Exploiting Multichannel Diversity in Spectrum Sharing Systems Using Optimal Stopping Rule

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This letter studies the problem of exploiting multichannel diversity in a spectrum sharing system, where the secondary user (SU) sequentially explores channel state information on the licensed channels with time consumption. To maximize the expected achievable throughput for the SU, we formulate this problem as an optimal stopping problem, whose objective is to choose the right channel to stop exploration based on the observed signal-to-noise ratio sequence. Moreover, we propose a myopic but optimal rule, called one-stage look-ahead rule, to solve the stopping problem.

Keywords: Cognitive radio, spectrum sharing systems, multichannel diversity, optimal stopping rule.

I. Introduction

Spectrum sharing (SS) has been regarded as a promising solution to lessen the spectrum shortage problem, where the secondary user (SU) can share the same spectrum with the primary user (PU) provided that the interference to the PU is kept below a threshold [1]. In this letter, we investigate the problem of exploiting multichannel diversity in an SS system where the SU sequentially explores the licensed channels with time consumption.

Currently, most existing work, for example [2], mainly focuses on analyzing the achievable multichannel gain by assuming that channel state information (CSI) is known. However, such an assumption does not hold in practice. In fact, to obtain multichannel diversity gain, the SU has to know the CSI on all the channels, which consumes resources (such as time, energy, and bandwidth) proportional to the number of channels [3]. Then, as the number of explored channels increases, the obtained diversity gain increases and so does the exploration overhead. Thus, there is a fundamental tradeoff between the multichannel diversity gain and the exploration overhead, which has not been considered yet in SS systems.

In this letter, we formulate the above tradeoff as an optimal stopping problem and propose a myopic rule, the one-stage look-ahead (1–SLA) rule, to solve it. The optimality of the 1–SLA rule is analytically validated, and the impact of the interference constraints imposed by the PU on the stopping time is also studied.

II. System Model

Let us consider an SS system where an SU transmitter sends data to its receiver on one of the N licensed channels. It is assumed that each licensed channel is occupied by a PU receiver. Moreover, denote the interference channel power gain from the SU transmitter to the PU receiver and the data channel power gain from the SU transmitter to its receiver on channel n (n=1,…, N) as α_n and β_n, respectively.

For simplicity of analysis, both the data channel power gain and interference channel power gain on each licensed channel are assumed to be independent and identically distributed (i.i.d.) exponential random variables with a unit mean [5]. Then, the common probability density function (PDF) of α_n and β_n is given by $g(x)=e^{-x}$, $x\geq 0$. Furthermore, it is assumed that both α_n and β_n are block-fixed for a duration of T and change randomly in the next period. Denote T_s as the length of the required time for CSI exploration on one channel and $\tau=T_s/T$ as the fraction

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of exploration overhead. Note that there are some approaches to obtain the interference channel gain α_n in practice, and the most well-known are i) direct feedback from the primary receiver [1] and ii) active learning without the cooperation of the PUs [4].

According to the policy of SS, the transmitting power of the SU on channel n is given by $P_n^{tx}=\min\{P, Q/\alpha_n\}$, where P denotes the peak power of the SU and Q denotes the interference threshold perceived by each PU receiver. Then, the received signal-to-noise ratio (SNR) on channel n is given by $\eta_n=P_n^{tx}\beta_n/\sigma^2$, where the variance of white Gaussian noise σ^2 is set to be one for simplicity of analysis. Thus, the SNR sequence $\{\eta_n\}_{n=1}^N$ is i.i.d. with the following common PDF [5]:

$$f(\eta)=\frac{1}{P}\left(1-e^{-\frac{Q}{P}}\right)e^{-\frac{\eta}{P}}+\frac{Q}{P}\frac{(Q+P+\eta)}{(Q+\eta)^2}e^{-\frac{Q+\eta}{P}}, \eta\geq 0. \quad (1)$$

