MMSE Transmit Optimization for Multiuser Multiple-Input Single-Output Broadcasting Channels in Cognitive Radio Networks

Huijin Cao[1], Yanhui Lu[1] and Jun Cai[2]
[1] School of Information Engineering, Zhengzhou University
Zhengzhou, 450001 - China
[e-mail: 871307578@qq.com ieyhlu@zzu.edu.cn]
[2] Electrical and Computer Engineering, University of Manitoba
Winnipeg, MB R3T 2N2 - Canada
[e-mail: jcai2004@gmail.com]
*Corresponding author: Yanhui Lu

Received May 20, 2013; revised August 10, 2013; revised August 20, 2013; accepted August 28, 2013; published September 30, 2013

Abstract

In this paper, we address the problem of linear minimum mean-squared error (MMSE) transmitter design for the cognitive radio (CR) multi-user multiple-input single-output (MU-MISO) broadcasting channel (BC), where the cognitive users are subject to not only a sum power constraint, but also a interference power constraint. Evidently, this multi-constraint problem renders it difficult to solve. To overcome this difficulty, we firstly transform it into its equivalent formulation with a single constraint. Then by utilizing BC-MAC duality, the problem of BC transmitter design can be solved by focusing on a dual MAC problem, which is easier to deal with due to its convexity property. Finally we propose an efficient two-level iterative algorithm to search the optimal solution. Our simulation results are provided to corroborate the effectiveness of the proposed algorithm and show that this proposed CR MMSE-based scheme achieves a suboptimal sum-rate performance compared to the optimal DPC-based algorithm with less computational complexity.

Keywords: Cognitive radio, broadcast channel, multiple access channel, multi-user MISO, MMSE.

This work was supported in part by the Program for New Century Excellent Talents in University under Grant No.NCET-12-0699, National Natural Science Foundation of China under Grant No.61271421, Program for Young Teachers in Colleges and University in Henan Province under Grand No.2011GGJS-002, and Henan Province Major science and technology project under Grant No.112102210507.

http://dx.doi.org/10.3837/tiis.2013.09.003

1. Introduction

Recently, the concept of cognitive radio (CR) has been proposed in order to improve the spectrum utilization efficiency [1]. In CR, a secondary (unlicensed) user (SU) is allowed to opportunistically or concurrently access the spectrum with the primary (licensed) users (PUs) as long as it won't introduce harmful interference. However, the introduction of CR raises new challenges in the network design. One of them is the transmitter design in multiuser CR networks with multiple-input single-output broadcasting channel (MU-MISO-BC). Different from the traditional wireless networks, in CR, SUs are subject to not only a sum power constraint at the transmitter, but also the interference power constraint at the PU. Such multiple-constraint property makes the solutions in traditional networks infeasible for CR. Although a dirty paper coding (DPC) [2] based algorithm was proposed in [3] that maximizes the weighted sum rate of the CR MU MIMO-BC, it is difficult to implement in practical systems due to its non-linear, since a high computational burden is inevitable. In order to avoid the high complexity of the DPC based nonlinear algorithm, many linear precoding schemes are attempted to be extended to the CR network [4-8]. Here, the linear precoder method based on minimum mean-squared error (MMSE) criterion is considered, which achieves a suboptimal sum-rate performance compared to the sum capacity found for CR DPC-based algorithm, but has much lower computational complexity.

In this paper, the problem of minimizing the sum of all normalized MSE for the K SUs in CR MISO-BC is discussed. Owing to the coupled structure of the transmitted signals, optimization problems associated with BC are typically non-convex, and are difficult to solve directly. The key technique used to overcome this difficulty is to transform the BC problem into its convex multiple access channel (MAC) problem via a BC-MAC duality relationship. The conventional BC-MAC duality is established via BC-MAC signal transformation, and has been successfully applied to solve beamforming optimization [9], signal-to-interference-plus-noise ratio (SINR) balancing [10], and capacity region computation [11-13]. However, this conventional duality approach is applicable only to the case with a single sum power constraint. Due to this limitation, the previous algorithms relying crucially on a single sum power constraint are not applicable to the problem with multiple linear constraints, which is the case of interest in this paper. Beginning with formulating the multi-constraint CR MISO-BC problem, we first transform it into an equivalent single-constraint optimization problem with multiple auxiliary variables. By fixing the auxiliary variables, a dual single-input multiple-output multiple access channel (SIMO-MAC) problem is derived based on the results in [14], which maintains the same MSE achievable region as that of the original MISO-BC. Next, we propose an *Iterative Power Allocation Algorithm* to solve the dual SIMO-MAC problem, and then map the results to the BC MMSE-based linear precoding. After that, a *Complete Iterative Algorithm* is proposed to update the auxiliary variables and solve the original optimization problem formulated.

The following notations are used in this paper. Bold upper and lower case letters denote matrices and vectors, respectively; $(\ )^*$ and $(\ )^T$ denote the conjugate transpose and transpose respectively; $\boldsymbol{I}_M$ denotes an $M \times M$ identity matrix; tr( ) denotes the trace of a matrix; $[x]^+$ denotes $\max(x, 0)$; $(\ )^b$ and $(\ )^m$ denote the quantities associated with a broadcast channel and a multiple access channel respectively; E[ ] denotes the expectation operator.

2. System Model and Problem Formulation

We consider a CR system as shown in Fig. 1, where the MU-MISO-BC consists of K SUs coexisting with one PU. The secondary base station (SBS) accesses the licensed spectrum to broadcast data to K SUs. The SBS has M antennas, while both SUs and the PU equip a single antenna.

