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Outage Analysis of CRNs with SC Diversity Over Nakagami-m Fading Environment

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Abstract

In this paper, we investigate the outage performance of a cognitive relay network. We consider mutual interference in an independent, non-identically distributed Nakagmai-m fading environment. We first derive the close-form outage probability expression, which provides an efficient means to evaluate the effects of several parameters. This allows us to study the impact of several parameters on the network's performance. We then derive the asymptotic expression and reveal that the diversity order is strictly determined by the fading severity of the cognitive system. It is not affected by the primary network. Moreover, the primary network only affects the coding gain of the cognitive system. Finally, Monte Carlo simulations are provided, which corroborate the analytical results.

Keywords: cognitive relay network (CRN), selection combining (SC), Nakagami-m fading, outage probability.

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1. Introduction

Radio spectrum is among the most heavily used and expensive natural resource in the world. Although almost all the spectrum suitable for wireless communications has been allocated, recent studies and observations indicate that many portions of the radio spectrum are not used for a significant amount of time or in certain geographical areas, while unlicensed spectrum bands are always crowded. Such spectral under-utilization has motivated cognitive radio (CR) technology [1]-[7]. Cognitive radios are those that can change the environment in which they operate. CR has built-in radio environment awareness and spectrum intelligence. CR was widely studied as a promising solution to the problem of spectrum shortage and low spectrum utilization by allowing for dynamic access of unused bands through spectrum sensing.

Relay communication is a promising technology for improving the throughput and coverage of wireless communication systems, and has also found applications in cognitive radio systems [8]-[16]. In [8]-[13], outage performance was analyzed for Rayleigh fading environments. In [13], the exact outage performance of an underlay cognitive network using decode-and-forward (DF) relaying with multiple primary users (PUs) in Rayleigh fading channels has been studied. However, in [14]-[16], the outage performance was analyzed in a more general environment. In [14], the outage performance of DF cognitive dual-hop systems was investigated, considering joint constraints on the peak and average interference powers at the primary receiver in Nakagami-m environment. The outage probability of dual-hop cognitive amplify-and-forward (AF) relay networks was examined in [15]. This probability was subject to independent, non-identically distriubted Nakagami-m fading. . In [16], the outage performance of dual-hop cognitive relay networks was derived, considering the direct link and interference from PU.

This prior work have improved our understanding on the performance of cognitive relay networks (CRNs). Most of them assumed Rayleigh fading. However, the prior related works ignored the interference from PU for Nakagami-m environment. To the best of our knowledge, the outage analysis of dual-hop CRN considering mutual interference in Nakagami-m fading environment is almost unexplored from the analytical point view. As such, the main focus of this paper is to fill this important gap. More specifically, our results reveal some important design insights and the impact of some key system parameters on the cognitive system, such as power constraints, and fading parameters.

The main contributions of this paper are outlined as follows: First, the cognitive system can obtain full diversity order. Specifically, the diversity order is only determined by the links of cognitive system including the direct link, the first hop link and the second hop link. Second, the diversity order of spectrum-sharing is in line with those obtained from traditional dual-hop system. More specifically, we conclude that the diversity-multiplexing tradeoff is independent of the primary network, and the primary network only affects the coding gain of the considered spectrum sharing system.

The remainder of this paper is organized as follows. Section II presents a brief description of the system and channel models considered in this paper. In Section III, we derive the exact outage probability expression that provides an efficient means to evaluate the effect of system papramevers. Results obtained numerically and via Monte Carlo simulations validate the theoretical results obtained in Section IV. Finally, concluding remarks are provided in Section V.

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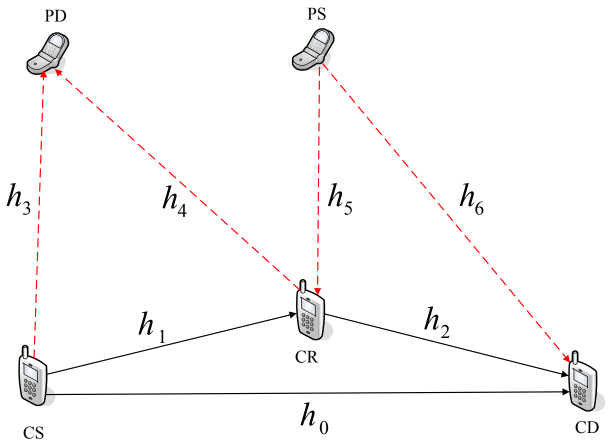

Fig. 1. System Model

2. System Model

We consider a CRN, as depicted in Fig.1, which consists of a pair of PUs, primary transmitter (PS) and primary destination (PD). The secondary system consists of a cognitive source (S), a cognitive relay (R), and a cognitive destination (D). The channel gains between any two nodes are Nakagami-m fading. Therefore, the channel gains follow gamma distribution with fading severity parameter m and average power Ω. Specifically, the communication in the secondary system occurs in two phases. In the first phase, S broadcasts the signal to R and D with transmit power P_S. In the second phase, R decodes and forwards the resulting signal to D. At the cognitive destination, the two signals are combined using a selection combining (SC) scheme.