III. Optimal Channel Exploration Rule

1. Problem Formulation

For presentation, we assume that the SU explores the licensed channels in a natural incremental order. After exploring the first n channels, the SU observes the SNR sequence $\{\eta_i\}_{i=1}^n$. Then, if the SU decides to stop channel exploration and begins to transmit on the explored channel with the strongest SNR, it obtains the following throughput:

$$R_n(\eta_1,\ldots,\eta_n)=s_n\log(1+\eta_n^{\max}), \quad (2)$$

where $s_n=1-n\tau$ represents the fraction of effective transmission time in a slot and $\eta_n^{\max}=\max_{1\leq i\leq n}\{\eta_i\}$ represents the strongest SNR among the explored channels. It is seen that as n increases, $\eta_n^{\max}$ increases but s_n decreases. Thus, the objective of the SU is to choose the right time to stop channel exploration based on the observed SNR sequence.

It is seen that this objective belongs to the optimal stopping problem with finite horizon as the SU must stop at the last channel. In principle, such a problem can be solved by the method of backward induction [6]. Since we must stop at channel N, we first find the optimal rule at channel N–1. Then, knowing the optimal rule at channel N–1, we find the optimal rule at channel N–2 and so on back to the first channel (channel 1). Specifically, we define

$$V_N(\eta_1,\ldots,\eta_N)=R_N(\eta_1,\ldots,\eta_N), \quad (3)$$

and then inductively for $n=N-1$, backward to $n=1$,

$$V_n(\eta_1,\ldots,\eta_n) =\max\{R_n(\eta_1,\ldots,\eta_n), \mathrm{E}_{\eta_{n+1}}[V_{n+1}(\eta_1,\ldots,\eta_n,\eta_{n+1})\,|\,\{\eta_i\}_{i=1}^n]\}, \quad (4)$$

where $\mathrm{E}_{\eta_{n+1}}$ is the expectation operation over η_{n+1}. Inductively, $V_n(\eta_1,\ldots,\eta_n)$ represents the maximum achievable throughput that the SU can obtain starting from channel n after observing the SNR sequence $\{\eta_i\}_{i=1}^n$. At the channel n, we compare the achieved throughput for stopping, that is, $R_n(\eta_1,\ldots,\eta_n)$, with the expected throughput by continuing exploration and using the optimal rule for channel n+1 through N, that is, $\mathrm{E}_{\eta_{n+1}}[V_{n+1}(\eta_1,\ldots,\eta_n,\eta_{n+1})\,|\,\{\eta_i\}_{i=1}^n]$. To maximize the expected throughput of the SU, it is optimal to stop if the former is no less than the latter; otherwise, it is optimal to continue [6]. Specifically, the optimal number of channels to explore before stopping is given by

$$N_{\mathrm{opt}}=\min\{1\leq n\leq N: R_n(\eta_1,\ldots,\eta_n)\geq \mathrm{E}_{\eta_{n+1}}[V_{n+1}\,|\,\{\eta_i\}_{i=1}^n]\}. \quad (5)$$

Note that N_{opt} is a random variable which is inherently determined by the realization of $\{\eta_i\}_{i=1}^n$ on the licensed channels. However, due to the extremely high complexity, its statistical distribution, that is, $\Pr\{N_{\mathrm{opt}}=n\}$, $n=1,\ldots, N$, is hard to obtain. Based on the above argument and analysis, the maximum achievable throughput of the SU is given by:

$$C_{\max}=\mathrm{E}_{\eta_1}[V_1(\eta_1)]. \quad (6)$$

It is seen from (4) and (5) that to make the optimal decision at channel n, the SU has to calculate the sequence $\{\mathrm{E}_{\eta_{k+1}}[V_{k+1}\,|\,\{\eta_i\}_{i=1}^k]\}_{k=n}^N$ backward from N–1 to n. However, such a backward induction solution is a type of dynamic programming which results in uncountable and infinite calculating space. Thus, the optimal stopping time specified by (5) is not feasible in practice.

