find the i^*-th PU that satisfies SU rate $R_{K_{i^*}}^s \geq R_{K_i}^s$, and interference channel coefficient $\left|g_{i^*,k}^{sp}\right| \leq \left|g_{i,k}^{sp}\right|$, for all $1 \leq k \leq K$;

(b) For the i^*-th PU, find the corresponding available subcarrier k^* for SU that satisfies $\left|\text{SINR}_{i^*,k^*}^{\text{PUR}}\right| \geq \left|\text{SINR}_{i^*,j}^{\text{PUR}}\right|$, $j \in A$;

(c) For the i^*-th PU and the corresponding available subcarrier k^*, let $K_{i^*} = K_{i^*} \cup \{k^*\}$, $A = A - \{k^*\}$, $R_{K_{i^*}}^s = B\log_2\left[1+\text{SINR}_{K_{i^*}}^{\text{SUR}}\right]$;

(d) Continue the above iteration steps until $A = \Phi$, then $R_{\text{total}}^s = \sum_{i^*=1}^{I} R_{K_{i^*}}^s$.

3.2 Bit Assignment via AM-GM Inequality

Next, we describe bit allocation scheme in the determined subcarriers. To find the optimal transmission bits for SU in the k-th available subcarrier, we define the function $P_k^s(b_k) = \sum_{l=1}^{b_k} \Delta P_{k,l}^s$, where $\Delta P_{k,l}^s$ indicates that given $(l-1)$ bits at the k-th subcarrier, the incremental power for transmitting one additional bit over this subcarrier. To be specific, when the number of bits loaded on the subcarrier is $(l-1)$, the additional required power to transmit one bit over the k-th subcarrier is $\Delta P_{k,l}^s$. Therefore, SU transmitting power in the k-th subcarrier can be expressed as the sum of the total required incremental power to transmit b_k bits over the k-th subcarrier. Therefore, the objective function expressed in Eq. (4) can be rewritten as

$$\underset{\Delta P_{k,l}^s, \rho_k}{\arg\min} \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{l=1}^{b_k} \rho_k \left(\left|g_{i,k}^{sp}\right|^2 \Delta P_{k,l}^s\right) \tag{10}$$

Lagrange polynomial expression of the above problem can be shown as

$$J\left(\Delta P_{k,l}^s\right) = \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{l=1}^{b_k} \rho_k \left|g_{i,k}^{sp}\right|^2 \Delta P_{k,l}^s - \lambda \sum_{k=1}^{K} \rho_k B \log_2\left(1+\text{SINR}_k^{\text{SUR}}\right) \tag{11}$$

where $\text{SINR}_k^{\text{SUR}}$ denotes the SINR at SUR when the k-th subcarrier is occupied by SUT, which is shown in Eq. (2). We also have $P_k^s = \sum_{l=1}^{b_k} \Delta P_{k,l}^s$. Next, we set the differential equation of

Eq.(11) equals to zero, that is

$$\frac{\partial J\left(\Delta P_{k,l}^{s}\right)}{\partial \Delta P_{k,l}^{s}}=\sum_{i=1}^{I}\sum_{k=1}^{K}\rho_k\left|g_{i,k}^{sp}\right|^2-\frac{\lambda B}{\ln 2}\sum_{k=1}^{K}\frac{\rho_k}{\left[1+\mathrm{SINR}_k^{\mathrm{SUR}}\right]}\cdot\frac{\left|h_k^{ss}\right|^2}{\sum_{\substack{m=1\\m\neq k}}^{K}\sum_{i=1}^{I}P_{i,m}^{p}\left|g_{i,m}^{ps}\right|^2+N_0B}=0,$$
$$k=1,2,\cdots\quad,\ l=1,2,\cdots \tag{12}$$

