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EFFECT OF POROSITY ON THE TRANSIENT MHD GENERALIZED COUETTE FLOW WITH HEAT TRANSFER IN THE PRESENCE OF HEAT SOURCE AND UNIFORM SUCTION AND INJECTION

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ABSTRACT. The transient magnetohydrodynamic (MHD) generalized Couette flow with heat transfer through a porous medium of an electrically conducting, viscous, incompressible fluid bounded by two parallel insulating porous plates is studied in the presence of uniform suction and injection and a heat source considering the Hall effect. A uniform and constant pressure gradient is imposed in the axial direction and an externally applied uniform magnetic field as well as a uniform suction and injection are applied in the direction perpendicular to the plates. The two plates are kept at different but constant temperatures while the Joule and viscous dissipations are included in the energy equation. The effect of the Hall current, the porosity of the medium and the uniform suction and injection on both the velocity and temperature distributions is investigated.

1. INTRODUCTION

The magnetohydrodynamic (MHD) flow between two parallel plates, known as Hartmann flow, is a classical problem that has many applications in MHD power generators, MHD pumps, accelerators, aerodynamic heating, electrostatic precipitation, polymer technology, petroleum industry, purification of crude oil and fluid droplets and sprays. Hartmann and Lazarus [1] studied the influence of a transverse uniform magnetic field on the flow of a conducting fluid between two infinite parallel, stationary, and insulated

+plates. Then, a lot of research work concerning the Hartmann flow has been obtained under different physical effects [2-10]. In most cases the Hall and ion slip terms were

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ignored in applying Ohm's law as they have no marked effect for small and moderate values of the magnetic field. However, the current trend for the application of MHD is towards a strong magnetic field, so that the influence of electromagnetic force is noticeable [6-7]. Under these conditions, the Hall current and ion slip are important and they have a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. Soudalgekar et al. [8-9] studied the effect of the Hall currents on the steady MHD Couette flow with heat transfer. The temperatures of the two plates were assumed either to be constant [8] or to vary linearly along the plates in the direction of the flow [9]. Attia [10-11] extended the problem to the unsteady state with heat transfer, with constant pressure gradient applied.

In the present work, the transient generalized Couette flow and heat transfer through a porous medium of an incompressible, viscous, electrically conducting fluid between two infinite insulating horizontal porous plates are studied with the consideration of the Hall current and in the presence of a heat source. The upper plate is moving with a uniform velocity while the lower plate remains stationary. The fluid is acted upon by a constant pressure gradient, a uniform suction and injection and a uniform magnetic field perpendicular to the plates. The flow through a porous medium deals with the analysis in which the differential equation governing the fluid motion is based on the Darcy's law which accounts for the drag exerted by the porous medium [12-14]. The two plates are maintained at two different but constant temperatures. The governing equations including the Joule and viscous dissipations are solved numerically using the method of finite difference. The effect of the magnetic field, the Hall current, the porosity of the medium and the suction and injection on both the velocity and temperature distributions is reported.

2. DESCRIPTION OF THE PROBLEM

The two insulating plates are located at the $y=\pm h$ planes and extend from $x=-\infty$ to ∞ and $z=-\infty$ to ∞. The upper plate is moving with a uniform velocity U_o while the lower plate remains fixed. The lower and upper plates are kept at the two constant temperatures T_1 and T_2, respectively, where $T_2>T_1$ and a heat source is included. The fluid flows between the two plates under the effect of a constant pressure gradient dP/dx in the axial x-direction, and a uniform suction from above and injection from below which are applied at $t=0$ with velocity v_o. The whole system is subjected to a uniform magnetic field B_o in the positive y-direction. This is the total magnetic field acting on the fluid since the induced magnetic field is neglected. The fluid flows between the two plates in a porous medium where the Darcy model is assumed [12-14]. From the geometry of the problem, it is evident that all quantities are independent of x and z-coordinates apart from the pressure gradient dP/dx. The existence of the Hall term results in a z-component of the velocity. Thus, the velocity vector of the fluid is $v(y,t) = u(y,t)i + v_o j + w(y,t)k$.

