# Optimal Energy-Efficient Power Allocation and Outage Performance Analysis for Cognitive Multi-Antenna Relay Network Using Physical-Layer Network Coding

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*Received August 5, 2013; revised September 27, 2013; accepted October 23, 2013; published December 27, 2013*

## Abstract

In this paper, we investigate power allocation scheme and outage performance for a physical-layer network coding (PNC) relay based secondary user (SU) communication in cognitive multi-antenna relay networks (CMRNs), in which two secondary transceivers exchange their information via a multi-antenna relay using PNC protocol. We propose an optimal energy-efficient power allocation (OE-PA) scheme to minimize total energy consumption per bit under the sum rate constraint and interference power threshold (IPT) constraints. A closed-form solution for optimal allocation of transmit power among the SU nodes, as well as the outage probability of the cognitive relay system, are then derived analytically and confirmed by numerical results. Numerical simulations demonstrate the PNC protocol has superiority in energy efficiency performance over conventional direct transmission protocol and Four-Time-Slot (4TS) Decode–and-Forward (DF) relay protocol, and the proposed system has the optimal outage performance when the relay is located at the center of two secondary transceivers.

**Keywords:** physical-layer network coding, energy-efficient, power allocation, cognitive relay system

This work has been supported by New Century Excellent Talents Program of China (No.NCET-11-0593), National Science & Technology Specific Program of China(No. 2010ZX03005-001-03), and International Science and Technology Cooperation Project (No. 2010DFA11590).

**http://dx.doi.org/10.3837/tiis.2013.12.004**

## 1. Introduction

Spectrum and energy limitations are considered as barriers of the develop of future wireless system. Due to the increasing popularity of wireless devices, the radio spectrum is becoming an increasingly scarce resource. However, most of the licensed spectrum remains under-utilized. Cognitive radio (CR) is an efficient way to improve spectrum utilization [1]. The basic idea of CR is to allow unlicensed or secondary users (SUs) to access the licensed spectrum originally allocated to primary users (PUs) without sacrificing the quality-of-service (QoS) of the PUs. Although cognitive radio can improve the utilization of the licensed spectrum, it is not enough to enhance the ability of combating channel fading. Cooperative relay technique can achieve full spatial diversity and is often used in CR systems to improve spectrum efficiency [2][3][4]. Therefore, cognitive relay networks have attracted significant attention in recent researches. In [5][6] , cooperative one-way relaying protocol is investigated. The research results show that it can improve the system performance in terms of the achievable rate and link reliability. In [7], an optimal relay selection and resource allocation scheme for cognitive radio systems is discussed. Moreover, to reduce complexity, a suboptimal approach for relay selection is proposed.

On the other hand, due to the bidirectional nature of communication networks, a promising relay technique, two-way relaying, has attracted much attention. In energy efficient wireless systems, it is important to minimize the number of utilized channel in the communication. Traditional two hop relay schemes consume four time slots for two way communication. Two-way relaying applies the principle of physical-layer network coding (PNC) at the relay node so as to mix the signals received from the two source nodes, and then employs self-interference (SI) cancelation at each destination to extract the desired information [8][9]. As a result, two-way relaying needs only two time slots to exchange information between two sources and has higher spectral efficiency than the traditional two hop relaying. It is thus natural to incorporate two-way relaying into CR networks to further enhance spectrum utilization.

As the pioneers in this area, authors in [10] firstly introduced analog network coding (ANC) protocol into cognitive relay system. They also proposed an optimal power allocation scheme to achieve the max-min transmit rate fairness between two SUs without violating the interference power constraint of PU receivers. An optimal power allocation scheme for the SU network employing PNC based two-way relaying is discussed in [11]. It proposed a spectrum efficient SU communication scheme to maximize the sum rate under a total power constraint and the interference power threshold (IPT) constraints to PU. In [12], under the dissimilar interference power case, the exact outage probability of the system is derived. It is shown how interference power affects the optimal power allocation between the source nodes when the relay power increases. In [13], a joint relay selection and optimal power allocation scheme was proposed to achieve maximum throughput with ANC protocol in cognitive two-way relay system. However, to the best of our knowledge, no work investigates energy-efficient power allocation scheme for cognitive two-way relaying. From the system point of view, it is more desirable to consider the problem of minimizing the energy consumption per bit with a predefined quality of service requirement, especially for the power-limited systems where energy efficiency is obviously a crucial factor. Meanwhile, the previously mentioned works consider the cognitive relay networks in which all the nodes are equipped with a single antenna. The spectrum sharing of multi-antenna cognitive relay networks, where the SU communications are assisted by the multi-antenna relay, have raised great interest due to its capability to improve the performance of SUs significantly [14][15][16]. Therefore, in this article, we consider the optimal energy-efficient power allocation problem for PNC based CMRN. Meanwhile, we evaluate the channel performance for a secondary (unlicensed) cooperative diversity operating within the constraint of the peak power received at the PUs.

The main contributions of our work include:

1) Energy-efficient power allocation problem for the PNC based CMRN is firstly considered in this paper. The optimal energy-efficient power allocation (OE-PA) scheme is proposed to minimize total energy consumption per bit with the sum rate constraint and IPT constraints. The closed-form expressions of optimal power allocation are derived.

2) The exact closed-form expression of outage probability for the PNC based CMRN is also derived. It indicates that the proposed system has the optimal outage performance when the relay is located at the center of two SU transceivers.

This paper is organized as follows. Section 2 presents the system model. In Section 3, the OE-PA scheme and outage performance are analyzed. Simulation results are presented in Section 4 and conclusions are drawn in Section 5.

## 2. System Model and Problem Formulation

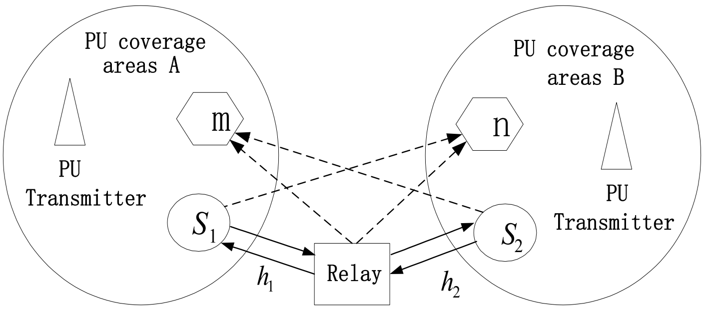



**Fig. 1.** PNC based SU communication, solid line illustrates the secondary communication, dash line illustrates the power interference to primary users

We consider two PU coverage areas $A$ and $B$ as shown in **Fig.1**. Users in $A$ use frequency set $I_a$ and users in $B$ use set $I_b$. Both frequency sets are orthogonal to each other. Secondary user nodes $S_1$ and $S_2$ communicate with each other in two timeslots with the use of relay node. $S_1$ and $S_2$ are equipped with single antenna, while the relay has L antennas.

The relay performs network coding operations and the detailed diagram of this secondary communications is shown in **Fig.2.** Here the frequency allocation of the system maintains a minimum interference to the $PUs$. We consider that $m$ and $n$ are the $PUs$ that experience the maximum interference from the secondary transmission. Additionally, we assume bandwidths of two frequency sets are equal to simplify the analysis.