Eq. (12) is a nonlinear optimization problem that can not be solved directly. Therefore, we consider another asymptotical solution. Referring to classical Hughes-Hartogs bit allocation algorithm [20], we propose an improved scheme that the required incremental power to transmit one additional bit $\Delta P_{k,l}^{s}$ can be acted as geometric progression through the k -th subcarrier [14,17]. Suppose BER requirement for SU cognitive link is Pr_b and SUT transmit symbol is modulated by M -ary quadrature amplitude modulation (MQAM) and Gray mapping. The incremental power $\Delta P_{k,l}^{s}$ can be expressed as

$$\Delta P_{k,l}^{s}=\frac{\left(f(l)-f(l-1)\right)}{\left|h_k^{ss}\right|^2} \tag{13}$$

where $f(b_k)=\frac{N_0}{3}\left[Q^{-1}\left(\frac{\mathrm{Pr}_b}{4}\right)\right]^2\left(2^{b_k}-1\right)$ indicates the lower bound of SU power requirement for transmitting b_k bits over the k -th available cognitive OFDM subcarrier.

Therefore, SU transmitting power P_k^s is related with the modulation method of transmit symbol, the number of allocated bits b_k, the threshold of PUR received SINR and the secondary BER requirement. Suppose MQAM modulation is implemented and M denotes the number of points in the signal constellation, namely, $M=2^b$. b is the number of bits carried by one MQAM symbol. BER performance of cognitive link has an upper union bound with the average Euclidean distance between MQAM signal points, which is tightly approximated by

$$\mathrm{Pr}_b<(M-1)Q\left(\sqrt{d^2/2N_0}\right) \tag{14}$$

where $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}e^{-t^2/2}dt$, and d denotes the average Euclidean distance between MQAM signal points. The bound may be loose when M is large. Hence, we may approximate Pr_b via replacing $M-1$ by M_n, where M_n is the largest number of neighboring points which are at distance d from any constellation point. For simplicity, M_n is constant and equals $M_n\approx 4$ [14].

The average energy of an MQAM symbol equals $(M-1)d^2/6$ [14]. Hence, the average required power for one MQAM symbol transmission can be written as

$$f(b)=(2^b-1)d^2/6 \tag{15}$$

From Eq. (14) and $M_n \approx 4$, we can get

$$d < \sqrt{2N_0}Q^{-1}\left(\frac{\Pr_b}{4}\right) \tag{16}$$

Substitute Eq. (16) into Eq. (15). It is indicated that, given secondary BER requirement $\Pr_b$, the lower bound of SU power for transmission $b_{i,k}$ bits by MQAM modulation over the k-th cognitive OFDM subcarrier, can be obtained as

$$f(b_k) = \frac{N_0}{3}\left[Q^{-1}\left(\frac{\Pr_b}{4}\right)\right]^2 \left(2^{b_k} - 1\right) \tag{17}$$

It is observed that $f(b_k)$ is a convex function with $f(0) = 0$. This condition essentially implies that, if the transmit bit is zero, no power is required. The required power for SU to transmit one additional bit increases with the assigned bit numbers b_k.

In the optimal target Eq. (10), the required SU transmit power for one bit increment over the k-th subcarrier is denoted as $\Delta P_{k,l}^{s}$, which can be acted as geometric progression in the k-th subcarrier [14]. Therefore, it is understood that the incremental sequences $\left|g_{i,k}^{sp}\right|^2 \Delta P_{k,1}^{s}$, $\left|g_{i,k}^{sp}\right|^2 \Delta P_{k,2}^{s}, \ldots, \left|g_{i,k}^{sp}\right|^2 \Delta P_{k,b_k}^{s}$ are the geometric series with initial term $\left|g_{i,k}^{sp}\right|^2 \frac{f(1)}{\left|h_k^{ss}\right|^2}$ and common ration 2. It has been proven that, if the sum of the last term $\sum_{k=1}^{K_i}\left|g_{i,k}^{sp}\right|^2 \Delta P_{k,b_k}^{s}$ approaches to its minimum value, the objective function shown in Eq. (10) can reach asymptotical minimization [14].

There exist several solutions to search for the asymptotical results of Eq. (10). Due to the fact that the incremental power can be expressed as geometric series in the determined subcarrier, we mainly apply arith-metric geo-metric (AM-GM) inequality to find its minimum value. Meanwhile, compared with conventional Hughes-Hartogs optimal bit allocation algorithm, the proposed approximate optimal bit assignment scheme can significantly reduce the computational complexity.