The initial and boundary conditions are: u=w=0 at t≤0, u=w=0 at y=-h for t>0 and u=U_o and w=0 at y=h for t>0. The temperature $T(y,t)$ at any point in the fluid satisfies both the initial and boundary conditions T=T_1 at t≤0, T=T_2 at y=+h, and T=T_1 at y=-h for t>0. The fluid flow is governed by the momentum equation

$$\rho \frac{Dv}{Dt} = \mu \nabla^2 v - \nabla P + J \wedge B_o \tag{1}$$

where ρ and μ are, respectively, the density and the coefficient of viscosity of the fluid. If the Hall term is retained, the current density $\boldsymbol{J}$ is given by

$$J = \sigma(v \wedge B_o - \beta(J \wedge B_o))$$

where σ is the electric conductivity of the fluid, and β is the Hall factor [8,9]. This equation may be solved in $\boldsymbol{J}$ yielding

$$J \wedge B_o = -\frac{\sigma B_o^2}{1+m^2}((u+mw)i + (w-mu)k) \tag{2}$$

where $m = \sigma \beta B_o$, is the Hall parameter [8,9]. Thus, in terms of Eq. (2), the two components of Eq. (1) read [15]

$$\rho \frac{\partial u}{\partial t} + \rho v_o \frac{\partial u}{\partial y} = -\frac{dP}{dx} + \mu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_o^2}{1+m^2}(u+mw) - \frac{\mu}{K_p} u, \tag{3}$$

$$\rho \frac{\partial w}{\partial t} + \rho v_o \frac{\partial w}{\partial y} = \mu \frac{\partial^2 w}{\partial y^2} - \frac{\sigma B_o^2}{1+m^2}(w-mu) - \frac{\mu}{K_p} w, \tag{4}$$

The temperature distribution is governed by the energy equation [15]

$$\rho c_p \frac{\partial T}{\partial t} + \rho c_p v_o \frac{\partial T}{\partial y} = k \frac{\partial^2 T}{\partial y^2} + Q(T-T_1) + \mu((\frac{\partial u}{\partial y})^2 + (\frac{\partial w}{\partial y})^2) + \frac{\sigma B_o^2}{1+m^2}(u^2+w^2), \tag{5}$$

where c_p and k are, respectively, the specific heat capacity and the thermal conductivity of the fluid, K_p is the Darcy's permeability and Q is the heat generation coefficient. The last terms in the right side of Eqs. (3) and (4) represent the porosity force. The second and third terms in the right side represent the viscous and Joule dissipations, respectively. Introducing the following non-dimensional quantities

$$\hat{x} = \frac{x}{h}, \hat{y} = \frac{y}{h}, \hat{z} = \frac{z}{h}, \hat{u} = \frac{u}{U_o}, \hat{w} = \frac{w}{U_o}, \hat{P} = \frac{P}{\rho U_o^2}, \hat{t} = \frac{tU_o}{h},$$

$\mathrm{Re} = \rho h U_O / \mu$, is the Reynolds number,

$S = v_O / U_O$, is the suction parameter,

$\mathrm{Pr} = \mu c_p / k$, is the Prandtl number,

$Ec = U_o^2 / c_p(T_2 - T_1)$, is the Eckert number,

$Ha^2 = \sigma B_O^2 h^2 / \mu$, where Ha is the Hartmann number,

$M = h\mu / (\rho U_0 Kp)$ is the porosity parameter,

$\hat{Q} = QU_0 / (\rho h c_p)$ is the dimensionless heat generation coefficient the basic Eqs. (3)-(5) are written as (the hats are dropped for convenience)

$$\frac{\partial u}{\partial t}+\frac{S}{\mathrm{Re}}\frac{\partial u}{\partial y}=-\frac{dP}{dx}+\frac{1}{\mathrm{Re}}\frac{\partial^2 u}{\partial y^2}-\frac{Ha^2}{\mathrm{Re}(1+m^2)}(u+mw)-Mu, \tag{6}$$

$$\frac{\partial w}{\partial t}+\frac{S}{\mathrm{Re}}\frac{\partial w}{\partial y}=\frac{1}{\mathrm{Re}}\frac{\partial^2 w}{\partial y^2}-\frac{Ha^2}{\mathrm{Re}(1+m^2)}(w-mu)-Mw, \tag{7}$$

$$\frac{\partial T}{\partial t}+\frac{S}{\mathrm{Re}}\frac{\partial T}{\partial y}=\frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2}+QT+Ec((\frac{\partial u}{\partial y})^2+(\frac{\partial w}{\partial y})^2)+\frac{EcHa^2}{(1+m^2)}(u^2+w^2), \tag{8}$$

