A Fuzzy TOPSIS Approach Based on Trapezoidal Numbers to Material Selection Problem

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Abstract

Material selection is a complex problem in the design and development of products for diverse engineering applications. This paper is aimed to present a fuzzy decision making approach to deal with the material selection in engineering design problems. A fuzzy multi criteria decision making model is proposed for solving the material selection problem. The proposed model makes use of fuzzy TOPSIS (Technique for Order reference by Similarity to Ideal Solution) with trapezoidal numbers for evaluating the criteria and ranking the alternatives. And result is compared with fuzzy VIKOR (VIseKriterijumska Optimizacija I Kompromisno Resenje in Serbian, means Multi criteria Optimisation and Compromise Solution) which is proposed by Jeya Girubha and Vinodh [2012]. The present paper is aimed to also improve literature of fuzzy decision making for material selection problem.

Keywords : Material Selection, Fuzzy TOPSIS, Trapezoidal Fuzzy Numbers

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1. Introduction

The ability to select optimal material candidates from a wide variety of materials seems like a very complicated process. It is a hard and often difficult task to choose a suitable material from the hundreds of materials used in modern industry applications. Many different factors must be taken into account during material selection, such as material properties, material cost and availability, processing and environment. Toward this end, multi-criteria decision making (MCDM) methods have gained importance in this field. These methods perform an important role in the decision-making process for both small and large problems. In the past, much work has been reported on the selection of materials using classical methods [1-2-3]. Recently, fuzzy-based MCDM methods are commonly used in material selection area because of that they use subjective attributes. Wang and Chang [4] proposed a fuzzy MCDM approach to assist in the selection of the most suitable steel for specific manufacturing applications. They gave the weights of individual criteria and the material suitability ratios of various alternatives under different criteria in linguistic terms. Liao [5] presented a fuzzy MCDM method to support material selection decisions in engineering design applications. Similarly, Chen [6] presented a fuzzy environment method to solve material selection problems. They used fuzzy numbers both in determining criteria and alternatives. Jee and Kang [7] presented a hybrid method of entropy and the technique of order preference by similarity to ideal solution (TOPSIS) to aid designers on material selection. They developed the procedure of optimal material selection. Milani et al. [8] used TOPSIS method to select gear materials for power transmission. Rathod and Kanzaria [3] solved material selection problems using TOPSIS and fuzzy TOPSIS methods. The criteria weights are determined by analytic hierarchy process (AHP). Jahan et al. [9] proposed a novel normalization technique as an extension of the TOPSIS method. Their proposed method considers criteria that contain cost, benefit and target value as part of the normalization technique. Shanian and Savadogo [10] presented an application of the TOPSIS as a MCDM method for solving the material selection problem of metallic bipolar plates for polymer electrolyte fuel cell applications. By introducing a new non-compensatory approach, they proposed ELECTRE for the material selection of mass-produced, non-heat-treatable cylindrical cover materials [11] and ELECTRE I for the material selection of bipolar plates [12].

Chauhan and Vaish [13] used TOPSIS and the Vise Kriterijumska Optimizacija Kompromisno Resenje (VIKOR) methods to find the optimal materials for magnetic applications. Rao [1] presented the VIKOR method for a logical material selection procedure. The weights of the attributes were assigned using AHP. Jahan et al. [14] proposed an improved VIKOR method for ranking and selecting optimum materials. Girubha and Vinodh [15] used a fuzzy-based VIKOR method to evaluate alternative materials for the instrument panels used in electric cars. Chatterjee et al. [16] presented VIKOR and ELECTRE methods to solve material selection problems. The results obtained were compared with each other.

Dweiri and Al-Oqla [17] applied the AHP and sensitivity analysis to increase the confidence in a given material choice. Cao et al. [18] integrated the grey relation analysis (GRA) with the AHP in a material selection problem for electronic device housings. Cicek et al. [19] proposed an integrated decision aid (IDEA) method for matching appropriate techniques with different problem cases based on six dimensions. Karande and Chakraborty [20] used the multi-objective optimization on the basis of ratio analysis (MOORA) method to solve some common material selection problems.

2. The Fuzzy TOPSIS with Trapezoidal Fuzzy Numbers

TOPSIS (Technique for Order Performance by Similarity to Ideal Solution), one of the classical MCDM methods, was proposed by Hwang and Yoon [21]. Chen and Hwang [22] applied fuzzy numbers to establish fuzzy TOPSIS. Chen [23] is proposed an extension TOPSIS method under fuzzy environment. Yong [24] used fuzzy TOPSIS for plant location selection and Chena et al. [25] used fuzzy TOPSIS for supplier selection. Kahraman et al. [26] utilized fuzzy TOPSIS for industrial robotic system selection. Kaya and Kahraman [27] proposed a modified fuzzy TOPSIS for selection of the best energy technology alternative. Kim et al. [28] used fuzzy TOPSIS for modeling consumer's product adoption process. Park et al. [29] proposed TOPSIS to determine library air-conditioning systems.