To search for the minimum value of the sum of geometric series last term $\sum_{k=1}^{K_i}\left|g_{i,k}^{sp}\right|^2 \Delta P_{k,b_k}^{s}$, we apply AM-GM inequality expressed as

$$\frac{1}{N}\sum_{i=1}^{N} x_i \geq \left(\prod_{i=1}^{N} x_i\right)^{\frac{1}{N}} \tag{18}$$

if and only if $x_1 = x_2 = \cdots$, the left-hand side equals to the right-hand side.

To achieve the minimum interference power, we let the term $x_{i,k} = \left|g_{i,k}^{sp}\right|^2 \frac{f(1)}{\left|h_k^{ss}\right|^2} 2^{b_k - 1}$, and the sum of the last term can be written as

$$\sum_{k=1}^{K_i}\left|g_{i,k}^{sp}\right|^2 \Delta P_{k,b_k}^s = \sum_{k=1}^{K_i}\left|g_{i,k}^{sp}\right|^2 \frac{2^{b_k-1}f(1)}{\left|h_k^{ss}\right|^2} \tag{19}$$

If Eq. (19) achieves its minimization, the objective function shown in Eq. (10) can approach to its minimum value. For this reason, we substitute $x_{i,k}=\left|g_{i,k}^{sp}\right|^2 \frac{f(1)}{\left|h_k^{ss}\right|^2}2^{b_k-1}$ into Eq. (18) to obtain the inequality shown as below

$$\sum_{k=1}^{K_i}\left(\frac{\left|g_{i,k}^{sp}\right|^2}{\left|h_k^{ss}\right|^2}2^{b_k-1}f(1)\right) \geq K_i\cdot\left(\left(\prod_{k=1}^{K_i}\frac{\left|g_{i,k}^{sp}\right|^2}{\left|h_k^{ss}\right|^2}\right)\cdot\left(f(1)\right)^{K_i}\cdot 2^{\left(\sum_{k=1}^{K_i}b_k-K_i\right)}\right)^{\frac{1}{K_i}},\ i=1,2,\cdots,I \tag{20}$$

If and only if each term on the left-side of Eq. (20) is equal, the sum achieves its global minimum value. Then, each term on the left-side and on the right-side has the same value shown as

$$\frac{\left|g_{i,k}^{sp}\right|^2}{\left|h_k^{ss}\right|^2}2^{b_k-1}f(1) = \left(\left(\prod_{k=1}^{K_i}\frac{\left|g_{i,k}^{sp}\right|^2}{\left|h_k^{ss}\right|^2}\right)\cdot\left(f(1)\right)^{K_i}\cdot 2^{\left(\sum_{k=1}^{K_i}b_k-K_i\right)}\right)^{\frac{1}{K_i}},\ i=1,2,\cdots,I \tag{21}$$

where K_i denotes the number of allocated subcarriers derived from the i -th PU for SU.

Therefore, the sum of the last term in Eq. (10) achieves its minimum value, and the interference power also approaches its minimization, which satisfies the optimal target under secondary transmission rate QoS requirements shown in Eqs. (4)-(9).

We take logarithmic transform for Eq. (21), which is shown as

$$2\log_2\left(\frac{\left|g_{i,k}^{sp}\right|}{\left|h_k^{ss}\right|}\right)+b_k = \frac{2}{K_i}\left(\sum_{k=1}^{K_i}\log_2\left(\frac{\left|g_{i,k}^{sp}\right|}{\left|h_k^{ss}\right|}\right)\right)+\frac{1}{K_i}\sum_{k=1}^{K_i}b_k,\ i=1,2,\cdots,I \tag{22}$$

Hence, the asymptotic optimal number of bits assigned in the k -th subcarrier can be written as

$$b_k = \frac{2}{K_i}\sum_{k=1}^{K_i}\log_2\frac{\left|g_{i,k}^{sp}\right|}{\left|h_k^{ss}\right|}+\frac{R_{K_i}}{K_i}-2\log_2\frac{\left|g_{i,k}^{sp}\right|}{\left|h_k^{ss}\right|} \tag{23}$$

where $R_{K_i}^s = \sum_{k=1}^{K_i}b_k,\ i=1,2,\cdots$, and $R_{\text{total}}^s = \sum_{i=1}^{I}R_{K_i}^s$.