The peak interference power constraint at the primary destination is denoted as Q, which is fixed as a constant to guarantee that the secondary signals do not violate the PU. As such, the transmit powers at S and R are expressed as:

$$P_S = \frac{Q}{|h_3|^2} \tag{1}$$

and,

$$P_R = \frac{Q}{|h_4|^2}, \tag{2}$$

respectively, where $|h_3|^2$ and $|h_4|^2$ represent the channel gains between S and PD, and R and PD, respectively. Specfically, all the channel gains[1] $|h_0|^2, |h_1|^2, |h_2|^2, |h_3|^2, |h_4|^2, |h_5|^2$ and $|h_6|^2$ follow a Nakagami-m distribution with fading parameter m_i[2] and $\Omega_i, i = 0,1,...,6$.

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1 In this paper, we assume that the secondary user knows the channel gains perfectly. The outdated channel gains[17]-[19] are not considered in this paper.

2 In this paper, we only consider the case when fading parameter m_i is an integer. The case when fading parameter m_i is not an integer is not in scope of this paper.

As such, the probability density function (PDF) and cumulative distribution function (CDF) of a gamma random variable with parameters m and Ω can be expressed as:

$$f_{|h|^2}(t)=\frac{\beta^m}{\Gamma(m)}t^{m-1}e^{-\beta t} \tag{3}$$

and,

$$F_{|h|^2}(t)=\frac{\gamma(m,\beta t)}{\Gamma(m)}, \tag{4}$$

respectively, where $\beta=\frac{m}{\Omega}$, $\gamma(a,t)$ denotes the incomplete gamma function [20], $\Gamma(m)$ represents the gamma function [20].

The received signals at R and D are impacted by interference from the PS. This is due to the co-existence of PS to PD transmission. Therefore, the received signal-to-interference ratio (SIR[3]) at D from the relay link and the direct link are denoted by:

$$\gamma_{DF}=\min\left(\frac{Q}{|h_3|^2}\frac{|h_1|^2}{P_P|h_5|^2},\frac{Q}{|h_4|^2}\frac{|h_2|^2}{P_P|h_6|^2}\right) \tag{5}$$

and,

$$\gamma_{DT}=\frac{Q}{|h_3|^2}\frac{|h_0|^2}{P_P|h_6|^2}, \tag{6}$$

respectively, where P_P denotes the transmit power of PU, $|h_0|,|h_1|,|h_2|,|h_5|$ and $|h_6|$ are the channel coefficients of $S\to D, S\to R, R\to D, PS\to R$ and $PS\to D$, respectively. As such, the end-to-end instantaneous SIR at the cognitive destination can be denoted as

$$\gamma_D=\max\{\gamma_{DF},\gamma_{DT}\}. \tag{7}$$

3. Outage Probability Analysis

3.1 Outage Probability

In this section, we derive the exact outage probability of the CRN impacted by the interference from PU. The outage probability, i.e., the probability that the end-to-end SIR falls below a certain threshold γ, can be expressed as:

$$P_{out}=\Pr\{\gamma_D\le\gamma\}=F_{\gamma_D}(\gamma). \tag{8}$$

Our aim is to derive the cumulative distribution function (CDF) of γ_D. From (5), (6), and (7), we can conclude that γ_{DF} and γ_{DT} are not independent due to the presence of two common random variables $|h_3|^2$ and $|h_6|^2$. To address this issue, we use the analytical approach proposed in [21]. For simplicity of analysis, we set $X=|h_3|^2$ and $Y=|h_6|^2$. Therefore, the CDF of γ_D, conditioned on X and Y can be written as

[3] In this paper, we focused on the interference-limited scenario where the interference power from the PU is dominant relative to the noise, and therefore noise effects can be nelected [14].

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$$F_{\gamma_D}(\gamma \mid X,Y) = F_{\gamma_{DF}}(\gamma \mid X,Y) F_{\gamma_{DT}}(\gamma \mid X,Y). \tag{9}$$

The main task is to derive the $F_{\gamma_{DF}}(\gamma \mid X,Y)$ and $F_{\gamma_{DT}}(\gamma \mid X,Y)$. The $F_{\gamma_{DF}}(\gamma \mid X,Y)$ can be expressed as:

$$F_{\gamma_{DF}}(\gamma \mid X,Y) = 1-[1-F_M(\gamma \mid X,Y)][1-F_N(\gamma \mid X,Y)], \tag{10}$$

where $M = \frac{\bar{\gamma} |h_1|^2}{X |h_5|^2}$ and $N = \frac{\bar{\gamma} |h_2|^2}{Y |h_4|^2}$, $\bar{\gamma} = \frac{Q}{P_P}$. Therefore, M is independent to Y, and N is independent to X. Consequently, $F_M(\gamma \mid X,Y)$ and $F_N(\gamma \mid X,Y)$ can be calculated as:

$$\begin{aligned} F_M(\gamma \mid X,Y) &= \int_0^\infty \frac{\beta_5^{m_5}}{\Gamma(m_5)} t^{m_5-1} e^{-\beta_5 t} \frac{\gamma(m_1, \frac{\beta_1 \gamma X t}{\bar{\gamma}})}{\Gamma(m_1)} dt \\ &= 1 - \int_0^\infty \frac{\beta_5^{m_5}}{\Gamma(m_5)} t^{m_5-1} e^{-\beta_5 t} e^{-\frac{\beta_1 \gamma X t}{\bar{\gamma}}} \sum_{k_1=0}^{m_1-1} \frac{(\beta_1 \gamma X)^{k_1}}{\bar{\gamma}^{k_1} k_1!} t^{k_1} dt \\ &= 1 - \frac{\beta_5^{m_5}}{\Gamma(m_5)} \sum_{k_1=0}^{m_1-1} \frac{(\beta_1 \gamma X)^{k_1}}{\bar{\gamma}^{k_1} k_1!} \int_0^\infty t^{m_5+k_1-1} e^{-(\beta_5 + \frac{\beta_1 \gamma X}{\bar{\gamma}})t} dt \\ &= 1 - \frac{\beta_5^{m_5}}{\Gamma(m_5)} \sum_{k_1=0}^{m_1-1} \frac{(\beta_1 \gamma X)^{k_1}}{\bar{\gamma}^{k_1} k_1!} (m_5+k_1-1)! (\beta_5 + \frac{\beta_1 \gamma X}{\bar{\gamma}})^{-(m_5+k_1)}, \end{aligned} \tag{11}$$

$$F_N(\gamma \mid X,Y) = 1 - \frac{\beta_4^{m_4}}{\Gamma(m_4)} \sum_{k_2=0}^{m_2-1} \frac{(\beta_2 \gamma Y)^{k_1}}{\bar{\gamma}^{k_2} k_2!} (m_4+k_2-1)! (\beta_4 + \frac{\beta_2 \gamma Y}{\bar{\gamma}})^{-(m_4+k_2)}. \tag{12}$$

Based on (11) and (12), $F_{\gamma_{DF}}(\gamma \mid X,Y)$ can be expressed as:

$$\begin{aligned} F_{\gamma_{DF}}(\gamma \mid X,Y) = 1 &- \frac{\beta_4^{m_4}}{\Gamma(m_4)} \sum_{k_2=0}^{m_2-1} \frac{(\beta_2 \gamma Y)^{k_1}}{\bar{\gamma}^{k_2} k_2!} (m_4+k_2-1)! (\beta_4 + \frac{\beta_2 \gamma Y}{\bar{\gamma}})^{-(m_4+k_2)} \\ &\frac{\beta_5^{m_5}}{\Gamma(m_5)} \sum_{k_1=0}^{m_1-1} \frac{(\beta_1 \gamma X)^{k_1}}{\bar{\gamma}^{k_1} k_1!} (m_5+k_1-1)! (\beta_5 + \frac{\beta_1 \gamma X}{\bar{\gamma}})^{-(m_5+k_1)}. \end{aligned} \tag{13}$$

Similarly, $F_{\gamma_{DT}}(\gamma \mid X,Y)$ can be calculated as:

$$F_{\gamma_{DT}}(\gamma \mid X,Y) = \frac{\gamma(m_0, \frac{\beta_0 XY\gamma}{\bar{\gamma}})}{\Gamma(m_0)}. \tag{14}$$

The $F_{\gamma_D}(\gamma \mid X,Y)$ can be written as:

$$F_{\gamma_D}(\gamma|X,Y)=\frac{\gamma(m_0,\frac{\beta_0XY\gamma}{\overline{\gamma}})}{\Gamma(m_0)}(1-\frac{\beta_4{}^{m_4}}{\Gamma(m_4)}\sum_{k_2=0}^{m_2-1}\frac{(\beta_2\gamma Y)^{k_1}}{\overline{\gamma}^{k_2}k_2!}(m_4+k_2-1)!(\beta_4+\frac{\beta_2\gamma Y}{\overline{\gamma}})^{-(m_4+k_2)} \\ \frac{\beta_5{}^{m_5}}{\Gamma(m_5)}\sum_{k_1=0}^{m_1-1}\frac{(\beta_1\gamma X)^{k_1}}{\overline{\gamma}^{k_1}k_1!}(m_5+k_1-1)!(\beta_5+\frac{\beta_1\gamma X}{\overline{\gamma}})^{-(m_5+k_1)})=I_1-I_2, \quad (15)$$

where

$$I_1=\frac{\gamma(m_0,\frac{\beta_0XY\gamma}{\overline{\gamma}})}{\Gamma(m_0)}, \quad (16)$$

$$I_2=\frac{\gamma(m_0,\frac{\beta_0XY\gamma}{\overline{\gamma}})}{\Gamma(m_0)}\frac{\beta_4{}^{m_4}}{\Gamma(m_4)}\sum_{k_2=0}^{m_2-1}\frac{(\beta_2\gamma Y)^{k_1}}{\overline{\gamma}^{k_2}k_2!}(m_4+k_2-1)!(\beta_4+\frac{\beta_2\gamma Y}{\overline{\gamma}})^{-(m_4+k_2)} \\ \frac{\beta_5{}^{m_5}}{\Gamma(m_5)}\sum_{k_1=0}^{m_1-1}\frac{(\beta_1\gamma X)^{k_1}}{\overline{\gamma}^{k_1}k_1!}(m_5+k_1-1)!(\beta_5+\frac{\beta_1\gamma X}{\overline{\gamma}})^{-(m_5+k_1)}. \quad (17)$$

Consequently, the unconditional CDF of γ_D marginalized with respect to X and Y is expressed as:

$$F_{\gamma_D}(\gamma)=\int_0^\infty\int_0^\infty F_{\gamma_D}(\gamma|X,Y)f_X(x)f_Y(y)dxdy=I_3-I_4, \quad (18)$$

where

$$I_3=\int_0^\infty\int_0^\infty I_1 f_X(x)f_Y(y)dxdy \quad (19)$$

and,

$$I_4=\int_0^\infty\int_0^\infty I_2 f_X(x)f_Y(y)dxdy. \quad (20)$$

I_3 can be calculated using (3) and (16) in (19) as:

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$$I_3=\int_0^\infty\int_0^\infty \frac{\gamma(m_0,\frac{\beta_0 xy\gamma}{\bar{\gamma}})}{\Gamma(m_0)}\frac{\beta_3^{m_3}}{\Gamma(m_3)}x^{m_3-1}e^{-\beta_3 x}\frac{\beta_6^{m_6}}{\Gamma(m_6)}y^{m_6-1}e^{-\beta_6 y}dxdy$$

$$=\int_0^\infty\int_0^\infty (1-e^{-\frac{\beta_0 xy\gamma}{\bar{\gamma}}}\sum_{m=0}^{m_0}\frac{(\frac{\beta_0 xy\gamma}{\bar{\gamma}})^m}{m!})\frac{\beta_3^{m_3}}{\Gamma(m_3)}x^{m_3-1}e^{-\beta_3 x}\frac{\beta_6^{m_6}}{\Gamma(m_6)}y^{m_6-1}e^{-\beta_6 y}dxdy$$

$$=1-\int_0^\infty\int_0^\infty e^{-\frac{\beta_0 xy\gamma}{\bar{\gamma}}}\sum_{m=0}^{m_0}\frac{(\frac{\beta_0 xy\gamma}{\bar{\gamma}})^m}{m!}\frac{\beta_3^{m_3}}{\Gamma(m_3)}x^{m_3-1}e^{-\beta_3 x}\frac{\beta_6^{m_6}}{\Gamma(m_6)}y^{m_6-1}e^{-\beta_6 y}dxdy$$

$$=1-\sum_{m=0}^{m_0}\frac{(\frac{\beta_0\gamma}{\bar{\gamma}})^m}{m!}\frac{\beta_3^{m_3}}{\Gamma(m_3)}\frac{\beta_6^{m_6}}{\Gamma(m_6)}\int_0^\infty\int_0^\infty e^{-(\frac{\beta_0\gamma}{\bar{\gamma}}y+\beta_3)x}x^{m+m_3-1}y^{m+m_6-1}e^{-\beta_6 y}dxdy$$

$$=1-\sum_{m=0}^{m_0}\frac{(\frac{\beta_0\gamma}{\bar{\gamma}})^m}{m!}\frac{\beta_3^{m_3}\beta_6^{m_6}(m+m_3-1)!}{\Gamma(m_3)\Gamma(m_6)}\underbrace{\int_0^\infty(\frac{\beta_0\gamma}{\bar{\gamma}}y+\beta_3)^{-(m+m_3)}y^{m+m_6-1}e^{-\beta_6 y}dy}_{I_5}, \tag{21}$$

According to the [20 (9.211.4)], the I_5 can be calculated as:

$$I_5=\beta_3^{-(m+m_3)}(\frac{\beta_3\bar{\gamma}}{\beta_0\gamma})^{m+m_6}\Gamma(m+m_6)\Psi(m+m_6,m_6-m_3+1;\beta_6\frac{\beta_3\bar{\gamma}}{\beta_0\gamma}). \tag{22}$$

where

$$\Psi(a,\gamma;z)=\frac{1}{\Gamma(a)}\int_0^\infty e^{-zt}t^{a-1}(1+t)^{\gamma-a-1}dt,\quad [\mathrm{Re}\,a>0,\mathrm{Re}\,z>0]. \tag{23}$$

Similarly, I_4 can be calculated as:

$$I_2=I_6+I_7, \tag{24}$$

where I_6 and I_7 are shown as:

$$I_6=\int_0^\infty\int_0^\infty \frac{\beta_4^{m_4}}{\Gamma(m_4)}\sum_{k_2=0}^{m_2-1}\frac{(\beta_2\gamma Y)^{k_1}}{\bar{\gamma}^{k_2}k_2!}(m_4+k_2-1)!(\beta_4+\frac{\beta_2\gamma Y}{\bar{\gamma}})^{-(m_4+k_2)}\frac{\beta_3^{m_3}}{\Gamma(m_3)}x^{m_3-1}e^{-\beta_3 x}$$
$$\frac{\beta_5^{m_5}}{\Gamma(m_5)}\sum_{k_1=0}^{m_1-1}\frac{(\beta_1\gamma X)^{k_1}}{\bar{\gamma}^{k_1}k_1!}(m_5+k_1-1)!(\beta_5+\frac{\beta_1\gamma X}{\bar{\gamma}})^{-(m_5+k_1)}\frac{\beta_6^{m_6}}{\Gamma(m_6)}y^{m_6-1}e^{-\beta_6 y}dxdy, \tag{25}$$