The basic steps of proposed fuzzy TOPSIS method can be described as follows.

Step 1 : In the first step, a panel of decision-makers (DMs) who are knowledgeable about material selection and evaluation process is established. In a group that has K decision-makers (i.e. D_1, D_2, ⋯, D_k) are responsible for ranking (y_{jk}) of each criterion (i.e. C_1, C_2, ⋯, C_n) in increasing order.

$$\alpha_j = \min\{y_{jk}\} \quad b_j = \frac{1}{K}\sum_{k=1}^{K} y_{jk}$$
$$c_j = \frac{1}{K}\sum_{k=1}^{K} y_{jk} \quad d_j = \max\{y_{jk}\} \tag{1}$$

Then, the aggregated fuzzy importance weight for each criterion can be described as fuzzy trapezoidal numbers.

$$\tilde{w}_j = \{w_{j1}, w_{j2,} w_{j3,} w_{j4,}\}\ for$$
$$k = 1, 2, \cdots, K \text{ and } j = 1, 2, \cdots, n \tag{2}$$

The aggregated fuzzy importance weight can be determined as follows :

$$\tilde{w}_j = \{w_{j1}, w_{j2,} w_{j3,} w_{j4,}\}$$
$$w_{j1} = \min\{w_{jk1}\} \quad w_{j2} = \frac{1}{K}\sum_{k=1}^{k} w_{jk2}$$
$$w_{j3} = \frac{1}{K}\sum_{k=1}^{k} w_{jk3} \quad w_{j4} = \min\{w_{jk4}\} \tag{3}$$

Step 2 : A decision matrix is formed.

$$\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \tag{4}$$

Step 3 : To avoid complexity of mathematical operations in a decision process, the linear scale transformation is used here to transform the various criteria scales into comparable scales. The set of criteria can be divided into benefit criteria (the larger the rating, the greater the preference) and cost criteria (the smaller the rating, the greater the preference). Therefore, the normalized fuzzy-decision matrix can be represented as

$$\tilde{R}_{ij} = [\tilde{r}_{ij}]_{mxn} \tag{5}$$

where B and C are the sets of benefit criteria and cost criteria, respectively, and

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{d_j^*}, \frac{b_{ij}}{d_j^*}, \frac{c_{ij}}{d_j^*}, \frac{d_{ij}}{d_j^*}, \right), \ j \in B,$$
$$\tilde{r}_{ij} = \left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}} \frac{a_j^-}{b_{ij}} \frac{a_j^-}{a_{ij}} \right), j \in C \tag{6}$$
$$d_j^* = \max_i \ d_{ij}, \ j \in B,$$
$$a_j^* = \min_i \ a_{ij}, \ j \in C,$$

Step 4 : Considering the different weights of each criterion, the weighted normalized decision matrix is computed by multiplying the importance weight of evaluation criteria and the values in the normalized decision matrix. The weighted normalized decision matrix $\tilde{V}$ for each criterion is defined as:

$$\tilde{V} = [\tilde{v_{ij}}]_{mxn} \ for \ i = 1, 2, \cdots, m \text{ and } j = 1, 2, \cdots, n \tag{7}$$

Where $\tilde{v}_{ij} = \tilde{r}_{ij} \times \tilde{w}_j$ here $\tilde{v}_{ij}$ denotes normalized positive trapezoidal fuzzy numbers.

Step 5 : Then fuzzy positive $\tilde{A}^*$ and fuzzy negative $\tilde{A}^-$ ideal solutions are determined. The fuzzy positive-ideal solutions (FPIS, $\tilde{A}^*$) and the fuzzy negative ideal solution (FNIS, $\tilde{A}^-$) can be defined for beneficial criteria.

$$\tilde{A}^* = (\tilde{v}_1^*, \tilde{v}_2^*, \cdots, \tilde{v}_n^*) \text{ and } \tilde{v}_j^* = (\tilde{r}_1^+, \tilde{r}_2^+, \tilde{r}_3^+, \tilde{r}_4^+)$$
$$\tilde{A}^- = (\tilde{v}_1^-, \tilde{v}_2^-, \cdots, \tilde{v}_n^-) \text{ and } \tilde{v}_j^- = (\tilde{r}_1^-, \tilde{r}_2^-, \tilde{r}_3^-, \tilde{r}_4^-)$$
$$for \ i = 1, \ \cdots, \ n$$
$$\tilde{r}_i^+ = (1, 1, 1, 1) \text{ and } \tilde{r}_i^- = (0, 0, 0, 0) \ for \ i = 1, \cdots, n \tag{8}$$

Step 6 : Then the separation measures of each alternative from the positive ideal solution d_i^+ and the negative ideal solution d_i^- are calculated. The separation between alternatives can be measured by Hamming distance or Euclidean distance. We considered several definitions proposed by Park et al. [29], consist of Hamming, Euclidean and their normalized version.