If b_k is not an integer in Eq. (23), it should be rounded off as an integer. That is, if $R_l^s = R_{K_i}^s - \sum_{k=1}^{K_i}\hat{b}_k \neq 0$, we must round off b_k to an integer. We apply mathematical optimal algorithm to add or subtract one additional bit from the assigned subcarriers. Bit rounding procedure is expressed as follows [14,20].

(a) Round off each b_k, and calculate $R_l^s = R_{K_i}^s - \sum_{k=1}^{K_i} \hat{b}_k$.

(b) If $R_l^s = 0$, bit rounding procedure is not required.

(c) If $R_l^s > 0$, select R_l^s subcarriers based on a descending order of bit difference $b_k - \hat{b}_k$, and add one bit at each subcarrier.

(d) If $R_l^s < 0$, select $|R_l^s|$ subcarriers based on a ascending order of bit difference $b_k - \hat{b}_k$, and subtract one bit from each subcarrier.

In addition, we analyze computational complexity for the proposed scheme. It is easily found that the proposed bit allocation algorithm doesn't require an iteration process. Therefore, the complexity of the proposed bit assignment algorithm is about $O(K_i + K_i \log_2 K_i)$, while the conventional Hughes-Hartogs optimal bit allocation algorithm has the complexity about $O\left(R_{K_i}^s K_i \log_2 K_i\right)$ [14,20].

The purpose of joint subcarrier and bit allocation for SU with PU Coop is to take the advantage of SU's available subcarriers for its dynamic access and opportunistic resource utilization. To be specific, when the cognitive channel at the k -th cognitive subcarrier is good ($|h_k^{ss}|$ is large) and interference channel from SUT to PUR is poor ($|g_{ik}^{sp}|$ is small), less bits will be allocated to the k -th cognitive subcarrier, due to the fact that better channel quality guarantees low bit rate transmission. On the contrary, as cognitive quality at the k -th subcarrier degrades while interference channel gain increases ($|h_k^{ss}|$ is small and $|g_{ik}^{sp}|$ is large), more bits should be assigned to this subcarrier in order to meet certain rate requirements for SU and to satisfy SINR constraint for PU transmission.

4. Simulation Results and Analysis

In order to exaimine the validity of our proposed scheme, we perform computer simulations using Matlab. In the simulation, we set the total bandwidth in cognitive OFDM $B = 5\text{MHz}$, cognitive transmission BER requirement is supposed to be $\Pr_b = 10^{-4}$. SU's available subcarriers K are assumed from 64 to 1024, whereas the total bits are ranged from 64bits to 256bits. Cognitive channel noise power is assumed to be $\sigma^2 = 0.01$, and PU Coop numbers I are set to be $2/3/4$ respectively. Our performance analysis mainly focuses on cognitive spectral efficiency with SINR constraint at PUR, and SU interference power level with different SU's available subcarriers and allocated bits in the proposed joint subcarrier and bit allocation scheme [14, 19-21].

PU Coop can adjust PUT transmit power level to satisfy SINR constraint at PUR. It improves the anti-interference performance in CRN significantly, which mitigates the mutual interference and enhances cognitive spectral efficiency [9, 21]. Therefore, we first investigate the relationship of SU access ability (cognitive spectral efficiency) to SINR constraint at PUR. Simulation result of this relationship is shown in **Fig. 3**. We observe that the SU access ability (cognitive spectral efficiency) increases with the increasing of SINR in a nonlinear trend. Moreover, cognitive spectral efficiency increases with the increasing of PU Coop numbers. The highest spectral efficiency gain can be obtained from one PU (non-coop) to two PU Coops, whereas PU Coop numbers increasing from three to four provides less gain. In general, the relationship between SU access ability and PU Coop numbers keeps this trend. The dashed line illustrates the asymptotical curve of cognitive spectral efficiency upper bound, which presents "the maximum throughput" for SU access ability with PU Coop [22]. It is shown that, a small PU Coop numbers would be sufficient for the improvement of cognitive link. Moreover, primary cooperative diversity is improved at a higher SINR region via PU Coop, which shows that cognitive spectral efficiency achieved by PU Coop is gained from primary cooperative diversity and PU link QoS improvement in underlay spectrum sharing scenario [23].