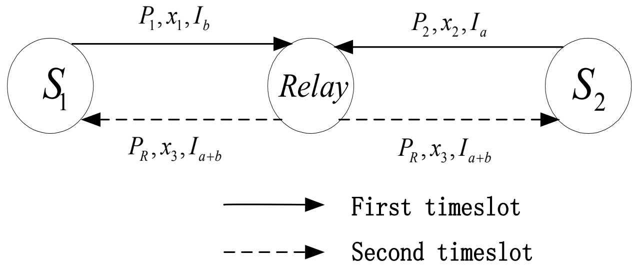



**Fig. 2.** Physical-layer network coding protocol with two time slots. Transmission configuration given as (transmit power, transmit symbol, frequency set).

PNC requires two timeslots to complete the two way transmission. As shown in **Fig. 2**, during the first timeslot, both $S_1$ and $S_2$ transmit their symbols to the relay node. More specifically, $S_1$ transmits $x_1$ with the frequency set $I_b$ and $S_2$ transmits $x_2$ with $I_a$ to the relay. Therefore, the transmissions do not interfere with the nearest neighbors. *PUs* have interference power threshold (IPT) values which they can tolerate. We consider that the relay combines the received signal with maximal-ratio combining (MRC). Therefore, the combined signal at the relay from $S_1$ and $S_2$ can be depicted respectively as follows

$$y_{1,R} = w_{1,R}^H \left( \sqrt{P_1} h_{1,R} x_1 + n_{1,R} \right) = \sqrt{P_1} \left\| h_{1,R} \right\| x_1 + \tilde{i} \tag{1}$$

$$y_{2,R} = w_{2,R}^H \left( \sqrt{P_2} h_{2,R} x_2 + n_{2,R} \right) = \sqrt{P_2} \left\| h_{2,R} \right\| x_2 + \tilde{i} \tag{2}$$

where $w_{1,R} = h_{1,R} / \left\| h_{1,R} \right\|, w_{2,R} = h_{2,R} / \left\| h_{2,R} \right\|$ denote the weighing vectors , $\tilde{i}$ $_{1,R} / \left\| h_{1,R} \right\|$ , $\tilde{i}$ $_{2,R} / \left\| h_{2,R} \right\|$ are the combined noise at the relay. $\left\| \cdot \right\|$ denotes the vector norm. $h_{1,R} = \left[ h_{1,R_1}, h_{1,R_2}, \ldots \right.$ $\ldots$ represent the channel gain for the links $S_1 \to$Relay and $S_2 \to$Relay respectively. $P_1$, $P_2$ are the transmit powers of $S_1, S_2$ respectively. In this paper, the subscripts 1 and 2 denote $S_1$ and $S_2$, R denotes the relay, subscripts m and n denote primary user m and primary user n.

During the second timeslot, the relay converts the received signal into a PNC modulated signal. PNC mapping follows the method given in [9] (relay does not perform any decoding and re-encoding processes for $x_1$ and $x_2$ separately). Then the PNC-modulated signal, $x_3$, is broadcast to $S_1$ and $S_2$. The relay uses both $I_a$ and $I_b$ spectrum sets in the second timeslot to transmit $x_3$ separately. We consider that frequency set as $I_{a+b}$. Therefore, $S_1$ can detect the signal with frequency $I_b$ and $S_2$ can detect the signal with frequency $I_a$. Similarly, the maximal-ratio combining (MRC) technique is used in the second timeslot, and thus the combined signal at $S_1$ and $S_2$ can be presented respectively as the following

$$y_{R,1} = \sqrt{P_R} \left\| h_{R,1} \right\| x_3 + \tilde{i} \tag{3}$$

$$y_{R,2} = \sqrt{P_R} \left\| h_{R,2} \right\| x_3 + \tilde{i} \tag{4}$$

where $h_{R,1} = \left[ h_{R_1,1}, h_{R_2,1}, \ldots, h_{R_L,1} \right]^T$, $h_{R,2} = \left[ h_{R_1,2}, h_{R_2,2}, \ldots, h_{R_L,2} \right]^T$ represent the channel gain of link Relay$\to S_1$ and Relay$\to S_2$ respectively. $P_R$ is the transmit powers of the relay node.

$\tilde{\imath}$ , $\tilde{\imath}$ are the combined noise at $S_1$ and $S_2$. The weakest link of the path determines the achievable rate. Therefore, the sum rate of the SU communication can be given as

$$R_{sum} = \frac{1}{2}\min(R_{1,R}, \bar{R}_{R,2}) + \frac{1}{2}\min(R_{2,R}, \bar{R}_{R,1}) \tag{5}$$

where $R_{i,j}$ is the achievable rate from the $i$th node to the $j$th node. $\bar{R}_{i,j}$ denotes the rate from the $i$th node to the $j$th node when PNC-modulated symbol is transmitted. There is a factor $1/2$ due to the fact that the two channels are used. To facilitate the study, we assume the noise variance is $\sigma^2$ at all the receivers. Therefore, we can write the achievable rate as

$$R_{i,j} = \log_2(1 + \left\|h_{i,j}\right\|^2 P_i / \delta^2) \tag{6}$$

where $h_{i,j}$ denotes the channel gain between node $i$ and node $j$. $\|\cdot\|^2$ denotes the squared Frobenius norm. $P_i$ is the transmit power of node $i$. We assume all the channels are reciprocal, let $h_1$ represent the channel between $S_1$ and relay, and $h_2$ represents the channel between $S_2$ and relay. Basically, PNC operation combines both received symbols into one symbol, which ultimately contains all the information. That means the relay broadcast rates should always be greater than $R_{1,R}$ and $R_{2,R}$. Otherwise, the relay cannot transmit all the received data, higher rates in the first timeslot are useless. Therefore, the relationship among four link capacities in (5) can be written as the following inequalities:

$$P_1\|h_1\|^2 \le P_R\|h_2\|^2 \quad ; \quad P_2\|h_2\|^2 \le P_R\|h_1\|^2 \tag{7}$$

Besides, during the two timeslots, the system should fulfill the following IPT constraints to ensure stable PU communication:

$$\begin{aligned} P_1\left|h_{1,n}\right|^2 \le Q_n \quad &; \quad P_2\left|h_{2,m}\right|^2 \le Q_m \\ P_R\left\|h_{R,n}\right\|^2 \le Q_n \quad &; \quad P_R\left\|h_{R,m}\right\|^2 \le Q_m \end{aligned} \tag{8}$$

where $Q_m$ and $Q_n$ are the IPT constraints of primary user $m$ and primary user $n$. The aim of this paper is to minimize the total energy consumption per bit under a sum rate constraint and IPT constraints to $PUs$. Energy consumption ratio (ECR) [17] is defined as the transmit energy per delivered information bit. Consider that $S_1$ intends to transmit information to $S_2$ with rate $r_{12}$ bit/s ($r_{12} > 0$), while $S_2$ intends to transmit information to $S_1$ with rate $r_{21}$ bit/s ($r_{21} > 0$), the sum rate requirement is $r$ bit/s ($r = r_{12} + r_{21}$). To facilitate the study, we assume that the total transmit time $T = 1s$ with equality for each slot, $T_i$ denotes the $i$th slot, i.e. $T_1 = T_2 = \frac{1}{2}s$. Then, energy efficiency $E_b$ which measured as energy-per-bit can be expressed as

$$E_b = \frac{\sum_{i=1}^{2} P_T(i)T_i}{r_{12} + r_{21}} = \frac{P_1 + P_2 + P_R + 2P_C}{2r} \tag{9}$$

where $P_T(i)$ is the total transmit power of the $i-th$ slot, $P_C$ denotes the basic circuit power consumption which has been discussed in [18].