2. One-Stage Look-Ahead (1–SLA) Rule

To reduce the complexity, we consider a truncated version of the optimal stopping rule as given in (5). The simplest truncation is the 1–SLA rule, with which the number of channels to explore before stopping is given as

$$N_1=\min\{1\leq n\leq N: R_n(\eta_1,\ldots,\eta_n)\geq \mathrm{E}_{\eta_{n+1}}[U_{n+1}\,|\,\{\eta_i\}_{i=1}^n]\}, \quad (7)$$

where $U_{n+1}=s_{n+1}\log(1+\max\{\eta_n^{\max},\eta_{n+1}\})$ represents the achievable throughput that the SU can achieve by proceeding to explore channel n+1 and then stopping. Specifically, we have

$$\mathrm{E}_{\eta_{n+1}}[U_{n+1}\,|\,\{\eta_i\}_{i=1}^n]=s_{n+1}\mathrm{E}_{\eta_{n+1}}[\log(1+\max\{\eta_n^{\max},\eta_{n+1}\})] =s_{n+1}\int_0^\infty \log(1+\max\{\eta_n^{\max},\eta\}f(\eta)d\eta. \quad (8)$$

It is noted from (7) that 1–SLA calls for stopping at channel n if the achievable throughput obtained at channel n is at least as great as the expected throughput of proceeding to explore channel n+1 and then stopping. In practice, $\mathrm{E}_{\eta_{n+1}}[U_{n+1} \mid \{\eta_i\}_{i=1}^n]$ can be calculated by numerical methods with much lower complexity than that of calculating (5).

3. Optimality Analysis

In this subsection, we prove that 1–SLA is optimal for the channel exploration problem in SS systems.

Definition 1. A stopping problem is said to be monotone if $A_1 \subset A_2 \subset \ldots \subset A_N$, where A_n denotes the following event [6]:

$$\{\eta_n^{\max} : R_n(\eta_1,\ldots,\eta_n) \geq \mathrm{E}_{\eta_{n+1}}[U_{n+1} \mid \{\eta_i\}_{i=1}^n]\}. \tag{9}$$

Proposition 1. In a finite horizon monotone stopping problem, the 1–SLA rule is optimal [6].

Theorem 1. The problem of exploiting multichannel diversity in SS systems is a finite horizon monotone stopping problem.

Proof. We firstly define the following function:

$$\begin{aligned} F_n(\eta_n^{\max}) &= R_n(\eta_1,\ldots,\eta_n) - \mathrm{E}_{\eta_{n+1}}[U_{n+1} \mid \{\eta_i\}_{i=1}^n] \\ &= s_n \log(1+\eta_n^{\max}) - s_{n+1}\mathrm{E}_\eta[\log(1+\max\{\eta_n^{\max},\eta\})]. \end{aligned} \tag{10}$$

After some manipulations, we have

$$F_n(\eta_n^{\max}) = G(\eta_n^{\max}) - F_n^*(\eta_n^{\max}), \tag{11}$$

where $G(\eta_n^{\max})$ and $F_n^*(\eta_n^{\max})$ are defined as

$$\begin{aligned} G(\eta_n^{\max}) &= \tau \mathrm{E}_\eta[\log(1+\max\{\eta_n^{\max},\eta\})], \\ F_n^*(\eta_n^{\max}) &= s_n \mathrm{E}_\eta[\log(1+\max\{\eta_n^{\max},\eta\}) - \log(1+\eta_n^{\max})], \end{aligned} \tag{12}$$

with exploration overhead $\tau = T_s/T$ and $s_n = 1 - n\tau$.