$$I_7=\int_0^\infty\int_0^\infty e^{-\frac{\beta_0 xy\gamma}{\bar{\gamma}}}\sum_{m=0}^{m_0}\frac{(\frac{\beta_0 xy\gamma}{\bar{\gamma}})^m}{m!}\frac{\beta_4^{m_4}}{\Gamma(m_4)}\sum_{k_2=0}^{m_2-1}\frac{(\beta_2\gamma Y)^{k_1}}{\bar{\gamma}^{k_2}k_2!}(m_4+k_2-1)!(\beta_4+\frac{\beta_2\gamma Y}{\bar{\gamma}})^{-(m_4+k_2)}\frac{\beta_3^{m_3}}{\Gamma(m_3)} \tag{26}$$
$$x^{m_3-1}e^{-\beta_3 x}\frac{\beta_5^{m_5}}{\Gamma(m_5)}\sum_{k_1=0}^{m_1-1}\frac{(\beta_1\gamma X)^{k_1}}{\bar{\gamma}^{k_1}k_1!}(m_5+k_1-1)!(\beta_5+\frac{\beta_1\gamma X}{\bar{\gamma}})^{-(m_5+k_1)}\frac{\beta_6^{m_6}}{\Gamma(m_6)}y^{m_6-1}e^{-\beta_6 y}dxdy.$$

After some mathematical manipulations, I_6 can be derived as:

$$I_6=\frac{\beta_3^{m_3}}{\Gamma(m_3)}\frac{\beta_4^{m_4}}{\Gamma(m_4)}\frac{\beta_5^{m_5}}{\Gamma(m_5)}\frac{\beta_6^{m_6}}{\Gamma(m_6)}\sum_{k_2=0}^{m_2-1}\frac{(\beta_2\gamma)^{k_1}}{\bar{\gamma}^{k_2}k_2!}(m_4+k_2-1)!\sum_{k_1=0}^{m_1-1}\frac{(\beta_1\gamma)^{k_1}}{\bar{\gamma}^{k_1}k_1!}(m_5+k_1-1)!I_8, \quad (27)$$

where

$$\begin{aligned}I_8&=\int_0^\infty\int_0^\infty e^{-\beta_3 x}x^{k_1+m_5-1}(\beta_5+\frac{\beta_1\gamma}{\bar{\gamma}}x)^{-(k_1+m_5)}e^{-\beta_6 y}y^{k_2+m_6-1}(\beta_4+\frac{\beta_2\gamma}{\bar{\gamma}}y)^{-(k_2+m_4)}dxdy\\&=\int_0^\infty e^{-\beta_3 x}x^{k_1+m_5-1}(\beta_5+\frac{\beta_1\gamma}{\bar{\gamma}}x)^{-(k_1+m_5)}dx\int_0^\infty e^{-\beta_6 y}y^{k_2+m_6-1}(\beta_4+\frac{\beta_2\gamma}{\bar{\gamma}}y)^{-(k_2+m_4)}dy\\&=(\frac{\bar{\gamma}}{\beta_1\gamma})^{k_1+m_5}\Gamma(k_1+m_5)\Psi(k_1+m_5,1;\frac{\beta_3\beta_5\bar{\gamma}}{\beta_1\gamma})(\frac{\bar{\gamma}}{\beta_2\gamma})^{k_2+m_4}\Gamma(k_2+m_4)\Psi(k_2+m_4,1;\frac{\beta_6\beta_4\bar{\gamma}}{\beta_2\gamma}).\end{aligned} \quad (28)$$

To this end, the last task is to derive I_7. I_7 can be written as:

$$I_7=\frac{\beta_3^{m_3}}{\Gamma(m_3)}\frac{\beta_4^{m_4}}{\Gamma(m_4)}\frac{\beta_5^{m_5}}{\Gamma(m_5)}\frac{\beta_6^{m_6}}{\Gamma(m_6)}\sum_{m=0}^{m_0-1}\frac{(\beta_2\gamma)^{m}}{\bar{\gamma}^{m}m!}\sum_{k_2=0}^{m_2-1}\frac{(\beta_2\gamma)^{k_1}}{\bar{\gamma}^{k_2}k_2!}(m_4+k_2-1)!\sum_{k_1=0}^{m_1-1}\frac{(\beta_1\gamma)^{k_1}}{\bar{\gamma}^{k_1}k_1!}(m_5+k_1-1)!I_9, \quad (29)$$

where

$$I_9=\int_0^\infty\int_0^\infty e^{-\frac{\beta_0\gamma}{\bar{\gamma}}xy}e^{-\beta_3 x}x^{m+k_1+m_5-1}(\beta_5+\frac{\beta_1\gamma}{\bar{\gamma}}x)^{-(k_1+m_5)}e^{-\beta_6 y}y^{m+k_2+m_6-1}(\beta_4+\frac{\beta_2\gamma}{\bar{\gamma}}y)^{-(k_2+m_6)}dxdy. \quad (30)$$

After some mathematical manipulations, I_9 can be re-written as:

$$I_9=\sum_{k_3=0}^{k_1+m_5}\sum_{k_4=0}^{k_2+m_6}\frac{(m+k_3-1)!(\frac{\beta_3\bar{\gamma}}{\beta_0\gamma})^{m_4+m}\Psi(m+m_4,m_4-k_3+1;\frac{\beta_6\beta_3\bar{\gamma}}{\beta_0\gamma})}{C_{k_1+m_5}^{k_3}\beta_5^{k_3}(\frac{\beta_1\gamma}{\bar{\gamma}})^{k_1+m_5-k_3}C_{k_2+m_6}^{k_4}\beta_4^{k_4}(\frac{\beta_2\gamma}{\bar{\gamma}})^{k_2+m_6-k_4}}. \quad (31)$$