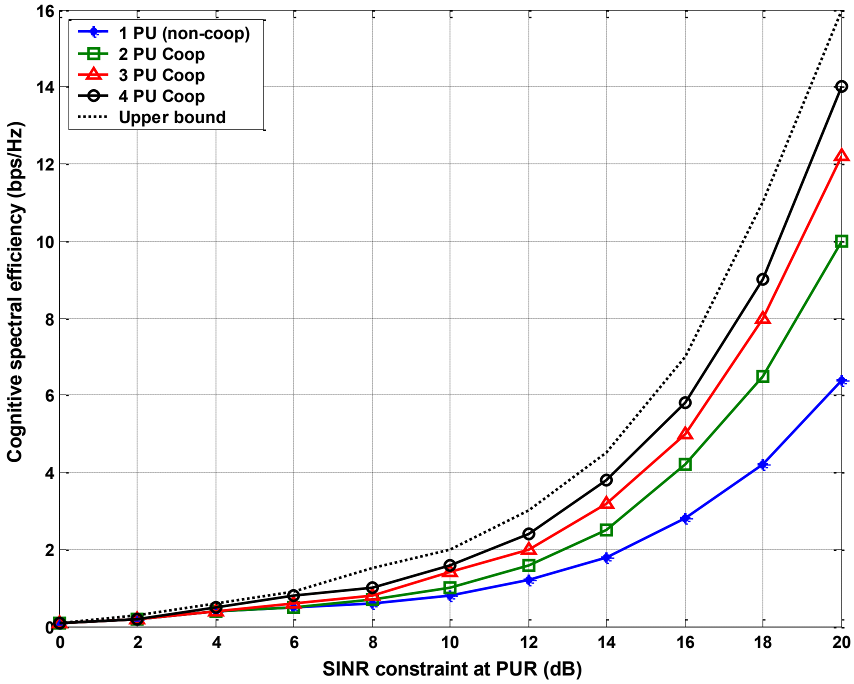


Fig. 3. Relationship of cognitive spectral efficiency to SINR constraint at PUR

Fig. 4 illustrates the relationship of available subcarriers for SU opportunistic access to average interference power level at PUR in a 4 PU Coop scenario. We observe that the interference power level is decreased dramatically with the increasing of SU's available subcarriers, which is consistent with the analysis in subsection 3.1. The proposed joint subcarrier and bit allocation scheme outperforms conventional Hughes-Hartogs bit allocation algorithm about 1dB interference power mitigation with a total of 64 allocated bits, which is due to the fact that Hughes-Hartogs algorithm acts as conventional optimal bit allocation algorithm. The proposed joint subcarrier and bit allocation scheme is based on the improved Hughes-Hartogs algorithm by the application of AM-GM inequality at each determined subcarrier, which is asymptotically optimal [19,20]. Meanwhile, the computational complexity of the proposed scheme is lower than the conventional Hughes-Hartogs algorithm. In addition, the average inteference power level increases with the increasing of allocated bit numbers. For SU's available 256 subcarriers, the average interference power level is about 8dB for 256 allocated bits, and 0dB for 64 allocated bits. This result shows that more assigned bits will result in higher interference power level, which will affect primary transmission QoS. Therefore, it is important to consider tradeoffs between bit assignment and power control for cognitive transmission.