The initial and boundary conditions for the velocity become

$$t\le 0: u=w=0, t>0: u=w=0, y=-1, u=1, w=0, y=1, \tag{9}$$

and the initial and boundary conditions for the temperature are given by

$$t\le 0: T=0, t>0: T=1, y=+1, T=0, y=-1. \tag{10}$$

3. NUMERICAL SOLUTION OF THE GOVERNING EQUATIONS

Equations (6)-(8) are solved numerically using the method of finite difference [16] under the initial and boundary conditions (9) and (10) to determine the velocity and temperature distributions for different values of the parameters *Ha*, *m*, *M*, *S* and *Q*. The Crank-Nicolson implicit method is applied and the finite difference equations are written at the mid-point of the computational cell and the different terms are replaced by their second-order central difference approximations in the *y*-direction. The diffusion term is replaced by the average of the central differences at two successive time levels. The viscous and Joule dissipation terms are evaluated using the velocity components and their derivatives in the *y*-direction which are obtained from the exact solution. Finally, the block tri-diagonal system is solved using Thomas' algorithm. All computations are carried out for $dP/dx=5$, $Re=1$, $Pr=1$ and $Ec=0.2$.

4. RESULTS AND DISCUSSION

Figure 1 presents the profiles of the velocity components *u* and *w* and the temperature *T* for different values of time *t* and for $Ha=1$, $m=3$, $M=2$, $S=1$ and $Q=0.4$. It is clear from the figure that the velocity components and temperature reaches the steady state monotonically with time. Also the velocity component *u* reaches the steady state faster than *w* which, in turn, reaches the steady state faster than *T* because *u* is the source of *w*, while both *u* and *w* act as sources for the temperature.

Figure 2 indicates that the time progression of *u* and *w* at the centre of the channel $y=0$ for different values of the Hall parameter *m* and for $Ha=1$, $M=2$, $S=0$ and $Q=0.4$. It is clear from Fig. 2a that increasing the parameter *m* increases *u* because the effective conductivity ($\sigma/(1+m^2)$) decreases with increasing *m* which reduces the magnetic resistive force on *u*. In Fig. 2b, the velocity component *w* increases with increasing the parameter *m* slightly (*m*=0 to 1), since increasing *m* increases the driving force term ($mHa^2u/(1+m^2)$) in Eq. (7) which affects the flow in the z-direction. However, increasing *m* more decreases the effective conductivity that results in a reduced driving force and then, decreases *w*. It is clear from Fig. 2c that increasing *m* decreases *T* for all *t* due to decreasing the effect of the Joule dissipation.

Figure 3 presents the time progression of *u*, *w* and *T* at the centre of the channel for different values of the Hartmann number *Ha* and for *m*=3, *M*=0, *S*=0 and *Q*=0.4. Figure 3a indicates that increasing *Ha* decreases *u* as a result of increasing the damping force on *u*. Figure 3b indicates that increasing *Ha* increases *w* since it increases the driving force on *w*. Figure 3c depicts that for small *t*, increasing *Ha* increases *T* due to the increment in the Joule dissipation. But, for large *t*, increasing *Ha* decreases *T* as a result of decreasing the velocities *u* and *w* and consequently decreases the viscous and Joule dissipations.

Figure 4 presents the time progression of *u*, *w* and *T* at the centre of the channel for different values of the suction parameter *S* and for *Ha*=1, *M*=2, *m*=3 and *Q*=0. Figures 4a and 4b indicate that increasing the suction decreases both *u* and *w* due to the convection of the fluid from regions in the lower half to the centre which has higher fluid speed. Figure 4c shows that increasing *S* decreases the temperature at the centre of the channel due to the influence of convection in pumping the fluid from the cold lower half towards the centre of the channel.

Figure 5 presents the time progression of *u*, *w* and *T* at the centre of the channel for different values of the porosity parameter *M* and for *Ha*=1, *m*=3, *S*=0 and *Q*=0. Figure 5a and 5b indicate that increasing *M* decreases *u* and *w* as a result of increasing the damping force. Figure 5c depicts that increasing *M* decreases *T* due to the decrement in the Joule and viscous dissipations. Figure 6 presents the time progression of *T* at the centre of the channel for different values of the parameter *Q* and for *Ha*=1, *m*=3, *M*=2 and *S*=1. The figure indicates that increasing *Q* increases the temperature at the centre of the channel and its steady state time.