It is seen that $G(\eta_{n+1}^{\max})$ is a strictly monotone increasing function of $\eta_n^{\max}$ and $F_n^*(\eta_n^{\max})$ is a strictly monotone decreasing function of $\eta_n^{\max}$. Now, we compare $F_{n+1}(\eta_{n+1}^{\max})$ and $F_n(\eta_n^{\max})$ as

$$\begin{aligned} &F_{n+1}(\eta_{n+1}^{\max}) - F_n(\eta_n^{\max}) \\ &\quad = G(\eta_{n+1}^{\max}) - G(\eta_n^{\max}) + F_n^*(\eta_n^{\max}) - F_{n+1}^*(\eta_{n+1}^{\max}). \end{aligned} \tag{13}$$

Since $\eta_{n+1}^{\max} = \max\{\eta_n^{\max}, \eta_{n+1}\} \geq \eta_n^{\max}$, and $s_n > s_{n+1}$ are always true for all n, we have

$$\begin{cases} G(\eta_{n+1}^{\max}) \geq G(\eta_n^{\max}), \\ F_n^*(\eta_n^{\max}) \geq F_n^*(\eta_{n+1}^{\max}) > F_{n+1}^*(\eta_{n+1}^{\max}). \end{cases} \tag{14}$$

Thus, the following inequality can be obtained immediately from (13) and (14):

$$F_{n+1}(\eta_{n+1}^{\max}) - F_n(\eta_n^{\max}) > 0. \tag{15}$$

Let us rewrite the event A_n as $\{\eta_n^{\max} : F_n(\eta_n^{\max}) \geq 0\}$. By using (15), we have

$$F_{n+1}(\eta_{n+1}^{\max}) > 0, \eta_n^{\max} \in A_n, \tag{16}$$

which is equivalent to $A_n \subset A_{n+1}$. Finally, the following can be inductively obtained:

$$A_1 \subset A_2 \subset \ldots \subset A_N. \tag{17}$$

By definition 1, theorem 2 follows. □

Proposition 1 and theorem 1 validate the optimality of the 1–SLA rule. Then, according to the 1–SLA rule specified by (7) and (8), the optimal channel exploration stopping time is easy to obtain and can be described in the following:

Step 1. Set n=1.

Step 2. The SU explores the CSI on the channel n and observes the SNR sequence $\{\eta_i\}_{i=1}^n$. Then, it calculates $R_n(\eta_1,\ldots,\eta_n)$ using (2) and $\mathrm{E}_{\eta_{n+1}}[U_{n+1} \mid \{\eta_i\}_{i=1}^n]$ using (8).

Step 3. If $R_n(\eta_1,\ldots,\eta_n) \geq \mathrm{E}_{\eta_{n+1}}[U_{n+1} \mid \{\eta_i\}_{i=1}^n]$, the SU stops channel exploration and selects the one with the highest SNR for transmission; otherwise, set n=n+1 and go to step 2, that is, it continues to explore the next channel.

IV. Simulation Results and Discussion

First, we compare the expected achievable throughput by using the optimal stopping rule and the explore-all scheme in Fig. 1. In the explore-all scheme, the SU explores all the channels and selects the one with the highest SNR for transmission. It is noted that there is a peak of the achievable throughput for the explore-all scheme. The reason is that the exploration overhead increases linearly with N, whereas the multichannel diversity gain increases significantly for small N but increases slightly for large N. This result shows that there indeed exists a tradeoff between the multichannel diversity gain and the exploration overhead. Hence, an improved approach, called the improved explore-all scheme, is proposed only to explore the first five channels even when N>5, as depicted by the middle curve in Fig. 1. It is noted from the figure that the expected achievable throughput obtained by the optimal stopping rule is always better than that of the improved explore-all scheme. The optimal stopping rule is always better because its decision is based on the observations so far. Also from the figure, it is seen that the achievable throughput moderates when the number of channels increases as can be expected in any multichannel diversity system.

Secondly, we investigate the impact of interference constraints Q on the achievable throughput in Fig. 2. It is noted

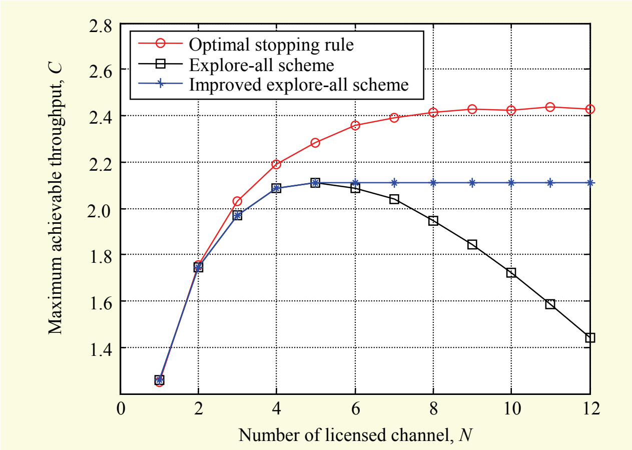

Fig. 1. Comparison with explore-all scheme (τ=0.05, P=10 dB, Q=0 dB).