The transmit-receive signal model from the SBS to the i^{th} SU, denoted by SU_i, for i= 1,..., K, can be expressed as

$$y_i = \boldsymbol{h}_i \boldsymbol{x} + z_i \,, \tag{1}$$

where y_i is the received signal, h_i is the $1 \times M$ channel vector from SBS to SU_i, and z_i is the Gaussian noise with zero mean and variance σ^2. $\boldsymbol{x} = \boldsymbol{UQd}$ is the $M \times 1$ transmitted signal vector, where $\boldsymbol{d} = [d_1,...,d_K]^T$ denotes unity-energy random transmit symbols with E{$\boldsymbol{dd}$*}=$\boldsymbol{I}_K$; $\boldsymbol{U}$ = [$\boldsymbol{u}_1$,...,$\boldsymbol{u}_K$] ($\|\boldsymbol{u}_i\|_2$=1) denotes the normalized beamforming matrix for the transmitted symbols, and $\boldsymbol{u}_i$ maps the transmitted signal d_i for SU_i onto M transmit antennas; $\boldsymbol{Q} = diag\{\boldsymbol{q}\}$ and the vector $\boldsymbol{q} = [q_1,...,q_K]$ denotes the transmission power for each SU. The received signal y_i is scaled by $\beta_i / \sqrt{q_i}$, where β_i adds additional degree of freedom which can be used for MSE optimization， and $\boldsymbol{\beta} = [\beta_1, \beta_2, ..., \beta_K]$. For simplifying the notation, we define $\boldsymbol{h}_0$ to represent the PU's interference channel (PUI) caused by SBS, which is a $M \times 1$ channel gain vector between the transmitters of SBS and the PU. We further assume that $\boldsymbol{h}_i$ for i = 1,...,K, and $\boldsymbol{h}_0$ are known to the SBS and SU_i.

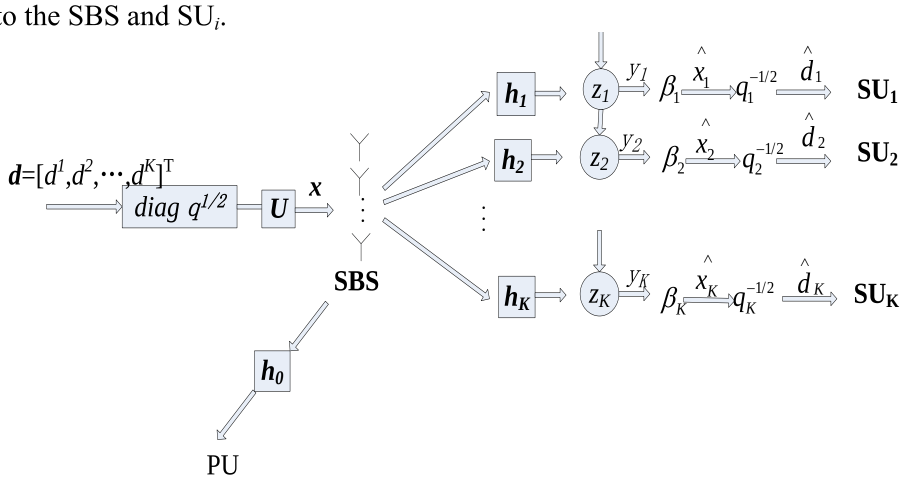

Fig. 1. System model

Based on the aforementioned system model, the problem of MMSE transmitter design for CR MU-MISO-BC can be formulated as:

Problem 1 (Main Problem):

$$\min_{\{\boldsymbol{Q}_i^b\}_{i=1}^K : \boldsymbol{Q}_i^b \geq 0, \boldsymbol{\beta}} \sum_{i=1}^{K} \varepsilon_i^b$$

$$\text{s.t.} \sum_{i=1}^{K} \boldsymbol{h}_0^* \boldsymbol{Q}_i^b \boldsymbol{h}_0 \leq P_t, and \sum_{i=1}^{K} tr(\boldsymbol{Q}_i^b) \leq P_u \,,$$

where ε_i^b is the individual normalized MSE, i.e., $\varepsilon_i^b = E[\left|\hat{d}_i - d_i\right|^2], \forall i \in \{1, 2, ..., K\}$. $\boldsymbol{Q}_i^b$ is the $M \times M$ transmit signal covariance matrix for SU_i and is semidefinite. P_t denotes the interference threshold of the PU, and P_u denotes the sum power constraint at the SBS. Compared with the similar problem under a non-CR setting, the key difference is that in addition to the sum power constraint, an interference power constraint is required.

3. CR MU-MISO Linear Processing

3.1 Equivalence

We transform Problem 1 into the following problem with a single constraint:

Problem 2 (MISO-BC)

$$g(q_t, q_u)_{q_t, q_u \geq 0} = \max_{\{\boldsymbol{Q}_i^b\}_{i=1}^K : \boldsymbol{Q}_i^b \geq 0, \beta} \sum_{i=1}^{K} -\varepsilon_i^b$$

$$\text{s.t.} \quad q_t\Big(\sum_{i=1}^{K} \boldsymbol{h}_0^* \boldsymbol{Q}_i^b \boldsymbol{h}_0 - P_t\Big) + q_u\Big(\sum_{i=1}^{K} tr(\boldsymbol{Q}_i^b) - P_u\Big) \leq 0,$$

where q_t and q_u are the two auxiliary variables.The relationship between Problem 1 and Problem 2 can be summarized as follows.

Proposition 1: the optimal solution of Problem 1 is equal to that of the problem $\min_{q_t, q_u} g(q_t, q_u)$.

Proof: Evidently, if $\boldsymbol{Q}_i^b, i = 1,\ldots,K$ are feasible for Problem 1, then it is also feasible for Problem 2. That is to say, the feasible region of Problem 1 is a subset of that of Problem 2. Therefore, the optimal solution of Problem 2 is an upper bound on that of Problem 1. Furthermore, we can prove that the upper bound is tight.

The KKT condition of the Problem 1 with respect to $\boldsymbol{Q}_i^b$ can be listed as follows:

$$\frac{\partial \sum_{i=1}^{K} -\varepsilon_i^b}{\partial \boldsymbol{Q}_i^b} = \mu_1 \boldsymbol{h}_0 \boldsymbol{h}_0^* + \mu_2 \boldsymbol{I}_M + \boldsymbol{\Omega}_i, \forall i \in \{1, 2, \ldots, K\}, \tag{2}$$

$$\mu_1\Big(\sum_{i=1}^{K} \boldsymbol{h}_0^* \boldsymbol{Q}_i^b \boldsymbol{h}_0 - P_t\Big) = 0, \tag{3}$$

$$\mu_2\Big(\sum_{i=1}^{K} tr(\boldsymbol{Q}_i^b) - P_u\Big) = 0, \tag{4}$$

where μ_1 and μ_2 are the Lagrange multipliers for the interference power constraint and the sum power constraint respectively; $\boldsymbol{\Omega}_i$ is the Lagrange multiplier associated with the constraint $\boldsymbol{Q}_i^b \geq 0$. When the optimal solution of Problem 1 is achieved, we assume that the corresponding optimal variables are $\boldsymbol{Q}_i^{b*} (i = 1,\ldots,K), \mu_1^*, \mu_2^*$ and $\boldsymbol{\Omega}_i^*$.