3.2 Asymptotic Analysis

We derive the asymptotic analysis to understand the impacts of the parameters on the outage performance of the secondary network. The coding gain and diversity can be obtained from this information. We note the following asymptotic behavior of an incomplete gamma function near zero:

$$F_{|h|^2}(t)=\frac{\gamma(m,\beta t)}{\Gamma(m)}=\frac{(\beta t)^m}{\Gamma(m+1)},\quad t\to 0. \quad (32)$$

As such, $F_M(\gamma\,|\,X,Y)$ can be re-written as:

$$\begin{aligned}F_M(\gamma\,|\,X,Y)&=\int_0^\infty\frac{\beta_5^{m_5}}{\Gamma(m_5)}t^{m_5-1}e^{-\beta_5 t}\frac{\gamma(m_1,\frac{\beta_1\gamma Xt}{\bar{\gamma}})}{\Gamma(m_1)}dt\\&=\int_0^\infty\frac{\beta_5^{m_5}}{\Gamma(m_5)}t^{m_5-1}e^{-\beta_5 t}\frac{(\frac{\beta_1\gamma Xt}{\bar{\gamma}})^{m_1}}{\Gamma(m_1+1)}dt\\&=\frac{\beta_1^{m_1}(m_1+m_5-1)!}{\beta_5^{m_1}\Gamma(m_1+1)\Gamma(m_5)}(\frac{\gamma}{\bar{\gamma}})^{m_1}x^{m_1}.\end{aligned} \quad (33)$$

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Similarly, $F_N(\gamma \mid X, Y)$ can be re-written as:

$$F_N(\gamma \mid X, Y) = \frac{\beta_2^{m_2}(m_2 + m_4 - 1)!}{\beta_4^{m_2}\Gamma(m_2 + 1)\Gamma(m_4)}(\frac{\gamma}{\bar{\gamma}})^{m_2} y^{m_2}. \quad (34)$$

Omitting the higher-order terms, we obtain:

$$F_{\gamma_{DF}}(\gamma \mid X, Y) = F_M(\gamma \mid X, Y) + F_N(\gamma \mid X, Y). \quad (35)$$

To this end, $F_{\gamma_{DT}}(\gamma \mid X, Y)$ can be re-calculated as:

$$F_{\gamma_{DT}}(\gamma \mid X, Y) = \frac{\beta_0^{m_0}}{\Gamma(m_0 + 1)}(\frac{\gamma}{\bar{\gamma}})^{m_0} x^{m_0} y^{m_0}. \quad (36)$$

Therefore, the CDF of γ_D at the high transmit power can be re-written as:

$$F_{\gamma_D}(\gamma) = \int_0^\infty \int_0^\infty F_{\gamma_D}(\gamma \mid X, Y) f_X(x) f_Y(y) dxdy = I_{10} + I_{11}, \quad (37)$$

where

$$I_{10} = \int_0^\infty \int_0^\infty F_M(\gamma \mid X, Y) F_{\gamma_{DT}}(\gamma \mid X, Y) f_X(x) f_Y(y) dxdy \quad (38)$$

and,

$$I_{11} = \int_0^\infty \int_0^\infty F_N(\gamma \mid X, Y) F_{\gamma_{DT}}(\gamma \mid X, Y) f_X(x) f_Y(y) dxdy. \quad (39)$$

After substituting (3) and (33) into (38), I_{10} can be represented, after some algebraic manipulations by:

$$I_{10} = C_1(\frac{\gamma}{\bar{\gamma}})^{m_1 + m_0}, \quad (40)$$

where

$$C_1 = \frac{\beta_0^{m_0}\beta_1^{m_1}(m_1 + m_5 - 1)!(m_0 + m_3 + m_5 - 1)!(m_0 + m_6 - 1)!}{\beta_3^{m_0 + m_5}\beta_5^{m_1}\beta_6^{m_0}\Gamma(m_0 + 1)\Gamma(m_1 + 1)\Gamma(m_3)\Gamma(m_5)\Gamma(m_6)}. \quad (41)$$

Similarly, I_{11} can be expressed as:

$$I_{11} = C_2(\frac{\gamma}{\bar{\gamma}})^{m_2 + m_0}, \quad (42)$$

where

$$C_2 = \frac{\beta_0^{m_0}\beta_2^{m_2}(m_2 + m_4 - 1)!(m_0 + m_3 + m_4 - 1)!(m_0 + m_6 - 1)!}{\beta_3^{m_0 + m_4}\beta_4^{m_2}\beta_6^{m_0}\Gamma(m_0 + 1)\Gamma(m_2 + 1)\Gamma(m_3)\Gamma(m_4)\Gamma(m_6)}. \quad (43)$$

To this end, the asymptotic outage probability can be expressed as:

$$P_{out} = F_{\gamma_D}(\gamma) = C_D(\frac{\gamma}{\bar{\gamma}})^{\min(m_1, m_2) + m_0}, \quad (44)$$

where

$$C_D = \begin{cases} C_1, & m_1 < m_2 \\ C_1 + C_2, & m_1 = m_2 \\ C_2, & m_1 > m_2 \end{cases}. \quad (45)$$

Remarks: The diversity order of spectrum sharing is in line with those obtained from traditional dual-hop systems. Specifically, the cognitive system can obtain full diversity order of $\min(m_1, m_2) + m_0$, regardless of the primary network. As such, the diversity-multiplexing tradeoff is independent of the primary network. The coding gain of the spectrum sharing system under consideration is the only system affected by the primary network.