5. CONCLUSION

The transient MHD generalized Couette flow with heat transfer through a porous medium of an electrically conducting fluid under the influence of an applied uniform magnetic field has been studied considering the Hall effect in the presence of uniform suction and injection and a heat source. Introducing the Hall term gives rise to a velocity component *w* in the *z*-direction which affects the main velocity *u* in the *x*-direction. The effect of the magnetic field, the Hall parameter, the porosity parameter and the suction and injection velocity on the velocity and temperature distributions has been investigated. Both the magnetic field and the porosity of the medium have a damping effect on the velocity and temperature fields whereas the Hall parameter *m* increases the main velocity component *u*. On the other hand, increasing *m* increases the velocity component *w* for small *m* but decreases it for large *m*.

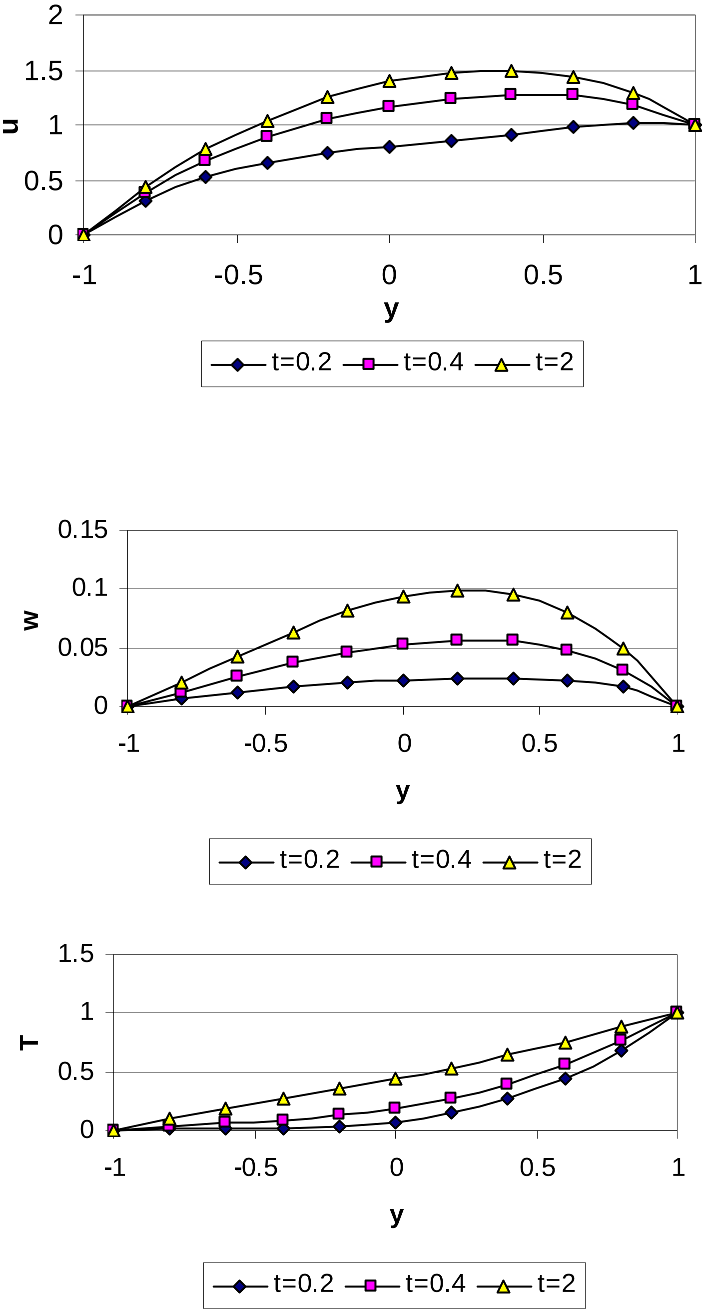

FIGURE 1. Time development of the profile of: (a) *u*; (b) *w*; and (c) *T* (*Ha*=1, *m*=3, *M*=2 and *S*=1, *Q*=0.4)