We now list the KKT conditions of Problem 2 as follows:

$$\frac{\partial \sum_{i=1}^{K} -\varepsilon_i^b}{\partial \boldsymbol{Q}_i^b} = \lambda(q_t \boldsymbol{h}_0 \boldsymbol{h}_0^* + q_u \boldsymbol{I}_M) + \boldsymbol{\Upsilon}_i, \forall i \in \{1,2,\ldots,K\}, \tag{5}$$

$$\lambda\Big(q_t \sum_{i=1}^{K} \boldsymbol{h}_0^* \boldsymbol{Q}_i^b \boldsymbol{h}_0 + q_u \sum_{i=1}^{K} tr(\boldsymbol{Q}_i^b) - q_t P_t - q_u P_u\Big) = 0, \tag{6}$$

where λ is the Lagrange multiplier, and γ_i is the Lagrange multiplier associated with the constraint $\boldsymbol{Q}_i^b \geq 0$. If we choose $\boldsymbol{Q}_i^b = \boldsymbol{Q}_i^{b*} (i = 1,\ldots,K), \lambda = 1, q_t = \mu_1^*, q_u = \mu_2^*, \gamma_i = \boldsymbol{\Omega}_i^*$, then the KKT conditions of Problem 2 are satisfied. In general, the KKT conditions are only necessary for a solution to be optimal for a non-convex problem. However, in the following we will show that for Problem 2, the KKT conditions are also sufficient for optimality.

According to Corollary 2 in [14], the achievable SINR region of the primal MISO-BC under the single constraint $q_t\sum_{i=1}^{K}\boldsymbol{h}_0^*\boldsymbol{Q}_i^b\boldsymbol{h}_0 + q_u\sum_{i=1}^{K}tr(\boldsymbol{Q}_i^b) \le q_tP_t + q_uP_u$ (see Fig. 2(a)), is equal to the achievable SINR region of its dual SIMO-MAC with a single weighted sum power constraint $\sum_{i=1}^{K}\sigma^2 tr(Q_i^m) \le q_tP_t + q_uP_u$ (see Fig. 2(b)), i.e., $SINR_i^b = SINR_i^m$, for i=1,2,...,K.

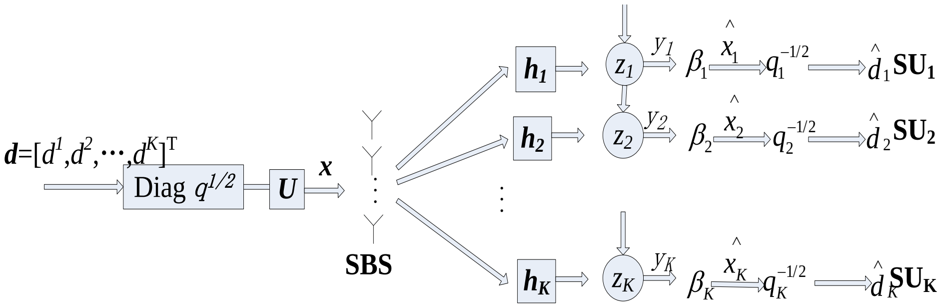

Fig. 2(a). MISO-BC system, the linear transmit covariance constraint is: $q_t\sum_{i=1}^{K}\boldsymbol{h}_0^*\boldsymbol{Q}_i^b\boldsymbol{h}_0 + q_u\sum_{i=1}^{K}tr(\boldsymbol{Q}_i^b) \le q_tP_t + q_uP_u$

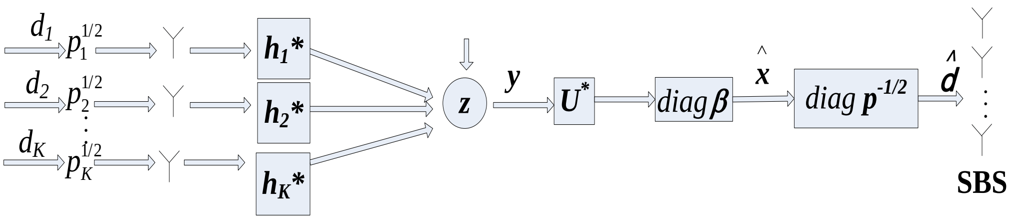

Fig. 2(b). dual SIMO-MAC system, $\sum_{i=1}^{K}\sigma^2 tr(Q_i^m) \le q_tP_t + q_uP_u$, $\boldsymbol{z} \sim N(0, q_t\boldsymbol{h}_0\boldsymbol{h}_0^* + q_u\boldsymbol{I}_M)$, $\boldsymbol{p}=[p_1,p_2,...,p_K]$ is the power allocation vector for the dual MAC

In Fig. 2(a),

$$SINR_i^b = \frac{q_i\boldsymbol{u}_i^*\boldsymbol{h}_i^*\boldsymbol{h}_i\boldsymbol{u}_i}{\sum_{j=1}^{K}q_j\boldsymbol{h}_i\boldsymbol{u}_j\boldsymbol{u}_j^*\boldsymbol{h}_i^* - q_i\boldsymbol{u}_i^*\boldsymbol{h}_i^*\boldsymbol{h}_i\boldsymbol{u}_i + \sigma^2}, \quad (7)$$

$$\begin{aligned}\varepsilon_i^b &= E[\left|\hat{d}_i - d_i\right|^2],\\ &= E[\left|\frac{\beta_i}{\sqrt{q_i}}(\boldsymbol{h}_i\boldsymbol{U}\sqrt{\boldsymbol{Q}}\boldsymbol{d} + z_i) - d_i\right|^2],\\ &= \frac{\beta_i^2}{q_i}\boldsymbol{h}_i\boldsymbol{U}\boldsymbol{Q}\boldsymbol{U}^*\boldsymbol{h}_i^* + \frac{\beta_i^2}{q_i}\sigma^2 - \beta_i\boldsymbol{h}_i\boldsymbol{u}_i - \beta_i\boldsymbol{u}_i^*\boldsymbol{h}_i^* + 1. \end{aligned} \quad (8)$$

Due to the coupled structure of the transmitted signals, we can get that ε_i^b is not differentiable on $\boldsymbol{Q}_i^b$ ($\boldsymbol{Q}_i^b = q_i\boldsymbol{u}_i\boldsymbol{u}_i^*$), that is why the optimization problems associated with the BC are usually non-convex.