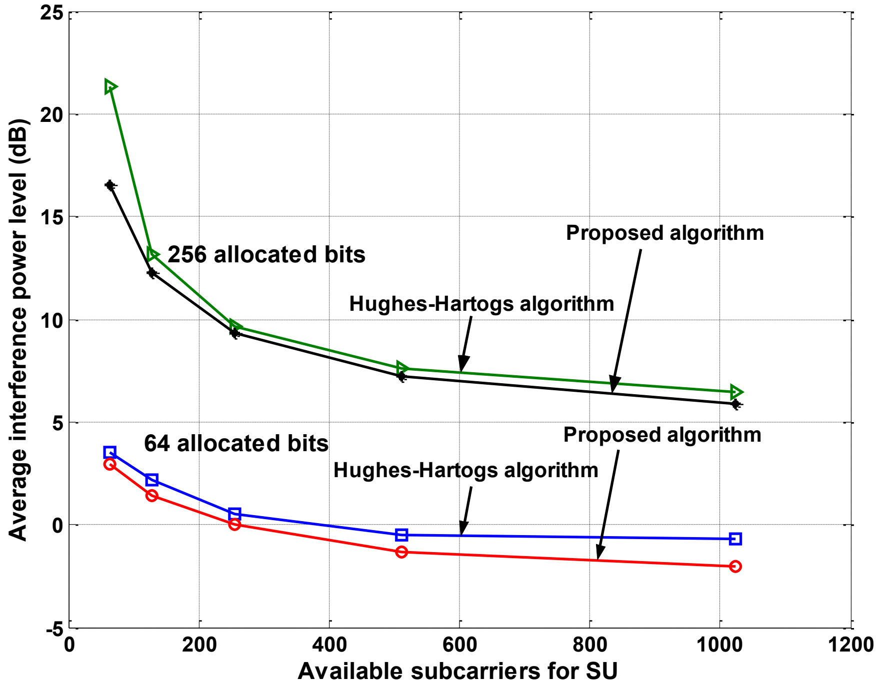


Fig. 4. Relationship of available subcarriers for SU to average interference power level

The relationship of the total allocated bits to average interference power level in a 4 PU Coop scenario is depicted in **Fig. 5**. It shows that the proposed joint subcarrier and bit allocation scheme outperforms traditional Hughes-Hartogs algorithm. In the proposed scheme, the interference power is reduced by about 1dB within the same allocated bits region. The interference power increases with the increasing of the total assigned bits for SU. Moreover, the interference power also increases with the decreasing of SU's available subcarriers. It is seen that, the increase of available subcarriers and the decrease of total assigned bits for SU reduce interference to PUs in underlay spectrum sharing scenario.

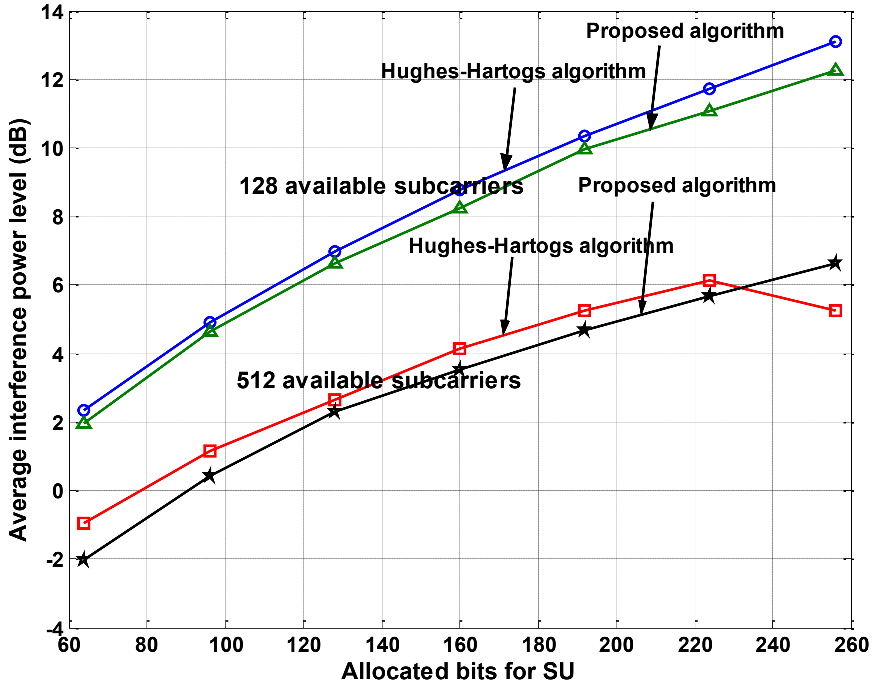


Fig. 5. Relationship of the total allocated bits for SU to average interference power level