# 3. Energy Efficiency Power Allocation and Outage Performance Analysis

## 3.1 Energy Efficiency Power Allocation

As discussed above, the optimization problem of minimizing total energy consumption per bit of the secondary transmissions can be formulated with sum rate constraint and IPT constraints. Therefore, we can formulate the optimization problem as below

$$\min_{P_1,P_2,P_R,r} \quad \mathrm{E}_b = \frac{P_1+P_2+P_R+2P_C}{2r}$$

$$st.\begin{cases} \frac{1}{2}\log_2(1+\frac{P_1\|h_1\|^2}{\delta^2})+\frac{1}{2}\log_2(1+\frac{P_2\|h_2\|^2}{\delta^2})=r \\ P_1\|h_1\|^2 \le P_R\|h_2\|^2 \\ P_2\|h_2\|^2 \le P_R\|h_1\|^2 \\ P_1|h_{1,n}|^2 \le Q_n \\ P_2|h_{2,m}|^2 \le Q_m \\ P_R\|h_{R,n}\|^2 \le Q_n \\ P_R\|h_{R,m}\|^2 \le Q_m \end{cases} \tag{10}$$

In order to make the mathematical treatment more tractable, we adopt the following high-SNR approximation to the sum rate constraint in (10):

$$\frac{1}{2}\log_2 P_1P_2\|h_1\|^2\|h_2\|^2 / \delta^4 = r \tag{11}$$

We consider different cases that are possible in this kind of cognitive radio network. The OE-PA scheme is discussed for each case in this section and their behavior with sum rate constraint and IPT constraints is illustrated with numerical results.

**A. Case I : Interferences not exceed IPT values**

Here the transmit powers of $S_1$, $S_2$ should not exceed the IPT levels of PUs. Therefore, we exclude IPT constraints to obtain the optimal power allocation. From (10), we can get $P_R \ge \max(P_1\|h_1\|^2\|h_2\|^{-2}, P_2\|h_2\|^2\|h_1\|^{-2})$. Then the optimal solution can be obtained for different transmit power of relay node.

1) If $P_R = P_1\|h_1\|^2\|h_2\|^{-2}$

We can rewrite the $E_b$ in (10) as

$$E_b = \frac{(1+\frac{\|h_1\|^2}{\|h_2\|^2})P_1 + \frac{2^{2r}\delta^4}{\|h_1\|^2\|h_2\|^2P_1} + 2P_C}{2r} \ge \frac{2^r\delta^2\frac{\sqrt{\|h_1\|^2+\|h_2\|^2}}{\|h_1\|\|h_2\|^2}+P_C}{r} \tag{12}$$

The equality is satisfied when

$$(1+\|h_1\|^2\|h_2\|^{-2})P_1 = \frac{2^{2r}\delta^4}{\|h_1\|^2\|h_2\|^2P_1} \tag{13}$$

Then the optimization problem is turned into the following unconstrained problem

$$\min_{r} \quad E_b = \frac{2^r \delta^2 \|h_1\|^{-1} \|h_1\|^{-2} \sqrt{\|h_1\|^2 + \|h_2\|^2} + P_C}{r} \tag{14}$$

We can differentiate (14) and isolate $r$ for the $E_b$ minimization of $E_b$. Let $r^{opt}$ be the optimal solution of (14). Note that $E_b$ has a unique minimum, which occurs at $r = r^{opt}$. Setting $\left.\frac{\partial E_b}{\partial r}\right|_{r=r^{opt}} = 0$ in (14), we obtain that

$$r^{opt} = \frac{W_0(P_c k_1^{-1} e^{-1}) + 1}{In2} \tag{15}$$

where $k_1 = \delta^2 \|h_1\|^{-1} \|h_1\|^{-2} \sqrt{\|h_1\|^2 + \|h_2\|^2}$ and $W_0$ denotes the real branch of the Lambert function $W$ which satisfies $W(z)e^{W(z)} = z$, where $z \in \mathbf{C}$ [19]. Then using (11), (13) and (15), the optimal solution can be obtained as

$$P_1^{opt} = 2^{r^{opt}} \delta^2 \|h_1\|^{-1} \left(\|h_1\|^2 + \|h_2\|^2\right)^{-\frac{1}{2}} \tag{16}$$

$$P_2^{opt} = 2^{r^{opt}} \delta^2 \|h_1\|^{-1} \|h_2\|^{-2} \left(\|h_1\|^2 + \|h_2\|^2\right)^{\frac{1}{2}} \tag{17}$$

$$P_R^{opt} = 2^{r^{opt}} \delta^2 \|h_1\| \|h_2\|^{-2} \left(\|h_1\|^2 + \|h_2\|^2\right)^{-\frac{1}{2}} \tag{18}$$

The following inequality should be satisfied to obtain the optimal solution:

$$P_1^{opt} \|h_1\|^2 \|h_2\|^{-2} > P_2^{opt} \|h_2\|^2 \|h_1\|^{-2} \tag{19}$$

By substituting (16), (17) into (19), we obtain the corresponding channel condition as

$$\|h_1\|^2 > \frac{\sqrt{5}+1}{2} \|h_2\|^2 \tag{20}$$

Meanwhile, the following power constraints should be satisfied

$$P_R^{opt} < \min\left\{ \frac{Q_m}{\|h_{R,m}\|^2}, \frac{Q_n}{\|h_{R,n}\|^2} \right\} \tag{21}$$

By substituting (18) into (21), the corresponding channel constraints can be easily obtained. Due to the page limitation, the channel condition is not listed here. We define C1 as the channel constraints in (20) and (21).

2) If $P_R = P_2 \|h_2\|^2 \|h_1\|^{-2}$

In the same way discussed above, we can obtain the optimal $r^{opt}$ as following

$$r^{opt} = \frac{W_0(P_c k_2^{-1} e^{-1}) + 1}{In2} \tag{22}$$

where $k_2 = \delta^2 \|h_1\|^{-2} \|h_2\|^{-1} \sqrt{\|h_1\|^2 + \|h_2\|^2}$, the optimal solution is given by

$$P_1^{opt} = \frac{2^{r^{opt}} \delta^2 \sqrt{(1 + \|h_1\|^{-2} \|h_2\|^2)}}{\|h_1\| \|h_2\|} \tag{23}$$

$$P_2^{opt} = \frac{2^{r^{opt}}\delta^2}{\|h_1\|\|h_2\|\sqrt{(1+\|h_1\|^{-2}\|h_2\|^2)}} \tag{24}$$

$$P_R^{opt} = \frac{2^{r^{opt}}\delta^2\|h_2\|}{\|h_1\|^3\sqrt{(1+\|h_1\|^{-2}\|h_2\|^2)}} \tag{25}$$

The following inequality should be satisfied.

$$P_2^{opt}\|h_2\|^2\|h_1\|^{-2} > P_1^{opt}\|h_1\|^2\|h_2\|^{-2} \tag{26}$$

Substituting (23) and (24) into (26), the corresponding channel condition is obtained as the following

$$\|h_1\|^2 < \frac{\sqrt{5}-1}{2}\|h_2\|^2 \tag{27}$$

Similarly, the constraint in (21) should be met. We define C2 as the constraints in (21) and (27).