While in Fig. 2 (b),

$$SINR_i^m = \frac{p_i \boldsymbol{u}_i^* \boldsymbol{h}_i^* \boldsymbol{h}_i \boldsymbol{u}_i}{\boldsymbol{u}_i^* (\sum_{j=1}^{K} p_j \boldsymbol{h}_j^* \boldsymbol{h}_j + q_t \boldsymbol{h}_0 \boldsymbol{h}_0^* + q_u \boldsymbol{I}_M - p_i \boldsymbol{h}_i^* \boldsymbol{h}_i) \boldsymbol{u}_i}, \tag{9}$$

$$\begin{aligned}\varepsilon_i^m &= E[\left|\hat{d}_i - d_i\right|^2], \\ &= E[\left|\frac{\beta_i}{\sqrt{p_i}} \boldsymbol{u}_i^* (\sum_{j=1}^{K} d_j \sqrt{p_j} \boldsymbol{h}_j^* + \boldsymbol{z}) - d_i\right|^2], \\ &= \frac{\beta_i^2}{p_i} \boldsymbol{u}_i^* (\sum_{j=1}^{K} p_j \boldsymbol{h}_j^* \boldsymbol{h}_j + q_t \boldsymbol{h}_0 \boldsymbol{h}_0^* + q_u \boldsymbol{I}_M) \boldsymbol{u}_i - \beta_i \boldsymbol{u}_i^* \boldsymbol{h}_i^* - \beta_i \boldsymbol{h}_i \boldsymbol{u}_i + 1. \end{aligned} \tag{10}$$

Since $SINR_i^b = SINR_i^m$, then

$$\frac{\boldsymbol{h}_i^* (\sum_{j=1}^{K} q_j \boldsymbol{u}_j \boldsymbol{u}_j^*) \boldsymbol{h}_i}{q_i} + \frac{\sigma^2}{q_i} = \frac{\boldsymbol{u}_i^* (\sum_{j=1}^{K} p_j \boldsymbol{h}_j^* \boldsymbol{h}_j) \boldsymbol{u}_i}{p_i} + \boldsymbol{u}_i^* \frac{q_t \boldsymbol{h}_0 \boldsymbol{h}_0^* + q_u \boldsymbol{I}_M}{p_i} \boldsymbol{u}_i$$

$$\beta_i^2 \frac{\boldsymbol{h}_i \boldsymbol{U} \boldsymbol{Q} \boldsymbol{U}^* \boldsymbol{h}_i^* + \sigma^2}{q_i} = \beta_i^2 \frac{\boldsymbol{u}_i^* (\sum_{j=1}^{K} p_j \boldsymbol{h}_j^* \boldsymbol{h}_j + q_t \boldsymbol{h}_0 \boldsymbol{h}_0^* + q_u \boldsymbol{I}_M) \boldsymbol{u}_i}{p_i}. \tag{11}$$

Combining (11) with (8) and (10), we can get that the primal MISO-BC and its dual SIMO-MAC also have the same MSE achievable region, which is shown in the following proposition.

Proposition 2: for fixed q_t and q_u, Problem 2 is equivalent to the following form:

Problem 3 (dual SIMO-MAC):

$$\max_{\boldsymbol{p}, \boldsymbol{U}, \boldsymbol{\beta}} \sum_{i=1}^{K} -\varepsilon_i^m$$

$$\text{s.t.} \sum_{i=1}^{K} p_i \le P, P = q_t P_t + q_u P_u .$$

According to the equation (10), we can see that ε_i^m is convex on p_i, u_i, and β_i. Thus, the Problem 3 is a convex optimization problem.

We now assume that $\ddot{\boldsymbol{Q}}_i^b, i = 1, 2, ..., K$ satisfy the KKT conditions in (5) (6), and achieve the sum MSE $\ddot{\varepsilon}$. Then, through BC-MAC dual mapping, we can obtain the corresponding solution $\ddot{\boldsymbol{p}} = [\ddot{p}_1, \ddot{p}_2, ..., \ddot{p}_K], \ddot{\boldsymbol{U}}$ for Problem 3 to achieve the same $\ddot{\varepsilon}$. We next assume that $\bar{\boldsymbol{Q}}_i^b, i = 1, 2, ..., K$ are the optimal solutions of the Problem 2 with the optimal sum MSE $\bar{\varepsilon}$, where $\bar{\varepsilon} < \ddot{\varepsilon}$. Thus, we can obtain the optimal solution of the Problem 3 $\bar{\boldsymbol{p}}, \bar{\boldsymbol{U}}$ by BC-MAC dual mapping. It is well known that Problem 3 is a convex optimization problem. Hence, we have $p_i^* = \ddot{p}_i + t(\bar{p}_i - \ddot{p}_i)$, $\boldsymbol{u}_i^* = \ddot{\boldsymbol{u}}_i + t(\bar{\boldsymbol{u}}_i - \ddot{\boldsymbol{u}}_i)$, $i = 1, ..., K$, where 0 < t < 1, is a better solution than $\ddot{p}_i$, $\ddot{\boldsymbol{u}}_i$, $i = 1, ..., K$ for Problem 3. Through MAC-BC dual mapping, we transform the dual MAC solution $p_i^*, \boldsymbol{u}_i^*, i=1,\ldots,K$ into its corresponding BC solution $\boldsymbol{Q}_i^{b*}, i=1,\ldots,K$. Since MAC-BC dual transformation is continuous, we can always find a t such that $\left\| \boldsymbol{Q}_i^{b*} - \dot{\boldsymbol{Q}}_i^b \right\| \le \varphi, i=1,2,\ldots,K$, for a given $\varphi > 0$. That is to say, $\dot{\boldsymbol{Q}}_i^b, i=1,2,\ldots,K$ is not the local optimal solution, which is contradicted with the KKT conditions. Therefore, the KKT conditions for Problem 2 are also sufficient for optimality, and the proof for equivalence between the Problem 1and the Problem 2 with $q_t = \mu_1^*, q_u = \mu_2^*$ follows.

3.2 MMSE Optimization for the Dual MAC

According to *Proposition* 2, for fixed q_t and q_u, Problem 2 is equivalent to Problem 3. In this subsection, we propose an efficient algorithm to solve Problem 3.