Separation measures based on the Hamming distance

(i) The extension of Burillo and Bustince's method,

$$A_{j^*}^{d_1} = \frac{1}{4} \sum_{i=1}^{n} \left[\begin{array}{l} |a_{ij}^* - a_i^+| + |b_{ij}^* - b_i^+| + \\ |c_{ij}^* - c_i^+| + |d_{ij}^* - d_i^+| \end{array} \right] \tag{9}$$
$$A_{j^-}^{d_1} = \frac{1}{4} \sum_{i=1}^{n} \left[\begin{array}{l} |a_{ij}^* - a_i^-| + |b_{ij}^* - b_i^-| + \\ |c_{ij}^* - c_i^-| + |d_{ij}^* - d_i^-| \end{array} \right]$$

(ii) The extension of modified Burillo and Bustince's method,

$$A_{j^*}^{d_2} = \frac{1}{4} \sum_{i=1}^{n} \left[\begin{array}{l} |a_{ij}^* - a_i^+| + |b_{ij}^* - b_i^+| + |c_{ij}^* - c_i^+| + \\ |d_{ij}^* - d_i^+| + \| a_{ij}^* - b_{ij}^* | - |a_i^+ - b_i^+ \| + \\ \| c_{ij}^* - d_{ij}^* | + |c_i^+ - d_i^+ \| \end{array} \right] \tag{10}$$
$$A_{j^-}^{d_2} = \frac{1}{4} \sum_{i=1}^{n} \left[\begin{array}{l} |a_{ij}^* - a_i^-| + |b_{ij}^* - b_i^-| + |c_{ij}^* - c_i^-| + \\ |d_{ij}^* - d_i^-| + \| a_{ij}^* - b_{ij}^* | - |a_i^- - b_i^- \| + \\ \| c_{ij}^* - d_{ij}^* | + |c_i^- - d_i^- \| \end{array} \right]$$

(iii) The extension of Grzegorzewski's method,

$$A_{j^*}^{d_H} = \frac{1}{2}\sum_{i=1}^{n}\begin{bmatrix}\max(|a_{ij}^*-a_i^+|,\ |b_{ij}^*-b_i^+|)+\\ \max(|c_{ij}^*-c_i^+|,\ |d_{ij}^*-d_i^+|)\end{bmatrix} \quad (11)$$

$$A_{j^-}^{d_H} = \frac{1}{2}\sum_{i=1}^{n}\begin{bmatrix}\max(|a_{ij}^*-a_i^-|,\ |b_{ij}^*-b_i^-|)+\\ \max(|c_{ij}^*-c_i^-|,\ |d_{ij}^*-d_i^-|)\end{bmatrix}$$

Separation measures based on the normalized Hamming distance

(i) The extension of Burillo and Bustince's method,

$$A_{j^*}^{l_1} = \frac{1}{4m}\sum_{i=1}^{n}\begin{bmatrix}|a_{ij}^*-a_i^+|+|b_{ij}^*-b_i^+|+\\ |c_{ij}^*-c_i^+|+|d_{ij}^*-d_i^+|\end{bmatrix} \quad (12)$$

$$A_{j^-}^{l_1} = \frac{1}{4m}\sum_{i=1}^{n}\begin{bmatrix}|a_{ij}^*-a_i^-|+|b_{ij}^*-b_i^-|+\\ |c_{ij}^*-c_i^-|+|d_{ij}^*-d_i^-|\end{bmatrix}$$

(ii) The extension of modified Burillo and Bustince's method,

$$A_{j^*}^{l_2} = \frac{1}{4m}\sum_{i=1}^{n}\begin{bmatrix}|a_{ij}^*-a_i^+|+|b_{ij}^*-b_i^+|+\\ |c_{ij}^*-c_i^+|+|d_{ij}^*-d_i^+|+\\ ||a_{ij}^*-b_{ij}^*|+|a_i^+-b_i^+||+\\ ||c_{ij}^*-d_{ij}^*|+|c_i^+-d_i^+||\end{bmatrix} \quad (13)$$

$$A_{j^-}^{l_2} = \frac{1}{4m}\sum_{i=1}^{n}\begin{bmatrix}|a_{ij}^*-a_i^-|+|b_{ij}^*-b_i^-|+\\ |c_{ij}^*-c_i^-|+|d_{ij}^*-d_i^-|+\\ ||a_{ij}^*-b_{ij}^*|+|a_i^--b_i^-||+\\ ||c_{ij}^*-d_{ij}^*|+|c_i^--d_i^-||\end{bmatrix}$$