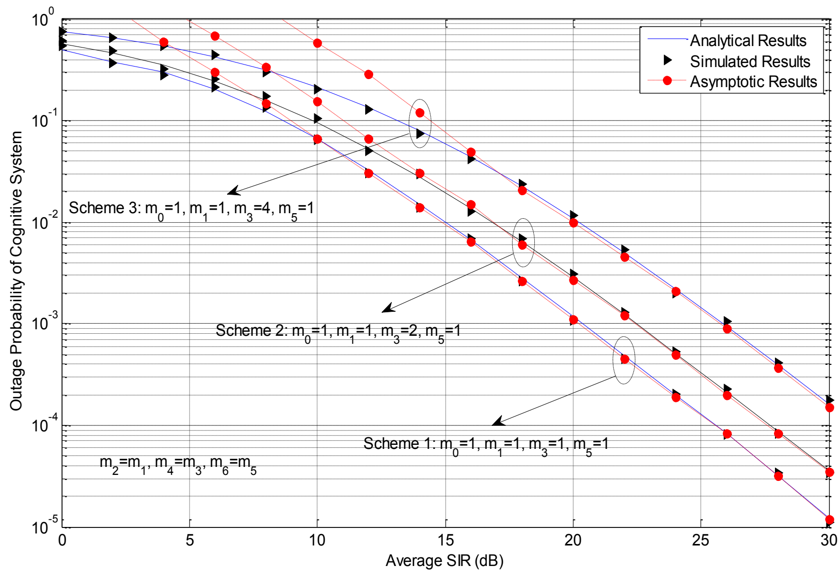

Fig. 2. Impacts of PU on the outage performance of cognitive relay networks.

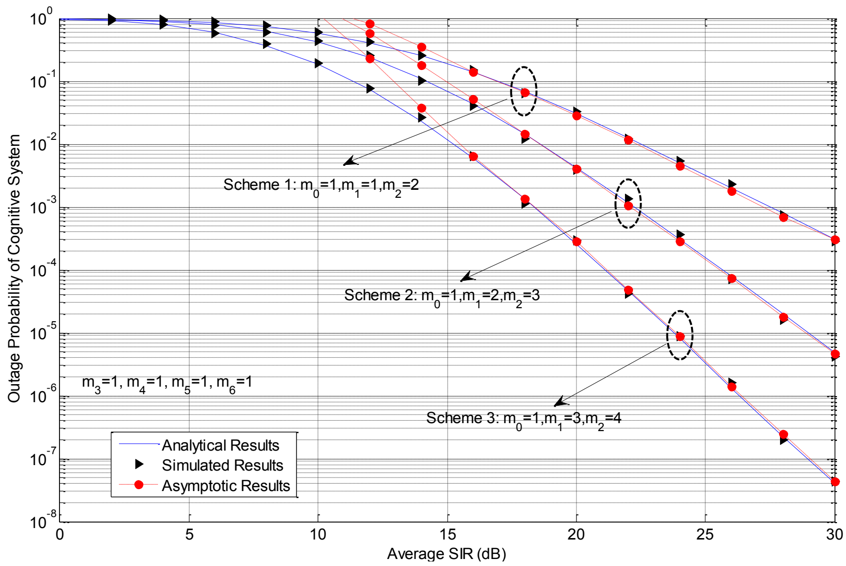

Fig. 3. Impacts of cognitive system on the outage performance of cognitive relay networks.

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4. Numerical Results

In this section, we numerically evaluate the outage probability of the considered system. The simulated results are obtained using the expectation over 10^9 independent trials.

Fig. 2 evaluates the impact of the primary network on the outage performance of the cognitive relay netowrk. The fading severity of the cognitive system remains fixed. Three schemes are presented. Specifically, we consider a symmetric system in the simulation. As such, the parameters are selected as Scheme 1: $\{m_i\}_{i=0,1,..,6} = \{1,1,1,1,1,1,1\}$, Scheme 2: $\{m_i\}_{i=0,1,..,6} = \{1,1,1,2,2,1,1\}$, Scheme 3: $\{m_i\}_{i=0,1,..,6} = \{1,1,1,4,4,1,1\}$. We clearly observe that the diversity is not affected by the fading parameters of the primary network. More specifically, the primary network only affects the coding gain of the considered system. This validates our analytical results.

Fig. 3 evaluates the impact of the fading severity of the cognitive system on the outage performance of cognitive relay networks. We keep the fading severity from the primary system fixed. Three schemes are presented. As such, the parameters are selected as: Scheme 1: $\{m_i\}_{i=0,1,..,6} = \{1,1,2,2,2,2,2\}$, Scheme 2: $\{m_i\}_{i=0,1,..,6} = \{1,2,3,2,2,2,2\}$, Scheme 3: $\{m_i\}_{i=0,1,..,6} = \{1,3,4,2,2,2,2\}$. Results indicate that the diversity order is strictly determined by the dual-hop links and the direct link of the cognitive relay network. More specifically, from Fig. 2 and Fig. 3, the analytical results and the Monte Carlo simulation results are very close. In addition, the asymptotic results are aligned with the analytical results in the high SIR regime. This indicates the validity of the analytical results.