3) If $P_R = P_1\|h_1\|^2\|h_2\|^{-2} = P_2\|h_2\|^2\|h_1\|^{-2}$

The optimal $r^{opt}$ is obtained as following

$$r^{opt} = \frac{W_0(P_c k_3^{-1}e^{-1})+1}{In2} \tag{28}$$

where $k_3 = \frac{\delta^2(\|h_1\|^4+\|h_1\|^2\|h_2\|^2+\|h_2\|^4)}{\|h_1\|^3\|h_2\|^3}$, the optimal solution can be easily obtained as

$$P_1^{opt} = 2^{r^{opt}}\delta^2\|h_1\|^{-3}\|h_2\| \tag{29}$$

$$P_2^{opt} = 2^{r^{opt}}\delta^2\|h_1\|\|h_2\|^{-3} \tag{30}$$

$$P_R^{opt} = 2^{r^{opt}}\delta^2\|h_1\|^{-1}\|h_2\|^{-1} \tag{31}$$

The corresponding channel condition is given as the following

$$\frac{\sqrt{5}-1}{2}\|h_2\|^2 < \|h_1\|^2 < \frac{\sqrt{5}+1}{2}\|h_2\|^2 \tag{32}$$

Meanwhile, the constraint in (21) should be met. We define C3 as the constraints in (21) and (32).

4) $P_R = \min\left\{Q_m\|h_{R,m}\|^{-2}, Q_n\|h_{R,n}\|^{-2}\right\}$

We can rewrite the $E_b$ in (10) as

$$E_b = \frac{P_1 + \frac{2^{2r}\delta^4}{P_1(\|h_1\|^2\|h_2\|^2)} + a + 2P_c}{2r} \geq \frac{\frac{2^r\delta^2}{\|h_1\|\|h_2\|} + 0.5a + P_c}{r} \tag{33}$$

where $a = \min\left\{\frac{Q_m}{\|h_{R,m}\|}, \frac{Q_n}{\|h_{R,n}\|}\right\}$, the optimal $r^{opt}$ is given as the following

$$r^{opt} = \frac{W_0\left((P_c+0.5a)k_4^{-1}e^{-1}\right)+1}{In2} \tag{34}$$

where $k_4 = \frac{\delta^2}{\|h_1\|\|h_2\|}$, the optimal solution is given by

$$P_1^{opt} = \frac{2^{r^{opt}}\delta^2}{\|h_1\|\|h_2\|} \tag{35}$$

$$P_2^{opt} = \frac{2^{r^{opt}}\delta^2}{\|h_1\|\|h_2\|} \tag{36}$$

$$P_R^{opt} = \min\left\{Q_m \left\|h_{R,m}\right\|^{-2}, Q_n \left\|h_{R,n}\right\|^{-2}\right\} \tag{37}$$

The corresponding channel condition is met when the constraints C1,C2 and C3 are all not satisfied.

### B. Case II : Interferences exceed IPT values

1) $S_1$ node power is limited due to IPT

If the transmit power of the $S_1$ node is limited due to IPT, the optimal transmit power of the $S_1$ node can be obtained as $P_1^{opt} = Q_n \left|h_{1,n}\right|^{-2}$. Substituting above equation into (16),(23),(29) and (35), we can get the optimal sum rate $r_1^*$ as following

$$r_1^* = \begin{cases} \log_2\left(\delta^{-2} Q_n \left|h_{1,n}\right|^{-2} \|h_1\| \sqrt{\|h_1\|^2 + \|h_2\|^2}\right) & C1 \\ \log_2\left(\frac{\delta^{-2} Q_n \left|h_{1,n}\right|^{-2} \|h_1\|^2 \|h_2\|}{\sqrt{\|h_1\|^2 + \|h_2\|^2}}\right) & C2 \\ \log_2\left(\frac{\delta^{-2} Q_n \left|h_{1,n}\right|^{-2} \|h_1\|^3}{\|h_2\|}\right) & C3 \\ \log_2\left(\delta^{-2} Q_n \left|h_{1,n}\right|^{-2} \|h_2\|\|h_1\|\right) & else \end{cases} \tag{38}$$

2) $S_2$ node power is limited due to IPT

If the $S_2$ node power is limited due to constraints, we can get the optimal transmit power of $S_2$ as $P_2^{opt} = Q_m \left|h_{2,m}\right|^{-2}$. Substituting above equation into (17),(24),(30) and (36), the optimal sum rate $r_2^*$ can be easily obtained as

$$r_2^* = \begin{cases} \log_2\left(\dfrac{Q_m \|h_1\| \|h_2\|^2}{\delta^2 |h_{2,m}|^2 \sqrt{\|h_1\|^2 + \|h_2\|^2}}\right) & C1 \\ \log_2\left(\dfrac{Q_m \|h_2\| \sqrt{\|h_1\|^2 + \|h_2\|^2}}{\delta^2 |h_{2,m}|^2}\right) & C2 \\ \log_2\left(\dfrac{Q_m \|h_2\|^3}{\delta^2 |h_{2,m}|^2 \|h_1\|}\right) & C3 \\ \log_2\left(\dfrac{Q_m \|h_2\| \|h_1\|}{\delta^2 |h_{2,m}|^2}\right) & else \end{cases} \tag{39}$$

3) Both $S_1$ and $S_2$ nodes power are limited due to IPT

If both the SUs transmit powers are limited by the IPT constraints, the optimal sum rate $r_3^*$ is given as follows

$$r_3^* = \min\left(r_1^*, r_2^*\right) \tag{40}$$

where $r_1^*$ and $r_2^*$ are defined in (38) and (39).

Finally, the optimal transmit power $P_1^{opt}, P_2^{opt}$ and $P_R^{opt}$ can be obtained by substituting $r^{opt}$ with $r_1^*$ or $r_2^*$ or $r_3^*$ into (16)-(18), (23)-(25), (29)-(31) and (35)-(37) for different channel conditions. Due to the page limitation, the optimal solutions are not listed here.

## 3.2 Outage Performance Analysis

In this section, we study the outage performance of the PNC based cognitive relay system depicted in **Fig. 1**. As defined in Section 2, we use $r_{12}$ and $r_{21}$ to denote the transmission rates of $S_1$ and $S_2$ respectively. An achievable rate region of the PNC protocol is the closure of the convex hull of the set of points $(r_{12}, r_{21})$ satisfying the following inequalities [20]: $r_{12} < I_1^{PNC}, r_{21} < I_2^{PNC}$ and $r_{12} + r_{21} < I_{sum}^{PNC}$ where $I_1^{PNC}, I_2^{PNC}$ and $I_{sum}^{PNC}$ are defined in (41)-(43) while satisfying the IPT constraints (8) for cognitive relay network.

$$I_1^{PNC} = \min\left\{\frac{1}{2}\log_2\left(1 + \frac{P_1\|h_1\|^2}{\delta^2}\right), \frac{1}{2}\log_2\left(1 + \frac{P_R\|h_2\|^2}{\delta^2}\right)\right\} \tag{41}$$

$$I_2^{PNC} = \min\left\{\frac{1}{2}\log_2\left(1 + \frac{P_2\|h_2\|^2}{\delta^2}\right), \frac{1}{2}\log_2\left(1 + \frac{P_R\|h_1\|^2}{\delta^2}\right)\right\} \tag{42}$$

$$I_{sum}^{PNC} = \frac{1}{2}\log_2\left(1 + \frac{P_1\|h_1\|^2 + P_2\|h_2\|^2}{\delta^2}\right) \tag{43}$$