(iii) The extension of Grzegorzewsk's method,

$$A_{j^*}^{l_H} = \frac{1}{2m}\sum_{i=1}^{n}\begin{bmatrix}\max(|a_{ij}^*-a_i^+|,\ |b_{ij}^*-b_i^+|)+\\ \max(|c_{ij}^*-c_i^+|,\ |d_{ij}^*-d_i^+|)\end{bmatrix} \quad (14)$$

$$A_{j^-}^{l_H} = \frac{1}{2m}\sum_{i=1}^{n}\begin{bmatrix}\max(|a_{ij}^*-a_i^-|,\ |b_{ij}^*-b_i^-|)+\\ \max(|c_{ij}^*-c_i^-|,\ |d_{ij}^*-d_i^-|)\end{bmatrix}$$

Separation measures based on the Euclidean distance

(i) The extension of Burillo and Bustince's method,

$$A_{j^*}^{e_1} = \left\{\frac{1}{4}\sum_{i=1}^{n}\begin{bmatrix}(a_{ij}^*-a_i^+)^2+(b_{ij}^*-b_i^+)^2+\\ (c_{ij}^*-c_i^+)^2+(d_{ij}^*-d_i^+)^2\end{bmatrix}\right\}^{\frac{1}{2}} \quad (15)$$

$$A_{j^-}^{e_1} = \left\{\frac{1}{4}\sum_{i=1}^{n}\begin{bmatrix}(a_{ij}^*-a_i^-)^2+(b_{ij}^*-b_i^-)^2+\\ (c_{ij}^*-c_i^-)^2+(d_{ij}^*-d_i^-)^2\end{bmatrix}\right\}^{\frac{1}{2}}$$

(ii) The extension of modified Burillo and Bustince's method

$$A_{j^*}^{e_2} = \left\{\frac{1}{4}\sum_{i=1}^{n}\begin{bmatrix}(a_{ij}^*-a_i^+)^2+(b_{ij}^*-b_i^+)^2+\\ (c_{ij}^*-c_i^+)^2+(d_{ij}^*-d_i^+)^2+\\ (|a_{ij}^*-b_{ij}^*|-|a_i^+-b_i^+|)^2+\\ (|c_{ij}^*-d_{ij}^*|+|c_i^+-d_i^+|)^2\end{bmatrix}\right\}^{\frac{1}{2}} \quad (16)$$

$$A_{j^-}^{e_2} = \left\{\frac{1}{4}\sum_{i=1}^{n}\begin{bmatrix}(a_{ij}^*-a_i^-)^2+(b_{ij}^*-b_i^-)^2+\\ (c_{ij}^*-c_i^-)^2+(d_{ij}^*-d_i^-)^2+\\ (|a_{ij}^*-b_{ij}^*|-|a_i^--b_i^-|)^2+\\ (|c_{ij}^*-d_{ij}^*|+|c_i^--d_i^-|)^2\end{bmatrix}\right\}^{\frac{1}{2}}$$

(iii) The extension of Grzegorzewski's method

$$A_{j^*}^{e_H} = \left\{\frac{1}{2}\sum_{i=1}^{n}\begin{bmatrix}(\max(|a_{ij}^*-a_i^+|,|b_{ij}^*-b_i^+|))^2+\\ (\max(|c_{ij}^*-c_i^+|,|d_{ij}^*-d_i^+|))^2\end{bmatrix}\right\}^{\frac{1}{2}} \quad (17)$$

$$A_{j^-}^{e_H} = \left\{\frac{1}{2}\sum_{i=1}^{n}\begin{bmatrix}(\max(|a_{ij}^*-a_i^-|,\ |b_{ij}^*-b_i^-|))^2+\\ (\max(|c_{ij}^*-c_i^-|,\ |d_{ij}^*-d_i^-|))^2\end{bmatrix}\right\}^{\frac{1}{2}}$$

Separation measures based on the normalized Euclidean distance

(i) The extension of Burillo and Bustince's method,

$$A_{j^*}^{q_1} = \left\{\frac{1}{4m}\sum_{i=1}^{n}\begin{bmatrix}(a_{ij}^*-a_i^+)^2+(b_{ij}^*-b_i^+)^2+\\ (c_{ij}^*-c_i^+)^2+(d_{ij}^*-d_i^+)^2\end{bmatrix}\right\}^{\frac{1}{2}} \quad (18)$$

$$A_{j^-}^{q_1} = \left\{\frac{1}{4m}\sum_{i=1}^{n}\begin{bmatrix}(a_{ij}^*-a_i^-)^2+(b_{ij}^*-b_i^-)^2+\\ (c_{ij}^*-c_i^-)^2+(d_{ij}^*-d_i^-)^2\end{bmatrix}\right\}^{\frac{1}{2}}$$