Fig. 4 evaluated the outage performance under the adjustable PU's transmit power with different fading parameters. Three schemes are presented. Therefore, the parameters are selected as: Scheme 1: $\{m_i\}_{i=0,1,..,6} = \{1,1,1,1,1,1,1\}$, Scheme 2: $\{m_i\}_{i=0,1,..,6} = \{1,2,2,1,1,1,1\}$, Scheme 3: $\{m_i\}_{i=0,1,..,6} = \{1,1,1,2,2,1,1\}$. Results illustrate that there is an exact match between the analytic results and the Monte Carlo simulation results. We also observe that the outage probability will increase as the transmit power of PU increases. Increasing the quality of links in the cognitive system will improve the performance of the system.

Fig. 5 evaluates the outage performance of cognitive system versus maximum interference power of the primary user in different fading parameters. Similarly, three schemes are considered: Scheme 1: $\{m_i\}_{i=0,1,..,6} = \{1,1,1,1,1,1,1\}$, Scheme 2: $\{m_i\}_{i=0,1,..,6} = \{1,2,2,1,1,1,1\}$, Scheme 3: $\{m_i\}_{i=0,1,..,6} = \{1,1,1,2,2,1,1\}$. Fig. 5 illustrates that the outage probability decreases with the primary user's increase in maximum interference power.

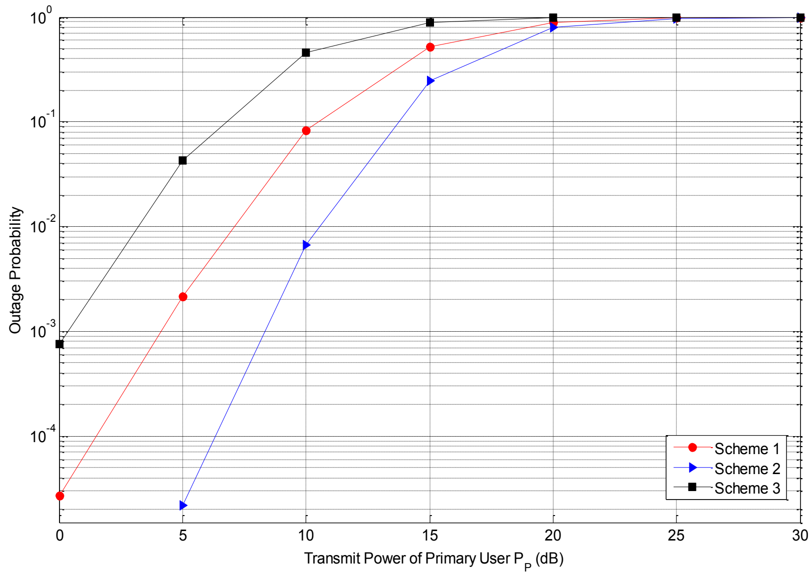

Fig. 4. Outage performance of cognitive relay networks: varying the transmit power of PU when $Q = 15$ dB.

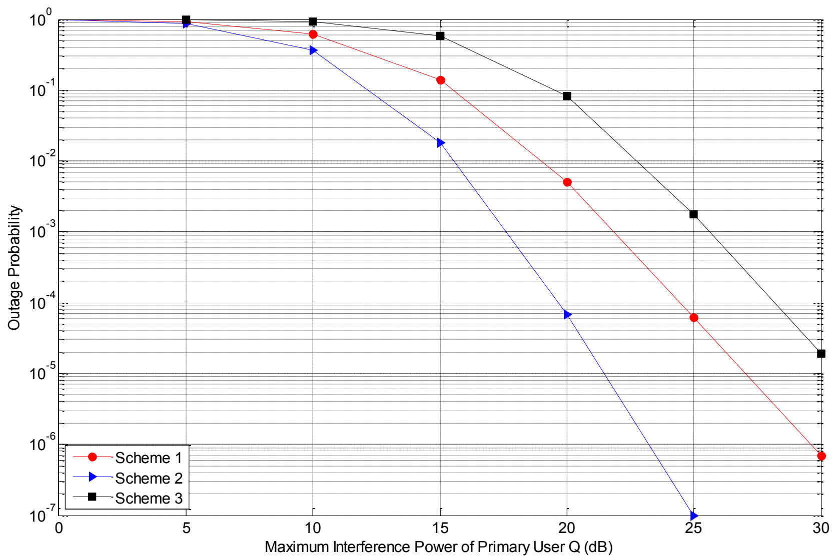

Fig. 5. Outage performance of cognitive relay networks: varying the maximum interference power constraint of PU when $P_P = 10\,dB$.

5. Conclusion and Future Work

In this paper, the outage performance of underlay cognitive relay networks with SC diversity was investigated in an independent, non-identical distributed Nakagami-m fading environment. The analytical results obtained proved effective in measuring the effects of system parameters. We derive the asymptotic expression in order to study the effect of the related parameters on the outage performance of CRNs. The diversity order of cognitive system is only determined by the fading severity of cognitive system, being

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therefore not affected by the primary network. Specifically, the cognitive relay network can obtain full diversity regardless of the transmit power constraint, and the primary networks only affect the coding gain of the cognitive system.

We have studied the effect of a single primary user on the outage performance of the cognitive relay network. In future work, we intend to extend and generalize this to cases of multiple relays and multiple primary users. Specifically, we will also accept a maximum allowable transmit power of the cognitive relay system into consideration.

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