In the dual SIMO-MAC (as shown in Fig. 2(b)), the symbol vector $\boldsymbol{d}$ is transmitted from K independent antennas over the SIMO channel $\boldsymbol{H} = [\boldsymbol{h}_1^*, \ldots, \boldsymbol{h}_K^*]$. The matrix $\boldsymbol{U^*}$ now acts as a multiuser receiver, which separates the data streams. We define the power allocation matrix $\boldsymbol{P}$=$diag[\boldsymbol{p}]$=$diag[p_1,p_2,\ldots,p_K]$. With the received signal $\boldsymbol{y} = \boldsymbol{H}\sqrt{\boldsymbol{P}}\boldsymbol{d} + \boldsymbol{z}$, the i^{th} estimated signal becomes

$$\hat{d}_i = \frac{\beta_i}{\sqrt{p_i}} \boldsymbol{u}_i^* (\boldsymbol{H}\sqrt{\boldsymbol{P}}\boldsymbol{d} + \boldsymbol{z}) , \tag{12}$$

and the normalized MSE is

$$\begin{aligned} \varepsilon_i^m &= E\left\{ \| \hat{d}_i - d_i \|^2 \right\}, \\ &= \frac{\beta_i^2}{p_i} \boldsymbol{u}_i^* (\boldsymbol{HPH}^* + \boldsymbol{R}_w)\boldsymbol{u}_i - \beta_i \boldsymbol{u}_i^* \boldsymbol{h}_i - \beta_i \boldsymbol{h}_i^* \boldsymbol{u}_i + 1, \end{aligned} \tag{13}$$

where $\boldsymbol{R}_w = q_t \boldsymbol{h}_0 \boldsymbol{h}_0^* + q_u \boldsymbol{I}_M$. It can be observed that $\varepsilon_1^m, \varepsilon_2^m, \ldots, \varepsilon_K^m$ can be optimized independently. Collecting all optimizers in a matrix $\tilde{\boldsymbol{U}}_{mmse}$, we have

$$\tilde{\boldsymbol{U}}_{mmse} = (\boldsymbol{HPH}^* + \boldsymbol{R}_w)^{-1} \boldsymbol{HP} , \tag{14}$$

where $\tilde{\boldsymbol{U}} = \boldsymbol{U} \times diag\boldsymbol{\beta}.$

Since $\boldsymbol{\varepsilon}^m = E\left\{ \| \hat{\boldsymbol{d}} - \boldsymbol{d} \|^2 \right\}$

$$= \| (diag\boldsymbol{\beta})\sqrt{\boldsymbol{P}}^{-1} \boldsymbol{U}^* \boldsymbol{H} \sqrt{\boldsymbol{P}} - \boldsymbol{I}_k \|_F^2 + tr((diag\boldsymbol{\beta})\sqrt{\boldsymbol{P}}^{-1} \boldsymbol{U}^* \boldsymbol{R}_w \boldsymbol{U} \sqrt{\boldsymbol{P}}^{-1} (diag\boldsymbol{\beta})) , \tag{15}$$

substituting (14) into (15), we obtain

$$\boldsymbol{\varepsilon}^m = tr[\boldsymbol{R}_w (\boldsymbol{HPH}^* + \boldsymbol{R}_w)^{-1}] + K - M . \tag{16}$$

As a result, the optimization Problem 3 can be reformulated as

Problem 4

$$\max_{p_1, p_2, \ldots, p_K \ge 0} -tr(\boldsymbol{R}_w [\boldsymbol{HPH}^* + \boldsymbol{R}_w]^{-1})$$

$$\text{s.t.} \sum_{i=1}^{K} p_i \le P, P = q_t P_t + q_u P_u$$

This problem is convex with respect to the power allocation, so it can be easily solved by Lagrangian methods. Correspondingly, the Lagrangian function is

$$L(p_1, p_2, ..., p_K, \lambda) = -tr(\boldsymbol{R}_w[\boldsymbol{HPH}^* + \boldsymbol{R}_w]^{-1}) - \lambda(\sum_{i=1}^{K} p_i - P), \tag{17}$$

where λ is the Lagrangian multiplier.

The dual objective function of Problem 4 is

$$g(\lambda) = \max_{p_1, p_2, ..., p_K \geq 0} L(p_1, p_2, ..., p_K, \lambda), \tag{18}$$

since the Problem 4 is convex, it is equivalent to the following minimization problem:

Problem 5 $\min_{\lambda} g(\lambda)$

$s.t. \lambda \geq 0$

We propose an efficient algorithm called *Iterative Power Allocation Algorithm* to solve Problem 5 and outline it as follows: We choose an initial λ and compute the value of $g(\lambda)$ in (18), then update λ according to the descent direction of $g(\lambda)$. The process repeats until the algorithm converges. It is easy to observe that once λ is fixed, the unique optimal set $\{p_1, p_2, ..., p_K\}$ can be obtained via the gradient ascent algorithm. We next need to determine the optimal λ. Since the Lagrangian function $g(\lambda)$ is convex over λ, the optimal λ can be obtained via the one-dimensional search. However, because $g(\lambda)$ is not necessarily differentiable, the gradient algorithm cannot be applied. Alternatively, the subgradient method can be used to find the optimal solution. According to [15], we can obtain that the subgradient of $g(\lambda)$ is $P - \sum_{i=1}^{K} p_i$, where p_i, i=1,...,K, are the corresponding optimal power allocation for a fixed λ in (18), and $P = q_t P_t + q_u P_u$.

3.3 MAC to BC Dual Mapping

According to the duality, although the quantities $\boldsymbol{H}$, $\boldsymbol{U}$, $\boldsymbol{\beta}$ in the MAC model are the same as those for the BC model, the power allocation may be different. In the following, we compute the BC power allocation vector $\boldsymbol{q}$ via the dual MAC power allocation vector $\boldsymbol{p}$.