(ii) The extension of modified Burillo and Bustince's method,

$$A_{j^*}^{q_2} = \left\{\frac{1}{4m}\sum_{i=1}^{n}\begin{bmatrix}(a_{ij}^*-a_i^+)^2+(b_{ij}^*-b_i^+)^2+\\ (c_{ij}^*-c_i^+)^2+(d_{ij}^*-d_i^+)^2+\\ (|a_{ij}^*-b_{ij}^*|+|a_i^+-b_i^+|)^2+\\ (|c_{ij}^*-d_{ij}^*|+|c_i^+-d_i^+|)^2\end{bmatrix}\right\}^{\frac{1}{2}} \quad (19)$$

$$A_{j^{=}}^{q_2} = \left\{ \frac{1}{4m} \sum_{i=1}^{n} \begin{bmatrix} (a_{ij}^{*} - a_i^{-})^2 + (b_{ij}^{*} - b_i^{-})^2 + \\ (c_{ij}^{*} - c_i^{-})^2 + (d_{ij}^{*} - d_i^{-})^2 + \\ (|a_{ij}^{*} - b_{ij}^{*}| + |a_i^{-} - b_i^{-}|)^2 + \\ (|c_{ij}^{*} - d_{ij}^{*}| + |c_i^{-} - d_i^{-}|)^2 \end{bmatrix} \right\}^{\frac{1}{2}}$$

(iii) The extension of Grzegorzewski's method,

$$A_{j^{*}}^{q_H} = \left\{ \frac{1}{2m} \sum_{i=1}^{n} \begin{bmatrix} (\max(|a_{ij}^{*} - a_i^{+}|, |b_{ij}^{*} - b_i^{+}|))^2 + \\ (\max(|c_{ij}^{*} - c_i^{+}|, |d_{ij}^{*} - d_i^{+}|))^2 \end{bmatrix} \right\}^{\frac{1}{2}} \quad (20)$$

$$A_{j^{-}}^{q_H} = \left\{ \frac{1}{2m} \sum_{i=1}^{n} \begin{bmatrix} (\max(|a_{ij}^{*} - a_i^{-}|, |b_{ij}^{*} - b_i^{-}|))^2 + \\ (\max(|c_{ij}^{*} - c_i^{-}|, |d_{ij}^{*} - d_i^{-}|))^2 \end{bmatrix} \right\}^{\frac{1}{2}}$$

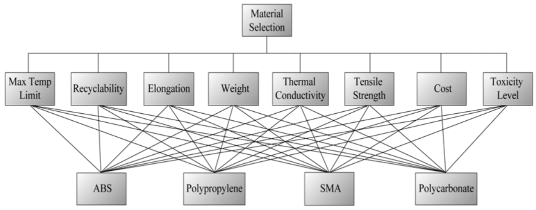
Step 7 : Then the fuzzy closeness coefficient CC_i is determined.

$$CC_i = d_i^{-} / (d_i^{-} + d_i^{+}) \quad (21)$$

Step 8 : Rank the preference order. For ranking alternatives using this index, we can rank alternatives in decreasing order.

3. Numerical Example

We take material selection problem for an automotive instrument panel applied by Girubha and Vinodh [18] as a numerical example. The alternatives consist of Acrylonitrile Butadiene Styrene (ABS), Polypropylene, Styrene Maleic Anhydride (SMA) and Polycarbonate. The materials are designated as M1 (SMA), M2 (Polycarbonate), M3 (Polypropylene) and M4 (ABS). And the criteria are determined as maximum temperature limit (C1), recyclability (C2), elongation (C3), weight (C4), thermal conductivity (C5), tensile strength (C6), cost (C7), toxicity level (C8). The criteria are non-beneficial except C1, C2 and C4.



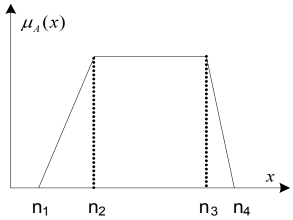
⟨Figure 1⟩ Hierarchical structure

The hierarchical structure of this decision problem is shown in <Figure 1>. The linguistics terms and corresponding fuzzy numbers for each criterion and materials assessed by decision makers are shown in <Table 1>. Linguistic variables were used to calculate the importance of the criteria and the ratings of alternatives with respect to various criteria.

A trapezoidal fuzzy number can be defined as $\{(n_1, n_2, n_3, n_4) \mid n_1, n_2, n_3, n_4 \in R;\ n_1 \le n_2 \le n_3 \le n_4\}$ which respectively, denotes the smallest possible, most promising, and largest possible values [15] and the membership function is defined and is shown in <Figure 2>.