The relationship among four link capacities is analyzed in (7). Similar to [20, Eq. (15)], we set the target data rate for each end-source as $r/2$ and assume the target data rate of the whole network is $r$. The system is in outage when the rate pair $(r_{12}, r_{21})$ falls out of the capacity region. Therefore, the outage probability for PNC based cognitive relay system is given by

$$\begin{aligned} P_{out}^{CR\_PNC} &= \Pr\,(I_1^{PNC} < \frac{r}{2} \quad \text{or} \quad I_2^{PNC} < \frac{r}{2} \quad \text{or} \quad I_{sum}^{PNC} < r) \\ &= \Pr\,(\min(I_1^{PNC}, I_2^{PNC}) < \frac{r}{2} \quad \text{or} \quad I_{sum}^{PNC} < r) \\ &= \Pr\,(\min(\gamma_{1,R}, \gamma_{2,R}) < 2^r - 1 \quad \text{or} \quad (\gamma_{1,R} + \gamma_{2,R}) < 2^{2r} - 1) \end{aligned} \tag{44}$$

where $\gamma_{1,R}$, $\gamma_{2,R}$ are the SNR for link $S_1 \rightarrow$ relay and $S_2 \rightarrow$ relay respectively. The allowed maximum instantaneous power of the secondary source $S_1$ is $Q_n \left|h_{1,p}\right|^{-2}$. Thus, the received SNR at secondary relay node is given by $\gamma_{1,R} = \frac{Q_n \left\|h_1\right\|^2}{\delta^2 \left|h_{1,p}\right|^2}$, where $\left|h_{1,p}\right|^2$ is exponentially distributed with parameter $\lambda_{1,p}$, and $\left\|h_1\right\|^2$ follows the chisquare distribution with $2L$ degrees. Thus, the probability density function (PDF) of the received SNR $\gamma_{1,R}$ can be obtained as

$$f_{\gamma_{1,R}}(x) = \int_0^\infty y f_Z(xy) f_Y(y)\,dy = \delta^{2L} \int_0^\infty y \frac{(xy)^{L-1}}{Q_n^L \Gamma(L)} e^{\frac{-\delta^2 xy}{Q_n}} \lambda_{1,p} e^{-\lambda_{1,p} y}\,dy = \frac{\delta^{2L} L Q_n \lambda_{1,p} x^{L-1}}{\left(\lambda_{1,p} Q_n + x\delta^2\right)^{L+1}} \tag{45}$$

And the cumulative density function (CDF) of $\gamma_{1,R}$ is given as

$$F_{\gamma_{1,R}}(x) = \int_0^x f_{\gamma_{1,R}}(z)\,dz = \delta^{2L} L Q_n \lambda_{1,p} \int_0^x \frac{z^{L-1}}{\left(z\delta^2 + \lambda_{1,p} Q_n\right)^{L+1}}\,dz = \frac{\delta^{2L} x^L}{\left(x\delta^2 + \lambda_{1,p} Q_n\right)^L} \tag{46}$$

Using the same way discussed above, we can get the PDF and CDF of $\gamma_{2,R}$ as the following

$$f_{\gamma_{2,R}}(x) = \frac{\delta^{2L} L Q_m \lambda_{2,p} x^{L-1}}{\left(\lambda_{2,p} Q_m + x\delta^2\right)^{L+1}} \qquad F_{\gamma_{2,R}}(x) = \frac{\delta^{2L} x^L}{\left(x\delta^2 + \lambda_{2,p} Q_m\right)^L} \tag{47}$$

**Theorem 1: For the PNC based cognitive multi-antenna relay system, an exact closed-form outage probability expression is given by**

$$\begin{aligned} P_{out}^{CR\_PNC} &= (-1)^{-L} \delta^{-2} L Q_m \lambda_{2,p} f(L+1, L, L-1, L, -\lambda_{2,p} Q_m / \delta^2, 2^{2r} - 1 + \lambda_{1,p} Q_n / \delta^2, 2^r - 1, 2^{2r} - 1) \\ &- \frac{\delta^{4L} (2^r - 1)^L (2^{2r} - 1)^L}{\left((2^r - 1)\delta^2 + \lambda_{1,p} Q_n\right)^L \left((2^{2r} - 1)\delta^2 + \lambda_{2,p} Q_m\right)^L} + \frac{\delta^{2L} (2^r - 1)^L}{\left((2^r - 1)\delta^2 + \lambda_{2,p} Q_m\right)^L} + \frac{\delta^{2L} (2^r - 1)^L}{\left((2^r - 1)\delta^2 + \lambda_{1,p} Q_n\right)^L} \end{aligned} \tag{48}$$

where $f(L+1, L, L-1, L, -\lambda_{2,p} Q_m / \delta^2, 2^{2r} - 1 + \lambda_{1,p} Q_n / \delta^2, 2^r - 1, 2^{2r} - 1)$ is defined in (52).

**Proof: See Appendix A.**

## 4. Numerical Simulations

In this section, we present the performance of the proposed OE-PA scheme and the outage performance for the cognitive relay system. The simulation topology is illustrated in **Fig.3**, where $S_1, S_2$ and the relay are assumed to be deployed in a line, m is right over $S_1$ and n is right over $S_2$, m and n are also in a line, the distance between the antennas is ignored. The channel coefficient is modeled as $\left|h_{i,j}\right|^2 = d_{i,j}^{-\varepsilon}$, where $\varepsilon$ is the path-loss factor, $d_{i,j}$ is the distance between node $i$ and node $j$. The relay position is measured as $\beta = d_{1,R}/d$, where $d$ is the distance between two SUs. We consider the basic circuit power consumption $P_c = 0.2W$ [18], the path-loss factor $\varepsilon = 4$ and the distance $d = 100m$. Furthermore, we set $Q_m = Q_n = Q$.