5. Conclusions

In this paper, we proposed a joint subcarrier and bit allocation scheme for SU with PU Coop in cognitive OFDM based CRN, and its performance is investigated. The scheme maintains PU SINR requirement and guarantees secondary reliable transmission by assignment of multiple resources in underlay spectrum sharing scenario. The joint subcarrier and bit allocation optimization scheme is investigated and formulated by MA criterion. This enables the minimum interference power to be found under the constraints of SU's transmission rate and SINR at PUR. Simulation results indicate that PU Coop enhances primary cooperative diversity and obtains higher cognitive spectral efficiency for SU's transmissions. The increase of SU's available subcarriers and the decrease of the total allocated bits reduce interference to PUs in underlay spectrum sharing scenario. Furthermore, the proposed scheme outperforms conventional Hughes-Hartogs bit allocation algorithm with lower computational complexity, and it provides an effective strategy for SU transmissions.

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Xiaorong Xu is with the College of Telecommunication Engineering, Hangzhou Dianzi University (HDU), Hangzhou, China. He was a research scholar with the Electrical and Computer Engineering Department, Stevens Institute of Technology (SIT), Hoboken, NJ, USA, during 2013-2014. He received the B. Eng. degree in Communication Engineering and M. Eng. degree in Communication and Information System from HDU, Hangzhou, China, in 2004 and 2007, respectively. He received Ph.D. degree major in Signal and Information Processing from Nanjing University of Posts and Telecommunications (NJUPT), Nanjing, China, in 2010. Previously, from 2011 to 2013, he was working as a postdoctoral researcher in the Institute of Information and Communication Engineering, Zhejiang University (ZJU), Hangzhou, China. Currently, he is working as a college teacher in HDU. Dr. Xu's research interests emphasize on cognitive radio networks, cooperative communications, CR enhanced VANET, CR based WSN, as well as compressed sensing theory, etc.

Yu-Dong Yao has been with Stevens Institute of Technology (SIT), Hoboken, NJ, USA, since 2000, and is currently a professor and department director of Electrical and Computer Engineering in SIT. He is a director of Stevens' Wireless Information Systems Engineering Laboratory (WISELAB). Previously, from 1989 to 1990, he was at Carleton University, Ottawa, Canada, as a research associate working on mobile radio communications. From 1990 to 1994, he was with Spar Aerospace Ltd., Montreal, Canada, where he was involved in research on satellite communications. From 1994 to 2000, he was with Qualcomm Inc., San Diego, CA, where he participated in research and development in wireless CDMA systems. Prof. Yao's research interests include wireless communications and networks, spread spectrum and CDMA, antenna arrays and beamforming, cognitive and software defined radio (CSDR), and digital signal processing for wireless systems, etc. Prof. Yao also serves as the journal reviewer for *IEEE Transactions on Communications*, *IEEE Transactions on Wireless Communications*, *IEEE Transactions on Vehicular Technology*, *IEEE Communications Letters* and several international journals.

Sanqing Hu is with the Electrical and Computer Engineering Department, Stevens Institute of Technology (SIT), Hoboken, NJ, USA, as assistant researcher. He received his B. Eng and M. Eng degree in Electrical and Information Engineering from Huazhong University of Science and Technology, Wuhan, China, in 2007 and 2009, respectively. In 2013, He received his Ph. D. degree in Electrical Engineering from Stevens Institute of Technology, Hoboken, NJ, USA. Dr. Hu's research interests emphasize on MAC protocols for wireless networks, cognitive radio networks, cellular telecommunications, machine learning and data mining in communication applications, etc. Dr. Hu also serves as the journal reviewer for *IEEE Transactions on Communications*, *IEEE Transactions on Wireless Communications*, *IEEE Transactions on Vehicular Technology*, *IEEE Communications Letters* and several international journals.

Yingbiao Yao is with the College of Telecommunication Engineering, Hangzhou Dianzi University (HDU), Hangzhou, China, as associate professor. He received the M. Eng. degree and Ph.D. degree in Communication and Information System from Zhejiang University (ZJU), Hangzhou, China, in 2003 and 2006, respectively. Dr. Yao's research interests emphasize on cognitive radio networks, CR based WSN, as well as embedded system design, etc.