Since the SINR in BC is given by

$$SINR_i^b = \frac{q_i \boldsymbol{h}_i^* \boldsymbol{u}_i \boldsymbol{u}_i^* \boldsymbol{h}_i}{\sum_{j=1, j \neq i}^{K} q_j \boldsymbol{h}_i^* \boldsymbol{u}_j \boldsymbol{u}_j^* \boldsymbol{h}_i + 1}, \tag{19}$$

$\boldsymbol{q}$ is characterized by

$$\boldsymbol{q} = (\boldsymbol{D}^{-1} - \boldsymbol{\psi})^{-1} \boldsymbol{1}_k, \tag{20}$$

where each component in $\boldsymbol{\psi}$ is

$$\psi_{ik} = \begin{cases} |\boldsymbol{u}_k^* \boldsymbol{h}_i|^2 & k \neq i \\ 0 & k = i \end{cases}, \tag{21}$$

and

$$\boldsymbol{D} = diag\left\{ [\frac{SINR_1^b}{|\boldsymbol{u}_1^* \boldsymbol{h}_1|^2}, ..., \frac{SINR_K^b}{|\boldsymbol{u}_K^* \boldsymbol{h}_K|^2}] \right\}. \tag{22}$$

The SINR in MAC is given by

$$SINR_i^m = \frac{p_i \boldsymbol{u}_i^* \boldsymbol{h}_i \boldsymbol{h}_i^* \boldsymbol{u}_i}{\boldsymbol{u}_i^* (\boldsymbol{HPH}^* + \boldsymbol{R}_w - p_i \boldsymbol{h}_i \boldsymbol{h}_i^*) \boldsymbol{u}_i}. \tag{23}$$

Therefore, given the condition $SINR_i^m = SINR_i^b$, the BC power allocation vector $\boldsymbol{q}$ is able to be obtained through the dual MAC power allocation vector $\boldsymbol{p}$.

3.4 A Complete Solution to the Problem of MMSE Transmit Optimization for CR MISO-BCMAC to BC

In the former subsections, we proposed an efficient algorithm to solve Problem 3. We are now ready to present a complete algorithm to solve Problem 1.

Since Problem 1 is equivalent to the following problem:

$$\min_{q_t, q_u} g(q_t, q_u),$$
$$\text{s.t. } q_t \geq 0 \text{ and } q_u \geq 0.$$

the remaining task is to determine the optimal q_t and q_u. Since $g(q_t, q_u)$ is not necessarily differentiable, the optimal q_t and q_u are searched via the subgradient algorithm. That is, in each iterative step, the vector $[q_t, q_u]$ is updated according to the subgradient direction $[P_t - \sum_{i=1}^{K} \boldsymbol{h}_0^* \boldsymbol{Q}_i^b \boldsymbol{h}_0, P_u - \sum_{i=1}^{K} tr(\boldsymbol{Q}_i^b)]$ of g(q_t, q_u), where $\boldsymbol{Q}_i^b$,i=1,...,K, are the corresponding optimal covariance matrices for the Problem 2 . We describe the *Complete Iterative Algorithm* to solve Problem 1 as follows:

1). Initialization: $q_t^{(1)}, q_u^{(1)}, n = 1,$

2). Repeat

2a) Find the optimal solution of the Problem 3 based on *Iterative Power Allocation Algorithm*;

2b) Find the solution of the Problem 2 via the MAC-to-BC mapping;

2c) Update $q_t^{(n)}$ and $q_u^{(n)}$ via the subgradient algorithm:

$$q_t^{(n+1)} = q_t^{(n)} + t(\sum_{i=1}^{K} \boldsymbol{h}_0^* \boldsymbol{Q}_i^b \boldsymbol{h}_0 - P_t), \quad q_u^{(n+1)} = q_u^{(n)} + t(\sum_{i=1}^{K} tr(\boldsymbol{Q}_i^b) - P_u),$$

where t denotes the step size of subgradient algorithm;

2d) n=n+1

3). Stop when $|q_t^{(n)}(\sum_{i=1}^{K} \boldsymbol{h}_0^* \boldsymbol{Q}_i^b \boldsymbol{h}_0 - P_t)| \leq \varepsilon$ and $|q_u^{(n)}(\sum_{i=1}^{K} tr(\boldsymbol{Q}_i^b) - P_u)| \leq \varepsilon$ are satisfied simultaneously.

As a summary, the flow chart of the complete algorithm is depicted in Fig. 3.

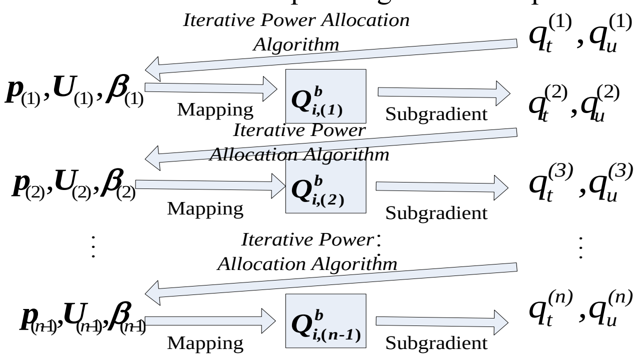

Fig. 3. The flow chart for the *Complete Iterative Algorithm*

4. Simulation Results and Analysis

In the simulation, the elements of $\boldsymbol{h}_0, \boldsymbol{h}_1, \boldsymbol{h}_2, \ldots, \boldsymbol{h}_K$ are independent and identically distributed (i.i.d.) complex Gaussian variables with mean zero and variance one. We define the SNR of the unlicensed MU-MISO system as $SNR = P_u / \sigma^2$.

Example 1: In this simulation, we consider a CR MISO-BC system with K=3, M=4, and SNR=10dB. Fig. 4 shows the convergence of the proposed *Complete Iterative Algorithm* for randomly selected initial q_t and q_u. It can be seen from the figure that for different settings of initial q_t and q_u, convergence can be guaranteed. Fig. 5 shows the sum power at the SBS and the interference power at the PU with P_u=10w, P_t=1w. From the figure, we can see that the sum power and the interference power approach to 10w and 1w, respectively, at convergence. This implies that the *Complete Iterative Algorithm* converges to the optimal point, since the sum power and interference power constraints are satisfied with equalities when the algorithm converges.