⟨Table 1⟩ Linguistic Terms and Corresponding Fuzzy Numbers

Linguistic Variable	Fuzzy Number	Linguistic Variable
Very Poor(VP)	(0,0; 0,0; 0,1; 0,2)	Very Low(VL)
Poor(P)	(0,1; 0,2; 0,2; 0,3)	Low(L)
Medium Poor(MP)	(0,2; 0,3; 0,4; 0,4)	Fairly Low(FL)
Fair(F)	(0,4; 0,5; 0,5; 0,6)	Medium(M)
Medium Good(MG)	(0,5; 0,6; 0,7; 0,8)	Fairly High(FH)
Good(G)	(0,7; 0,8; 0,8; 0,9)	High(H)
Very Good(VG)	(0,8; 0,9; 1;1)	Very High(VH)



⟨Figure 2⟩ Trapezoidal Fuzzy Number

〈Table 2〉 Importance Weight of Criteria (linguistic Variable)

	C1	C2	C3	C4	C5	C6	C7	C8
D1	G	G	G	G	G	G	G	MG
D2	G	G	G	MG	VG	MG	G	G
D3	VG	MG	MG	VG	G	G	G	MG
D4	G	VG	MG	G	VG	MG	VG	MG
D5	MG	G	G	G	G	MG	G	G

The linguistic variables for each criterion is shown in <Table 2> and the respective terms for describing the importance of material with respect to criteria assessed by decision makers are shown in <Table 3>.

〈Table 3〉 Importance of Material with Respect to Criteria (linguistic Variable)

		C1	C2	C3	C4	C5	C6	C7	C8
D1	M1	FH	H	FH	FH	H	FH	FH	FH
	M2	M	FH	H	H	H	FH	H	FH
	M3	VH	H	VH	H	H	VH	H	VH
	M4	H	VH	H	H	VH	H	VH	H
D2	M1	FH	H	FH	FH	FH	H	FH	FH
	M2	FH	M	FH	M	FH	M	M	M
	M3	H	VH	H	VH	H	VH	H	H
	M4	H	H	VH	H	VH	H	VH	VH
D3	M1	FH	H	FH	H	FH	FH	H	FH
	M2	FH	FH	H	H	H	H	FH	H
	M3	VH	VH	H	H	VH	VH	H	H
	M4	H	VH	H	H	H	VH	H	H
D4	M1	H	FH	H	FH	FH	H	FH	H
	M2	FH	FH	M	FH	M	FH	M	FH
	M3	H	VH	H	VH	H	H	VH	H
	M4	H	H	VH	H	H	H	VH	VH
D5	M1	FH	H	FH	H	FH	H	FH	FH
	M2	H	FH	H	FH	H	FH	M	M
	M3	VH	H	VH	H	VH	H	VH	H
	M4	H	VH	H	H	H	VH	H	M

The aggregated matrix for criterion weights and material ratings are calculated using Equation (3) and it is shown in <Table 4>.

In this step the normalization of the each criterion is calculated. The aggregated value is divided by minimum value if the criterion is cost (C). The aggregated value is divided by the maximum value if the criterion is benefiting (B) using Equation (6). The normalized values are shown in <Table 5>.

〈Table 4〉 The Aggregated Fuzzy Values of Materials

	M1	M2
C1	(0,5; 0,64; 0,72; 0,9)	(0,4; 0,62; 0,68; 0,9)
C2	(0,5; 0,76; 0,78; 0,9)	(0,4; 0,58; 0,66; 0,8)
C3	(0,5; 0,64; 0,72; 0,9)	(0,4; 0,64; 0,66; 0,9)
C4	(0,5; 0,68; 0,74; 0,9)	(0,4; 0,66; 0,7; 0,9)
C5	(0,5; 0,64; 0,72; 0,9)	(0,4; 0,7; 0,72; 0,9)
C6	(0,5; 0,72; 0,76; 0,9)	(0,4; 0,62; 0,68; 0,9)
C7	(0,5; 0,64; 0,72; 0,9)	(0,4; 0,58; 0,6; 0,9)
C8	(0,5; 0,64; 0,72; 0,9)	(0,4; 0,6; 0,64; 0,9)
	M3	**M4**
C1	(0,7; 0,86; 0,92; 1)	(0,7; 0,8; 0,8; 0, 9)
C2	(0,7; 0,86; 0,92; 1)	(0,7; 0,86; 0,92; 1)
C3	(0,7; 0,84; 0,88; 1)	(0,7; 0,84; 0,88; 1)
C4	(0,7; 0,84; 0,88; 1)	(0,7; 0,8; 0,8; 0, 9)
C5	(0,7; 0,84; 0,88; 1)	(0,7; 0,84; 0,88; 1)
C6	(0,7; 0,86; 0,92; 1)	(0,7; 0,84; 0,88; 1)
C7	(0,7; 0,84; 0,88; 1)	(0,7; 0,86; 0,92; 1)
C8	(0,7; 0,82; 0,84; 1)	(0,4; 0,78; 0,82; 1)