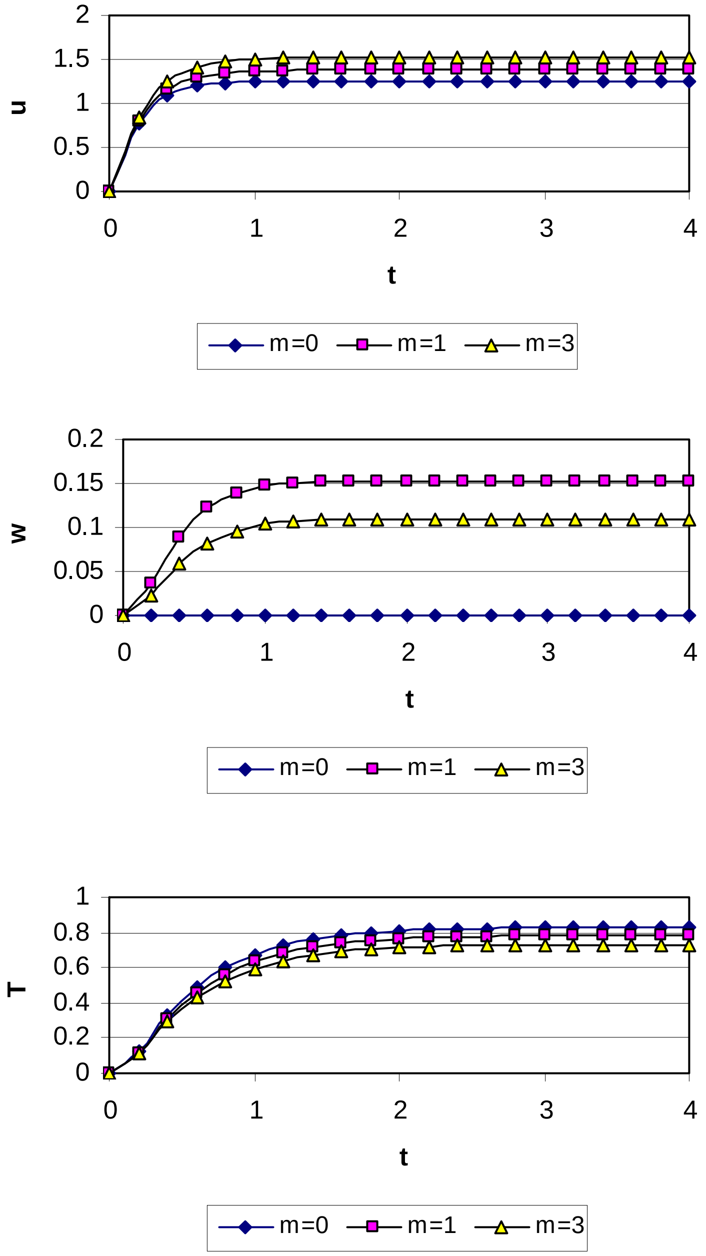

FIGURE 2. Effect of *m* on the time variation of: (a) *u* at *y*=0; (b) *w* at *y*=0 and (c) *T* at *y*=0. (*Ha*=1, *M*=2, *S*=0, and *Q*=0.4)