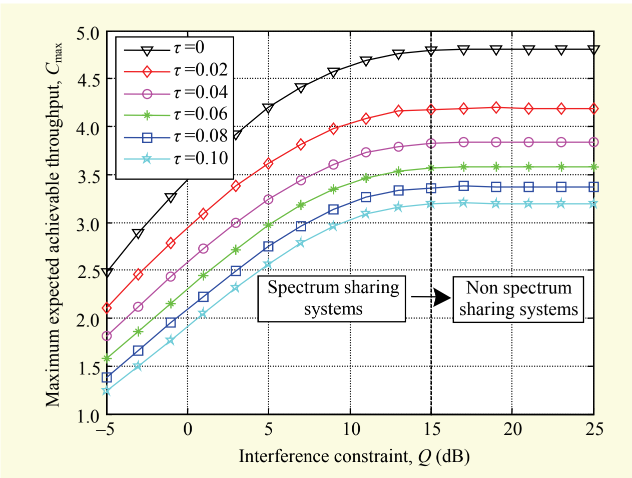

Fig. 2. Expected achievable throughput versus interference constraint (N=10, P=10 dB).

that for a given overhead τ, the expected throughput grows as Q increases. However, when $Q \rightarrow \infty$ (for example, $Q \geq 15$ dB for the simulated scenario), the achievable throughput remains almost unchanged, which implies that the SS system degenerates into a non-SS system. Also, it is noted from Fig. 2 that larger τ leads to lower achievable throughput as expected.

Thirdly, we study the impact of the interference constraints Q on the stopping time in Fig. 3. It is noted that for a given τ>0, the expected number of explored channels N_s decreases as Q increases. The reason is that as Q increases, the achievable throughput decreases by proceeding to explore the remaining channels, and then the SU is more likely to stop exploration. Also, when $Q \rightarrow \infty$ (for example, $Q \geq 15$ dB), N_s remains almost unchanged, which again implies that it degenerates into a non-SS system. It is noted from the figure that the SU always explores all the channels when τ=0 (that is, N_s=10) since there is no overhead. Moreover, larger τ leads to smaller N_s, as

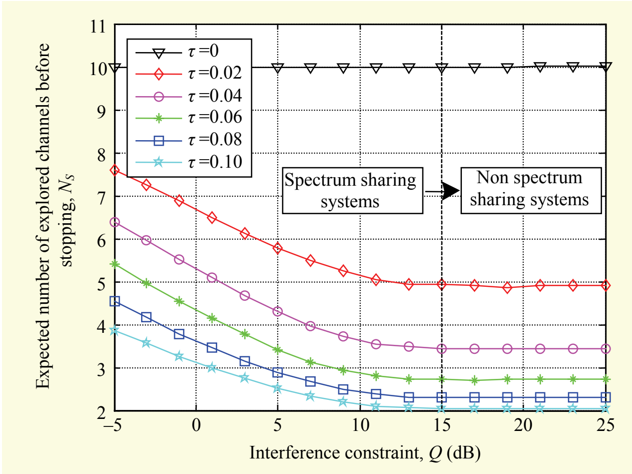

Fig. 3. Expected number of explored channels before stopping (N=10, P=10 dB).

expected.

V. Conclusion

We formulated the problem of exploiting multichannel diversity in spectrum sharing systems as an optimal stopping problem and proposed the myopic but optimal 1–SLA rule to solve it. Besides time consumption, we will investigate other resource consumption for channel exploration (for example, energy consumption) in further work.

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