〈Table 5〉 The Fuzzy Normalized Decision Matrix

	M1	M2
C1	(0,25; 0,5; 0,59; 0,9)	(0,2; 0,48; 0,56; 0,9)
C2	(0,25; 0,59; 0,64; 0,9)	(0,2; 0,45; 0,54; 0,8)
C3	(0,22; 0,4; 0,48; 0,72)	(0,22; 0,44; 0,48; 0,9)
C4	(0,25; 0,53; 0,61; 0,9)	(0,2; 0,51; 0,57; 0,9)
C5	(0,31; 0,47; 0,55; 0,8)	(0,31; 0,47; 0,5; 1)
C6	(0,22; 0,36; 0,41; 0,72)	(0,22; 0,4; 0,48; 0,9)
C7	(0,31; 0,46; 0,53; 0,8)	(0,31; 0,55; 0,58; 1)
C8	(0,22; 0,38; 0,46; 0,72)	(0,22; 0,43; 0,49; 0,9)
	M3	**M4**
C1	(0,35; 0,67; 0,75; 1)	(0,35; 0,62; 0,66; 0,9)
C2	(0,35; 0,67; 0,75; 1)	(0,35; 0,67; 0,75; 1)
C3	(0,2; 0,33; 0,36; 0,51)	(0,2; 0,33; 0,36; 0,51)
C4	(0,35; 0,66; 0,72; 1)	(0,35; 0,62; 0,66; 0,9)
C5	(0,28; 0,38; 0,42; 0,57)	(0,28; 0,38; 0,42; 0,57)
C6	(0,2; 0,3; 0,34; 0,51)	(0,2; 0,31; 0,35; 0,51)
C7	(0,28; 0,37; 0,4; 0,57)	(0,28; 0,36; 0,39; 0,57)
C8	(0,2; 0,32; 0,36; 0,51)	(0,2; 0,33; 0,38; 0,9)

The weighted normalized fuzzy decision matrix is constructed using Equation (7) and it is shown in <Table 6>.

〈Table 6〉 The Fuzzy Weighted Normalized Decision Matrix

	M1	M2
C1	(0,5; 0,64; 0,72; 0,9)	(0,4; 0,62; 0,68; 0,9)
C2	(0,5; 0,76; 0,78; 0,9)	(0,4; 0,58; 0,66; 0,8)
C3	(0,44; 0,556; 0,63; 0,8)	(0,44; 0,61; 0,63; 1)
C4	(0,5; 0,68; 0,74; 0,9)	(0,4; 0,66; 0,7; 0,9)
C5	(0,44; 0,56; 0,63; 0,8)	(0,44; 0,56; 0,57; 1)
C6	(0,44; 0,53; 0,56; 0,8)	(0,44; 0,59; 0,65; 1)
C7	(0,44; 0,56; 0,63; 0,8)	(0,44; 0,67; 0,69; 1)
C8	(0,44; 0,56; 0,63; 0,8)	(0,44; 0,63; 0,67; 1)
	M3	**M4**
C1	(0,7; 0,86; 0,92; 1)	(0,7; 0,8; 0,8; 0,9)
C2	(0,7; 0,86; 0,92; 1)	(0,7; 0,86; 0,92; 1)
C3	(0,4; 0,45; 0,48; 0,57)	(0,4; 0,45; 0,48; 0,57)
C4	(0,7; 0,84; 0,88; 1)	(0,7; 0,8; 0,8; 0,9)
C5	(0,4; 0,45; 0,48; 0,57)	(0,4; 0,45; 0,48; 0,57)
C6	(0,4; 0,43; 0,47; 0,57)	(0,4; 0,45; 0,48; 0,57)
C7	(0,4; 0,45; 0,48; 0,57)	(0,4; 0,43; 0,47; 0,57)
C8	(0,4; 0,48; 0,49; 0,57)	(0,4; 0,49; 0,51; 1)

Fuzzy positive ideal solution (FPIS, A^*) and fuzzy negative ideal solution (FNIS, A^-) is determined <Table 7>.