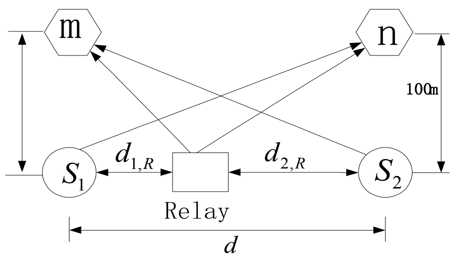



**Fig. 3.** Simulation topology

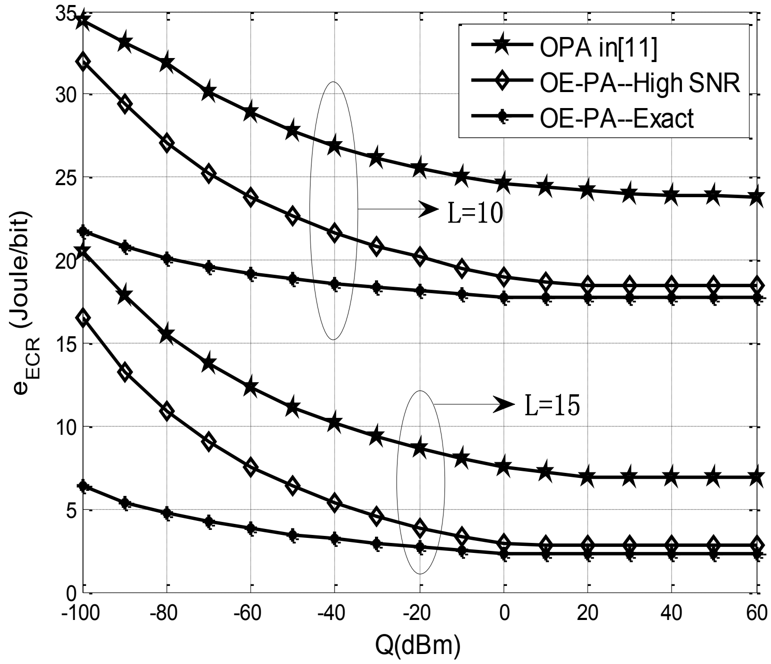
Energy efficiency comparison is demonstrated in **Fig. 4**, where the ECR of PNC protocol, direct transmission and 4TS DF protocol with OE-PA scheme are compared, we also make a comparision among our proposed OE-PA scheme, optimal PA (OPA) scheme for sum rate maximization in [11] and the exact OE-PA values solved by matlab directly.

**Fig. 4(a)** shows the ECR of different power allocation schemes where $Q$ ranges from -100dBm to 60dBm, $\delta^2 = -110dBm$, $\beta = 0.5$. **Fig. 4(b)** shows the ECR with different transmission protocols where $L$ ranges from 1 to 10, $\delta^2 = -110dBm$, $\beta = 0.5$. From the simulations, we have three conclusions:

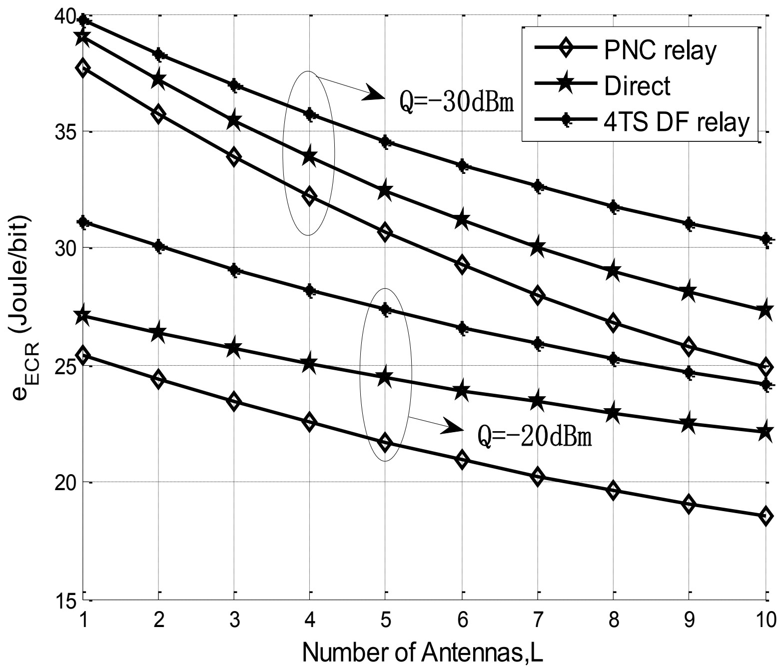
1) The gap between the curve of 'OE-PA-Exact' and the curve of 'OE-PA-High SNR' is small when Q is large, which indicates the tight fitness between the exact optimal values and our high-SNR approximation approach when SNR is high. Meanwhile, it is clear that the proposed OE-PA scheme based on the high-SNR approximation has obvious superiority over the OPA scheme proposed in [11] when Q is large. This is mainly due to allowing higher transmit power levels when $Q$ becomes larger.

2) Energy efficiency decreases as the IPT value increases. But it is saturated when parameter Q arrives at a certain level. (i.e. Q=20dBm in Fig. 4(a)). Since larger Q will make the transmit powers more close to the optimal values, but it will have no effect on the result when Q increases to a certain level.

3) As shown in **Fig. 4(b)**, energy efficiency decreases as the number of antenna increases in all three protocols due to higher diversity gain. PNC protocol has better energy efficiency performance than the other two protocols. Although the 4TS DF protocol has a little path-loss effect, its energy efficiency suffers because only half time is used to transmit original signal. The direct transmission doesn't waste time but its energy efficiency is poor because of the high path-loss effect. The PNC protocol achieves the best energy efficiency because it alleviates the path-loss effect and fully compensates the time consumption of relay using PNC.

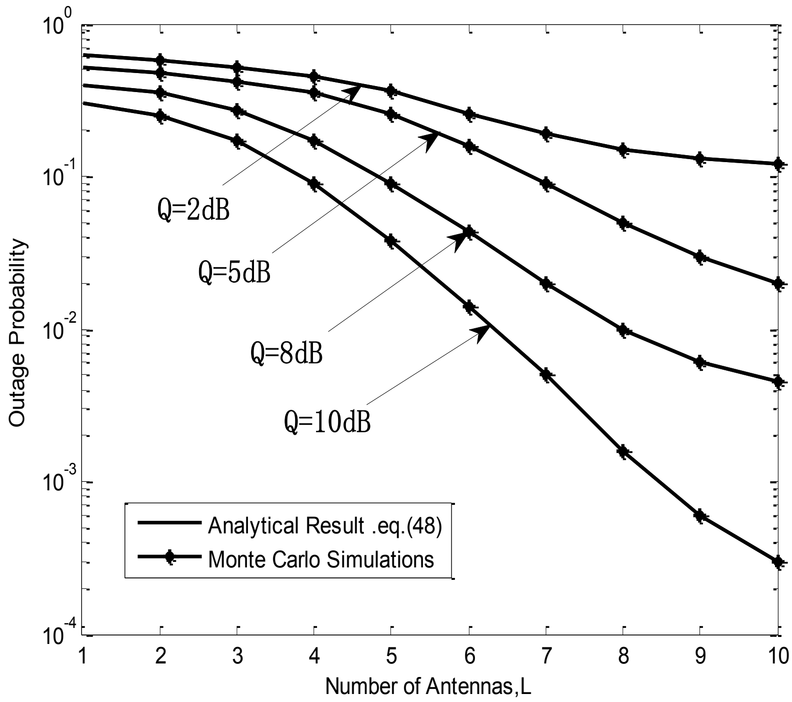



(a) Comparison among different power allocation schemes ( $\delta^2 = -110dBm,\ \beta = 0.5$ )