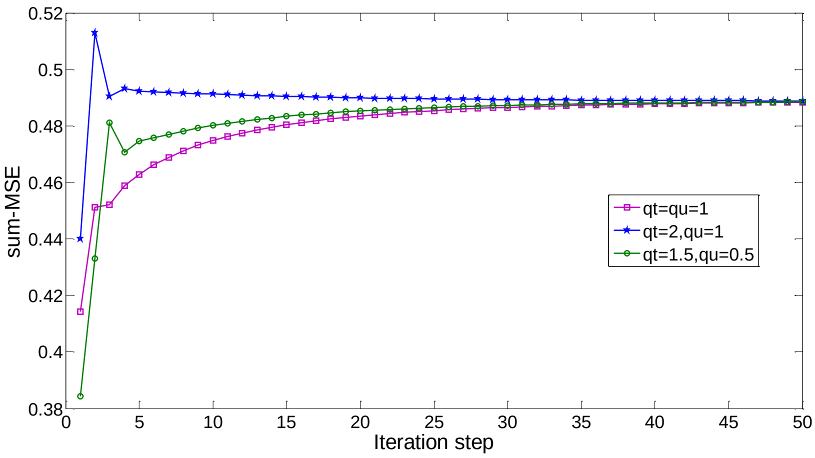

Fig. 4. The convergence behavior of the *Complete Iterative Algorithm*

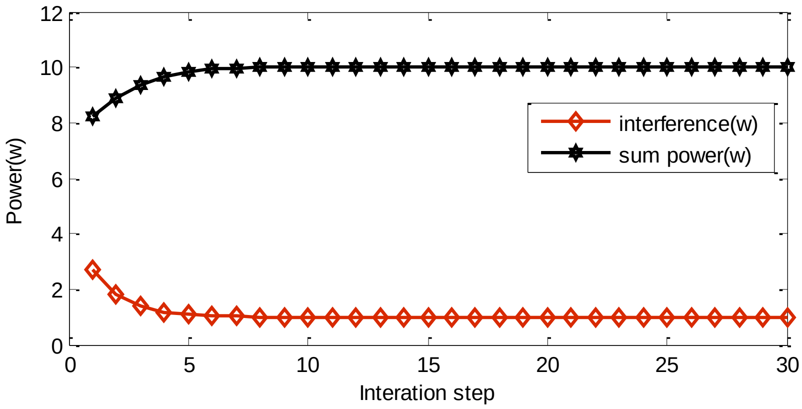

Fig. 5. The convergence behavior of the sum power at the BS and the interference at the PU for the *Complete Iterative Algorithm*

Example 2: In this simulation, we compare the performance of the proposed CR MMSE-based linear scheme with the optimal DPC based method in terms of the achievable sum-rate. In Fig. 6, we fix the number of antennas at SBS at 4. From the figure, it can be observed that as the number of SUs decreases, the performance gap between the proposed scheme and the DPC scheme decreases. Especially, in the setting of M=4, K=2, the difference is less then 3%. While we fix the number of SUs at 2, the performance gap between the proposed CR MMSE-based linear scheme and the DPC-based nonlinear scheme increases with the number of antennas at SBS increasing, as shown in Fig. 7. It is because with less SUs or less antennas at SBS, the multi-user interference becomes smaller so that the difference between MMSE-based linear precoding and DPC-based non-linear precoding tends to be negligible.

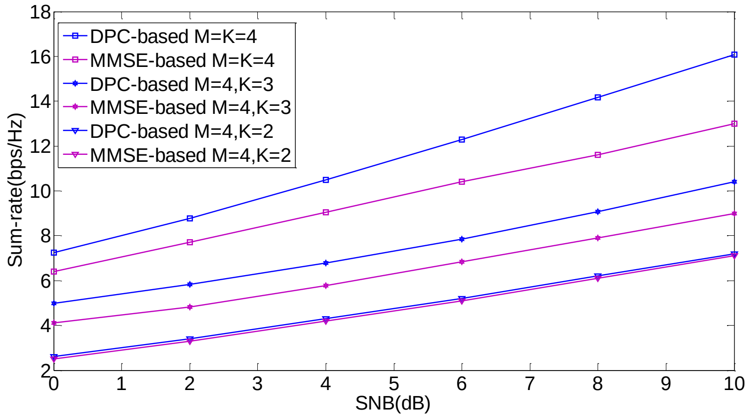

Fig. 6. Sum rate comparison for M=4, K=2, 3, 4

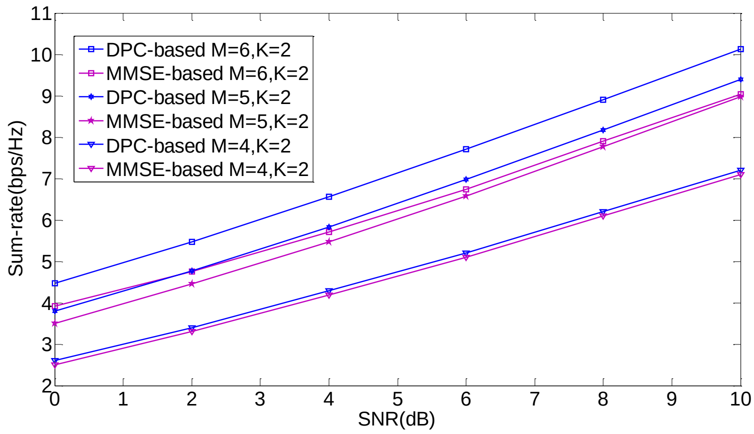

Fig. 7. Sum rate comparison for K=2, M=4, 5, 6

Example 3: In Fig. 8, we compare the computation complexity of the proposed CR MMSE linear scheme with that of CR nonlinear DPC scheme. In this simulation, we focus on comparing the overall time consumption by the two schemes. From the figure, we can figure out that the proposed CR MMSE scheme is less time-consuming than CR DPC scheme, and with the number of SUs increasing, the gap increases. By jointly considering the performance comparison in Figs. 6 and 7, we can conclude that the proposed CR MMSE scheme is more feasible for practical applications.

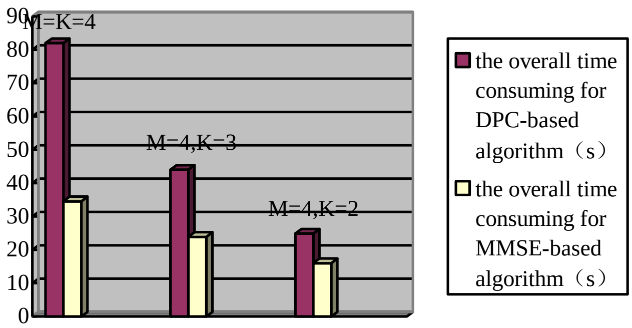

Fig. 8. The comparison of computational complexity

5. Conclusion

In this paper, the problem of linear MMSE transmitter design for the CR MU-MISO-BC system is investigated. By applying equivalent conversion, a two-level iterative algorithm is proposed to solve the non-convex CR MISO-BC problem. Simulation results show that the proposed CR MMSE-based scheme can provide a suboptimal sum-rate performance with much lower computational complexity. In the future, the extension of the work to address the comparison to other linear precoding schemes in MIMO systems will be considered.