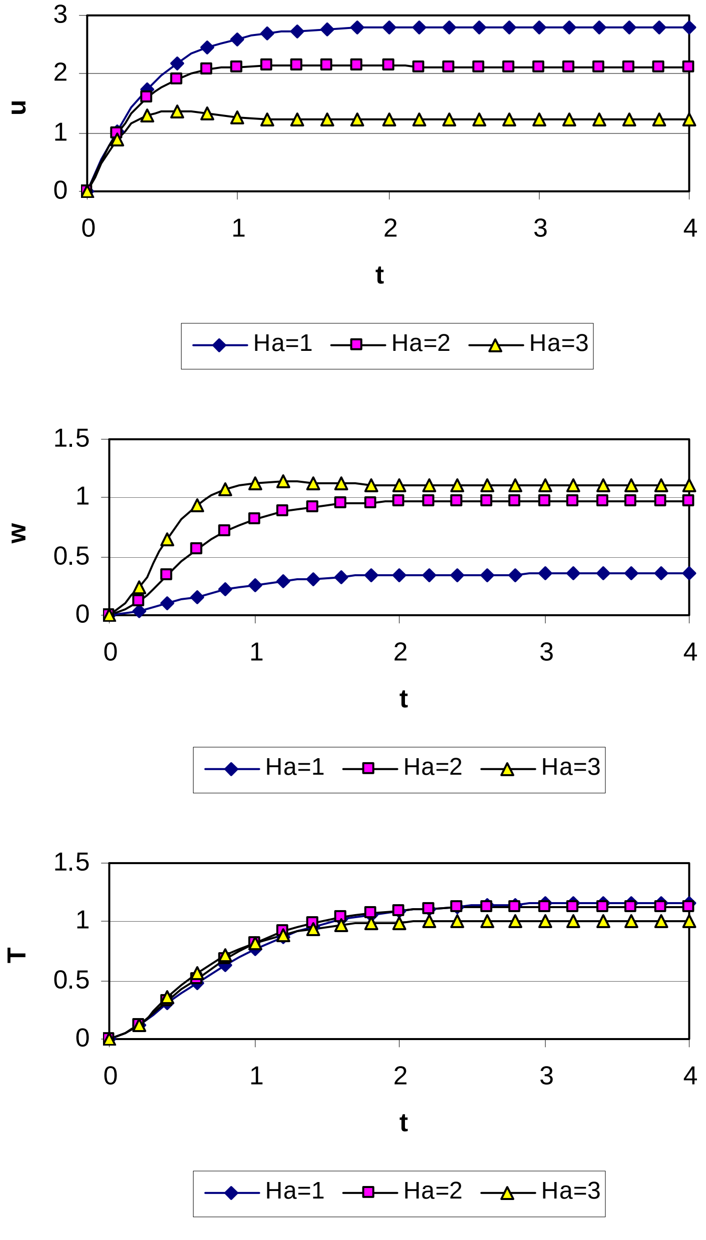

FIGURE 3. Effect of *Ha* on the time variation of: (a) *u* at *y*=0; (b) *w* at *y*=0 and (c) *T* at *y*=0. (*m*=3, *M*=0, *S*=0 and *Q*=0.4)

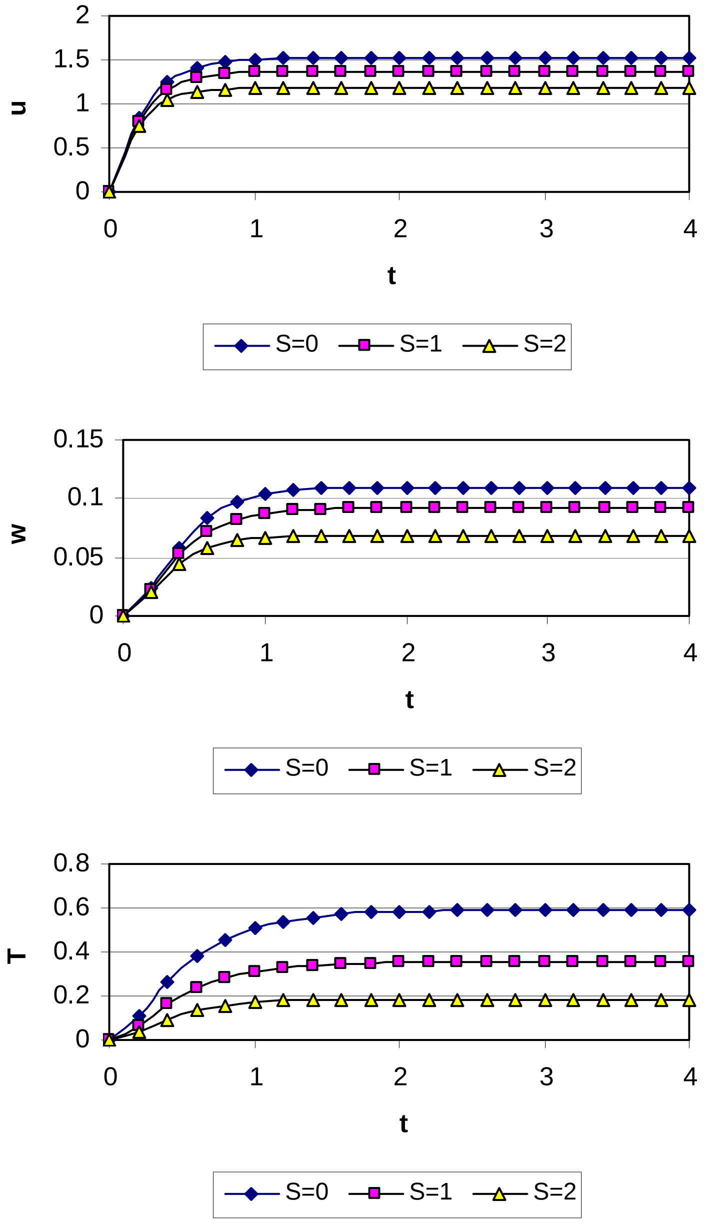

FIGURE 4. Effect of *S* on the time variation of: (a) *u* at *y*=0; (b) *w* at *y*=0; and (c) *T* at *y*=0. (*Ha*=1, *M*=2, *m*=3, and *Q*=0)