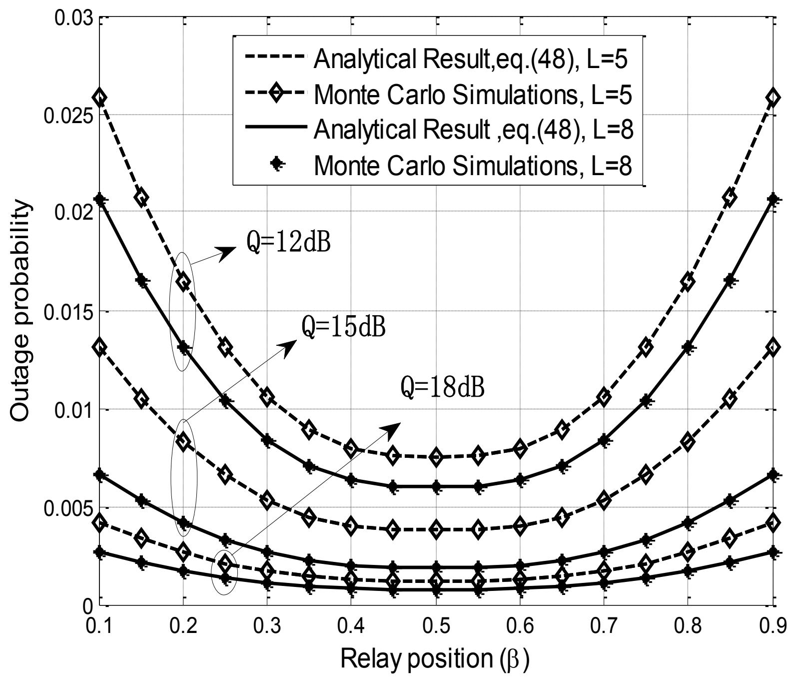



(b) Comparison among different transmission protocols ( $\delta^2 = -110dBm,\ \beta = 0.5$ )

**Fig. 4.** Energy efficiency versus IPT value Q (a) and versus number of antennas L (b)




(a) Impact of number of antennas L, and IPT value $Q$ on the outage probability ($\delta^2$=0dB, $\beta = 0.5$, r=1).




(b) Impact of relay position $\beta$, IPT value Q and number of antennas L on the outage probability $\left(\delta^2\text{=0dB, r=1}\right)$

**Fig. 5.** Outage probability versus number of antennas L (a) and versus relay position (b)

In Fig. 5, outage probability of the PNC based CMRN is analyzed. To simplify the outage performance analysis, the noise power is set as $\sigma^2 = 0dB$, the target data rate $r$ is set as $1\ bps/Hz$. Especially, it is assumed that $\lambda_{ij}^{-1} = d_{i,j}^{-\varepsilon}$ as [21]. From the simulations, we have two observations:

1.) Outage probability decreases as the interference threshold increases, as higher transmit power levels are allowed when Q becomes larger. Moreover, the cooperative spectrum sharing system gets higher diversity gain with a greater number of antennas.

2.) The smallest outage probability exits at $\beta = 0.5$, it denotes that the system has the optimal outage performance when the relay is located at the center of $S_1$ and $S_2$, as it can achieve the channel condition fairness between two SUs. Meanwhile, the simulated outage probability perfectly matches with the exact outage probability obtained by the derived expression.

## 5. Conclusion

In this paper, we propose an OE-PA scheme to minimize total energy consumption per bit with the sum rate constraint and IPT constraints for the PNC based CMRN. The closed-form solution for optimal transmit power among the SU nodes, as well as the outage probability of the CMRN are derived and confirmed by numerical results. It verifies that the PNC protocol has better energy efficiency performance than the 4TS DF protocol and direct transmission. And it also demonstrates the superiority of our proposed OE-PA scheme in energy efficiency performance when SNR is high. As expected, the outage probability improves when the interference threshold and the number of antennas increase, the system has the optimal outage performance when the relay is located at the center of two SU transceivers.

## APPENDIX A

We rewrite the outage probability $P_{out}^{CR\_PNC}$ out of (44) for the PNC based cognitive multi-antenna relay system as follows:

$$\begin{aligned} P_{out}^{CR\_PNC} &= \Pr\ (\min(\gamma_{1,R},\gamma_{2,R}) < 2^r - 1\ or\ (\gamma_{1,R}+\gamma_{2,R}) < 2^{2r}-1) \\ &= 1 - \Pr\ (\gamma_{1,R} > 2^r - 1, \gamma_{2,R} > 2^r - 1,\ (\gamma_{1,R}+\gamma_{2,R}) > 2^{2r}-1) \\ &= 1 + \Pr\ (\gamma_{1,R} > 2^r - 1, \gamma_{2,R} > 2^r - 1,\ (\gamma_{1,R}+\gamma_{2,R}) < 2^{2r}-1) - \Pr\ (\gamma_{1,R} > 2^r - 1, \gamma_{2,R} > 2^r - 1) \end{aligned} \tag{49}$$

The first term of (49) can be written as

$$\begin{aligned} &\Pr\left(\gamma_{1,R} > 2^r - 1, \gamma_{2,R} > 2^r - 1, \left(\gamma_{1,R}+\gamma_{2,R}\right) < 2^{2r}-1\right) \\ &= \int_{2^r-1}^{2^{2r}-1} f_{\gamma_{2,R}}\left(y\right) \int_{2^r-1}^{2^{2r}-1-y} f_{\gamma_{1,R}}\left(x\right) dxdy \\ &= \int_{2^r-1}^{2^{2r}-1} f_{\gamma_{2,R}}\left(y\right) F_{\gamma_{1,R}}\left(2^{2r}-1-y\right) dy - F_{\gamma_{1,R}}\left(2^r-1\right)\left[F_{\gamma_{2,R}}\left(2^{2r}-1\right) - F_{\gamma_{2,R}}\left(2^r-1\right)\right] \end{aligned} \tag{50}$$

Then, by inserting (45), (46) and (47) into (50), we can obtain

$$
\begin{aligned}
&\int_{2^r-1}^{2^{2r}-1} f_{\gamma_{2,R}}(y) F_{\gamma_{1,R}}\left(2^{2r}-1-y\right) dy \\
&= (-1)^{-L} \delta^{-2} L Q_m \lambda_{2,p} \int_{2^r-1}^{2^{2r}-1} \frac{y^{L-1}\left(2^{2r}-1-y\right)^L}{\left(\lambda_{2,p} Q_m / \delta^2 + y\right)^{L+1}\left(y-\left(2^{2r}-1+\lambda_{1,p} Q_n / \delta^2\right)\right)^L} dy \\
&\overset{(eq1)}{=} (-1)^{-L} \delta^{-2} L Q_m \lambda_{2,p} f(L+1, L, L-1, L, -\lambda_{2,p} Q_m / \delta^2, 2^{2r}-1+\lambda_{1,p} Q_n / \delta^2, 2^r-1, 2^{2r}-1)
\end{aligned}
\tag{51}
$$

where (eq1) uses the equality

$$
\begin{aligned}
&f(a,b,c,d,y_0,y_1,m,n) = \int_m^n \frac{y^c (n-y)^d}{(y-y_0)^a (y-y_1)^b} dy = \sum_{i=1}^{d} C_d^i n^{d-i} (-1)^i \int_m^n \frac{y^{c+i}}{(y-y_0)^a (y-y_1)^b} dy \\
&\overset{(eq2)}{=} \sum_{i=1}^{d} C_d^i n^{d-i} (-1)^i \left[ \sum_{j=1}^{a} \frac{d^{(a-j)} (y-y_1)^{-b}}{(a-j)! dy^{(a-j)}}\Big|_{y=y_0} \times \int_m^n \frac{y^{c+i}}{(y-y_0)^j} dy + \sum_{k=1}^{b} \frac{d^{(b-k)} (y-y_0)^{-a}}{(b-k)! dy^{(b-k)}}\Big|_{y=y_1} \times \int_m^n \frac{y^{c+i}}{(y-y_1)^k} dy \right] \\
&\overset{(eq3)}{=} \sum_{i=1}^{d} C_d^i n^{d-i} (-1)^i \left[ \sum_{j=1}^{a} \frac{d^{(a-i)} (y-y_1)^{-b}}{(a-i)! dy^{(a-i)}}\Big|_{y=y_0} \times \frac{1}{(-y_0)^j (c+i+1)} \left( n^{c+i+1} {}_2F_1\left(j, c+i+1; c+i+2; \frac{n}{y_0}\right) - m^{c+i+1} {}_2F_1\left(j, c+i+1; c+i+2; \frac{m}{y_0}\right) \right) \right. \\
&\left. + \sum_{k=1}^{b} \frac{d^{(b-k)} (y-y_0)^{-a}}{(b-k)! dy^{(b-k)}}\Big|_{y=y_1} \times \frac{1}{(-y_1)^k (c+i+1)} \left( n^{c+i+1} {}_2F_1\left(k, c+i+1; c+i+2; \frac{n}{y_1}\right) - m^{c+i+1} {}_2F_1\left(k, c+i+1; c+i+2; \frac{m}{y_1}\right) \right) \right]
\end{aligned}
\tag{52}
$$