References

[1] S. Haykin, “Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun*., vol. 23, no. 2, pp. 201-202, Feb. 2005. Article (CrossRef Link)

[2] M. H. M. Costa, “Writing on dirty paper,” *IEEE Trans. Inf. Theory*, vol. 29, no. 3, pp. 439–441, May 1983. Article (CrossRef Link)

[3] L. Zhang, Y. Xin, and Y.-C. Lia ng, “Weighted sum rate optimization for cognitive radio MIMO broadcast channels,” *IEEE Trans. Wireless Commun.*, vol. 8, no. 6, pp. 2950–2959, June 2009. Article (CrossRef Link)

[4] X in Gui, Guixia Kang, Ping Zhang, “Cooperative precoding with limited feedback in multi-user cognitive MIMO networks,” in *Proc. of 2013 IEEE 10th Consumer Communications and Networking Conference (CCNC)*, Las Vegas, NV, USA, 11-14 Jan. 2013. Article (CrossRef Link)

[5] Jaehyun Park, Yunju Park, Sunghyun Hwang, “Low-Complexity GSVD-Based Beamforming and Power Allocation for a Cognitive Radio Network,” *IEICE Transactions on Communications*, Volume:E95-B, Issue:11, Pages:3536-44, Nov.2012. Article (CrossRef Link)

[6] Khan, Faheem A.; Masouros, Christos; Ratnarajah, Tharmalingam, “Interference-Driven Linear Precoding in Multiuser MISO Downlink Cognitive Radio Network,” *IEEE TRANSACTIONS ON VEHICULAR TECHNOLOGY*, Volume: 61, Issue: 6, Pages: 2531-2543, JUL 2012. Article (CrossRef Link)

[7] Kyoung-Jae Lee, Hakja Sung, Inkyu Lee, “Linear Precoder Designs for Cognitive Radio Multiuser MIMO Downlink Systems,” in *Proc. of 2011 IEEE International Conference on Communications (ICC)*, Page(s):1-5,2011. Article (CrossRef Link)

[8] Samir M. Perlaza, Nadia Fawaz, Samson Lasaulce, Merouane Debbah, “From Spectrum Pooling to Space Pooling: Opportunistic Interference Alignment in MIMO Cognitive Networks,” *IEEE TRANSACTIONS ON SIGNAL PROCESSING*, VOL: 58, NO. 7, Pages: 3728-3741, JULY 2010. Article (CrossRef Link)

[9] W. Yu and T. Lan, “Transmitter optimization for the multi-antenna downlink with per-antenna power constraints,” *IEEE Trans. Signal Processing*, vol. 55, no. 6, pp. 2646–2660, June 2007. Article (CrossRef Link)

[10] M. Schubert and H. Boche, “Solution of the multiuser downlink beamforming problem with individual SINR constraints,” *IEEE Trans. Veh. Technol.*, vol. 53, no. 1, pp. 18–28, Jan. 2004. Article (CrossRef Link)

[11] H. Weingarten, Y. Steinberg, and S. Shamai, “The capacity region of the Gaussian multiple-input multiple-output broadcast channel,” *IEEE Trans. Inform. Theory*, vol. 52, no. 9, pp. 3936–64, Sept. 2006. Article (CrossRef Link)

[12] P. Viswanath and D. N. C. Tse, “Sum capacity of the vector Gaussian broadcast channel and uplink-downlink duality,” *IEEE Trans. Inform. Theory*, vol. 49, no. 8, pp. 1912–1921, Aug. 2003. Article (CrossRef Link)

[13] W. Yu and J. Cioffi, “Sum capacity of Gaussian vector broadcast channels,” *IEEE Trans. Inform. Theory*, vol. 50, no. 9, pp. 1875–1892, Sept. 2004. Article (CrossRef Link)

[14] L. Zhang, R. Zhang, Y.-C. Liang, Y. Xin, and H. V. Poor, “On gaussian MIMO BC-MAC duality with multiple transmit covariance constraints,” *IEEE Transactions on Information Theory*, Volume: 58,Issue:4,Page(s):2064-2078,2012. Article (CrossRef Link)

[15] S. Boyd, L. Xiao, and A. Mutapcic, “Subgradient methods," [Online]. Available. Article (CrossRef Link)

Cao Huijin is currently having her Ph.D study in Electrical and Computer Engineering, University of Manitoba. She received her B.S. and M.S. degrees in the School of Information Engineering, Zhengzhou University, Zhengzhou, China in 2010 and 2013, respectively. Her research interests focus on the optimization of wireless resources in Cognitive Radio.

Yanhui Lu is currently an associated professor with the School of Information Engineering, Zhengzhou University. She received the B.E. and M.S. degrees from Xi'an Jiaotong University, China, in 1994 and 1997, respectively, and a Ph.D degree from Beijing University of Posts and Telecommunications, China, in 2006, all in electrical engineering. She worked in National Digital Switching System Engineering & Technological Research Center from 1997 to 2002 as an engineer, and in Department of Computer Science, University of California, Davis, USA from 2010 to 2011 as a visiting scholar. Dr. Lu’s research are focusing on radio resource management in cognitive radio networks and next generation wireless communication systems, which has been funded partially by the National Natural Science Foundation of China and Chinese Government. In 2012, she was honored with one of the New Century Excellent Talents in University by Ministry of Education, China.

Jun Cai received the B.Sc. (1996) and the M.Sc. (1999) degrees from Xi'an Jiaotong University (China) and Ph.D. degree (2004) from University of Waterloo, Ontario (Canada), all in electrical engineering. From June 2004 to April 2006, he was with McMaster University as NSERC Postdoctoral Fellow. Since July 2006, he has been with the Department of Electrical and Computer Engineering, University of Manitoba, Canada, where he is an Associate Professor. His current research interests include multimedia communication systems, mobility and resource management in 3G beyond wireless communication networks, cognitive radios, and ad hoc and mesh networks. Dr. Cai serves as the Technical Program Committee Co-Chair for IEEE VTC 2012 – Fall Wireless Applications and Services Track, IEEE Globecom 2010 Wireless Communications Symposium, and IWCMC 2008 General Symposium, the

Publicity Co-Chair for IWCMC 2010, 2011 and 2013, and the Registration Chair for the First International Conference on Heterogeneous Networking for Quality, Reliability, Security and Robustness (QShine) 2005. Robustness (QShine) 2005. He also served the editorial board of Journal of Computer Systems, Networks, and Communications. Dr. Cai received the Best Paper award from Chinacom 2013, the Rh Award for outstanding contributions to research in applied sciences in 2012 from University of Manitoba, and Outstanding Service Award from IEEE Globecom 2010.