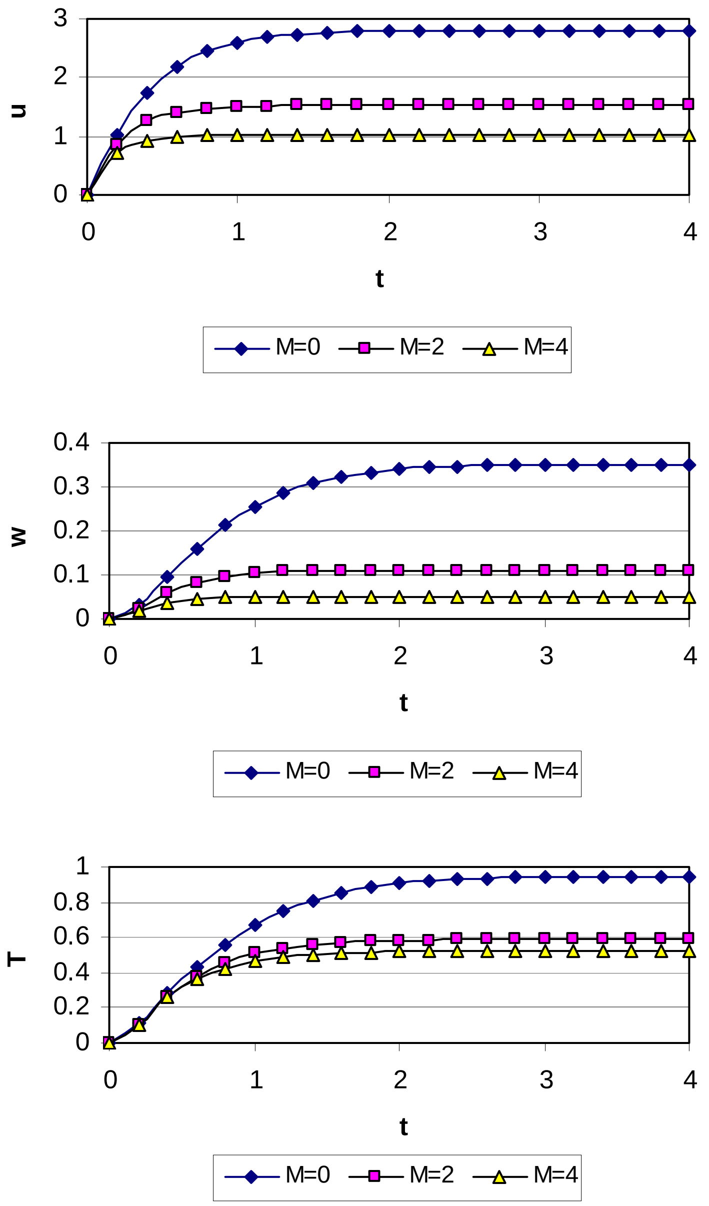

FIGURE 5. Effect of M on the time variation of: (a) u at y=0; (b) w at y=0; and (c) T at y=0. (Ha=1, m=3 and Q=0, S=0)

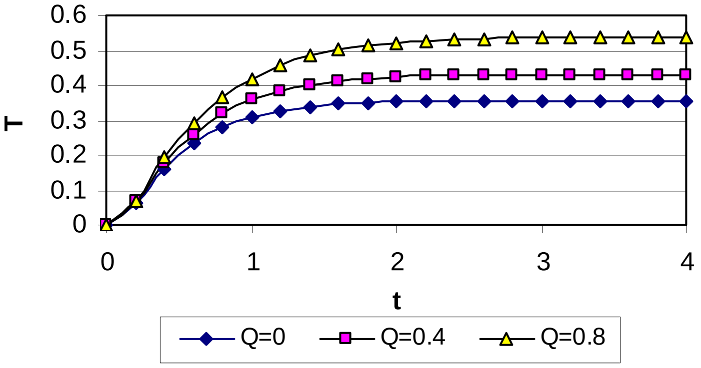

FIGURE 6. Effect of Q on the time variation of: T at y=0.
(Ha=1, m=3 and M=2, S=1)

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