With (eq2) follows the partial fraction in [22, eq.(3.326.2)] and (eq3) uses [22, eq. (3.194.1)]. Then, by inserting (46) , (47) and (51) into (50) and after simplifying, we can get

$$
\begin{aligned}
&\Pr\left(\gamma_{1,R} > 2^r-1, \gamma_{2,R} > 2^r-1, \left(\gamma_{1,R}+\gamma_{2,R}\right) < 2^{2r}-1\right) \\
&= (-1)^{-L} \delta^{-2} L Q_m \lambda_{2,p} f(L+1, L, L-1, L, -\lambda_{2,p} Q_m / \delta^2, 2^{2r}-1+\lambda_{1,p} Q_n / \delta^2, 2^r-1, 2^{2r}-1) \\
&- \frac{\delta^{4L} \left(2^r-1\right)^L \left(2^{2r}-1\right)^L}{\left(\left(2^r-1\right)\delta^2 + \lambda_{1,p} Q_n\right)^L \left(\left(2^{2r}-1\right)\delta^2 + \lambda_{2,p} Q_m\right)^L} + \frac{\delta^{4L} \left(2^r-1\right)^{2L}}{\left(\left(2^r-1\right)\delta^2 + \lambda_{1,p} Q_n\right)^L \left(\left(2^r-1\right)\delta^2 + \lambda_{2,p} Q_m\right)^L}
\end{aligned}
\tag{53}
$$

The last term of (49) can be derived as follows

$$
\begin{aligned}
&\Pr(\gamma_{1,R} > 2^r-1, \gamma_{2,R} > 2^r-1) = (1-F_{\gamma_{1,R}}(2^r-1))(1-F_{\gamma_{2,R}}(2^r-1)) \\
&= \left( \frac{((2^r-1)\delta^2 + \lambda_{1,p} Q_n)^L - \delta^{2L} (2^r-1)^L}{((2^r-1)\delta^2 + \lambda_{1,p} Q_n)^L} \right) \left( \frac{((2^r-1)\delta^2 + \lambda_{2,p} Q_m)^L - \delta^{2L} (2^r-1)^L}{((2^r-1)\delta^2 + \lambda_{2,p} Q_m)^L} \right)
\end{aligned}
\tag{54}
$$

Finally, by substituting (53) and (54) into (49), we can obtain the equation (48).

## APPENDIX B

**Table 1.** The Proposed OE-PA Scheme for the PNC based CMRN

| |
|---|
| 1) Initially, obtain the channel gain for each link. |
| 2) MainProcess:<br>If $\left\|h_1\right\|^2 > \frac{\sqrt{5}+1}{2}\left\|h_2\right\|^2$<br>a) Initialize $r^{\text{int}}$ and $P_R^{\text{int}}$ by Eq.(15) and Eq.(18). |

b) If $P_R^{\text{int}} < \min\left\{Q_m \left\|h_{R,m}\right\|^{-2}, Q_n \left\|h_{R,n}\right\|^{-2}\right\}$

Initialize $P_1^{\text{int}}, P_2^{\text{int}}$ *and* $P_R^{\text{int}}$ by Eq.(15) and Eq.(16)-(18).

else

Initialize $P_1^{\text{int}}, P_2^{\text{int}}$ *and* $P_R^{\text{int}}$ by Eq.(34) and Eq.(35)-(37).

end if

c) Calculate the optimal $P_1^{opt}, P_2^{opt}$ *and* $P_R^{opt}$ by the FinalProcess

else if $\left\|h_1\right\|^2 < \frac{\sqrt{5}-1}{2}\left\|h_2\right\|^2$

a) Initialize $r^{\text{int}}$ and $P_R^{\text{int}}$ by Eq.(22) and Eq.(25).

b) If $P_R^{\text{int}} < \min\left\{Q_m \left\|h_{R,m}\right\|^{-2}, Q_n \left\|h_{R,n}\right\|^{-2}\right\}$

Initialize $P_1^{\text{int}}, P_2^{\text{int}}$ *and* $P_R^{\text{int}}$ by Eq.(22) and Eq.(23)-(25).

else

Initialize $P_1^{\text{int}}, P_2^{\text{int}}$ *and* $P_R^{\text{int}}$ by Eq.(34) and Eq.(35)-(37).

end if

c) Calculate the optimal $P_1^{opt}, P_2^{opt}$ *and* $P_R^{opt}$ by the FinalProcess

else

a) Initialize $r^{\text{int}}$ and $P_R^{\text{int}}$ by Eq.(28) and Eq.(31).

b) If $P_R^{\text{int}} < \min\left\{Q_m \left\|h_{R,m}\right\|^{-2}, Q_n \left\|h_{R,n}\right\|^{-2}\right\}$

Initialize $P_1^{\text{int}}, P_2^{\text{int}}$ *and* $P_R^{\text{int}}$ by Eq.(28) and Eq.(29)-(31).

else

Initialize $P_1^{\text{int}}, P_2^{\text{int}}$ *and* $P_R^{\text{int}}$ by Eq.(34) and Eq.(35)-(37).

end if

c) Calculate the optimal $P_1^{opt}, P_2^{opt}$ *and* $P_R^{opt}$ by the FinalProcess

end if

3) FinalProcess:

a) Calculate $r_1^*$ and $r_2^*$ by Eq. (38) and Eq. (39) for different channel conditions.

b) If $P_1^{\text{int}} > Q_n \left|h_{1,n}\right|^{-2}$ and $P_2^{\text{int}} \le Q_m \left|h_{2,m}\right|^{-2}$

The optimal $r^{opt} = r_1^*$

else if $P_1^{\text{int}} \le Q_n \left|h_{1,n}\right|^{-2}$ and $P_2^{\text{int}} > Q_m \left|h_{2,m}\right|^{-2}$

The optimal $r^{opt} = r_2^*$

else if $P_1^{\text{int}} > Q_n \left|h_{1,n}\right|^{-2}$ and $P_2^{\text{int}} > Q_m \left|h_{2,m}\right|^{-2}$

The optimal $r^{opt} = \min\left(r_1^*, r_2^*\right)$

else

The optimal $r^{opt} = r^{\text{int}}$

end if

c) Calculate the optimal $P_1^{opt}, P_2^{opt}$ *and* $P_R^{opt}$ by substituting $r^{opt}$ into Eq. (16)-(18), Eq. (23)-(25), Eq. (29)-(31) and Eq. (35)-(37) for different channel conditions.

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