〈Table 7〉 FPIS and FNIS Value of Each Criteria

	$\tilde{A}^*$	$\tilde{A}^-$
C1	(1; 1; 1; 1)	(0; 0; 0; 0)
C2	(1; 1; 1; 1)	(0; 0; 0; 0)
C3	(0; 0; 0; 0)	(1; 1; 1; 1)
C4	(1; 1; 1; 1)	(0; 0; 0; 0)
C5	(1; 1; 1; 1)	(0; 0; 0; 0)
C6	(1; 1; 1; 1)	(0; 0; 0; 0)
C7	(1; 1; 1; 1)	(0; 0; 0; 0)
C8	(1; 1; 1; 1)	(0; 0; 0; 0)

The separation measures A^* and A^- of each alternative are calculated using Equations (9)-(20) from (FPIS, A^*) and (FNIS, A^-) respectively, based on the Hamming distance, the Euclidean distance and the normalized versions <Table 8>.

〈Table 8〉 Separation measures of each alternative

		d^1	d^2	d^h	l^1	l^2	l^h
A^*	M1	3,66	4,62	4,62	0,91	1,15	1,15
	M2	4,12	5,35	5,35	1,03	1,34	1,34
	M3	2,79	3,55	3,55	0,70	0,89	0,89
	M4	3,00	3,83	3,83	0,75	0,96	0,96
A^-	M1	4,34	5,30	5,30	1,09	1,33	1,33
	M2	3,88	5,12	5,12	0,97	1,28	1,28
	M3	5,21	5,98	5,98	1,30	1,49	1,49
	M4	5,00	5,83	5,83	1,25	1,46	1,46
		e1	e^2	e^h	q^1	q^2	q^h
A^*	M1	4,00	4,24	4,82	2,00	2,12	2,41
	M2	4,56	4,92	5,63	2,28	2,46	2,82
	M3	3,10	3,31	3,73	1,55	1,66	1,87
	M4	3,31	3,55	4,03	1,66	1,78	2,01
A^-	M1	4,63	4,84	5,42	2,31	2,42	2,71
	M2	4,35	4,73	5,26	2,18	2,36	2,63
	M3	5,37	5,49	6,04	2,69	2,75	3,02
	M4	5,19	5,36	5,89	2,60	2,68	2,94

The relative closeness of each alternative is calculated based on different separation measures and it is shown in <Table 9>.

〈Table 9〉 The Closeness Coefficient of Each Alternative

	Value	Rank	Value	Rank	Value	Rank
	CC^{d1}		CC^{d2}		CC^{dh}	
M1	0,5431	3	0,5347	3	0,5347	3
M2	0,4854	4	0,4888	4	0,4888	4
M3	0,6514	1	0,6271	1	0,6271	1
M4	0,6248	2	0,6034	2	0,6034	2
	CC^{l1}		CC^{l2}		CC^{lh}	
M1	0,5431	3	0,5347	3	0,5347	3
M2	0,4854	4	0,4888	4	0,4888	4
M3	0,6514	1	0,6271	1	0,6271	1
M4	0,6248	2	0,6034	2	0,6034	2
	CC^{e1}		CC^{e2}		CC^{eh}	
M1	0,5366	3	0,5328	3	0,5293	3
M2	0,4883	4	0,4903	4	0,4829	4
M3	0,6341	1	0,6239	1	0,6183	1
M4	0,6107	2	0,6014	2	0,5939	2
	CC^{q1}		CC^{q2}		CC^{qh}	
M1	0,5366	3	0,5328	3	0,5293	3
M2	0,4883	4	0,4903	4	0,4829	4
M3	0,6341	1	0,6239	1	0,6183	1
M4	0,6107	2	0,6014	2	0,5939	2

And alternatives are ranked according to closeness coefficient. For all separation measures M3 is determined as the most desirable alternative.

The different separation measures with TOPSIS give the same ranking with VIKOR. And the correlation coefficient between VIKOR and TOPSIS is very high <Table 10>. It means that the relationship between two methods is positive and strong.

⟨Table 10⟩ Correlation Coefficients of the Methods for Ranking

	CC^{d1}	CC^{d2}	CC^{e1}	CC^{e2}	CC^{eh}	$Q(\upsilon=0,5)$
CC^{d1}	1,00					
CC^{d2}	1,00	1,00				
CC^{e1}	1,00	1,00	1,00			
CC^{e2}	1,00	1,00	1,00	1,00		
CC^{eh}	1,00	1,00	1,00	1,00	1,00	
$Q(\upsilon=0,5)$	-0,98	-0,99	-0,99	-0,99	-0,99	1,00

4. Conclusion

We investigate material selection problem using TOPSIS as a MCDM method under fuzzy environment. The evaluation of alternatives with respect to criteria and the importance weights are more effective to use the linguistic variables versus numerical values in decision making problems. We follow a fuzzy trapezoidal number based methodology then we propose different separation measures to calculate fuzzy positive ideal solutions and negative ideal solutions of TOPSIS. The proposed different separation measures and method can be applied to other multi-criteria decision making problems.

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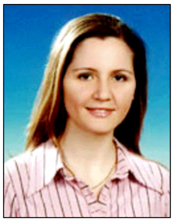

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