한국소음진동공학회논문집 제 22 권 제 8 호, pp. 777~783, 2012.

http://dx.doi.org/10.5050/KSNVE.2012.22.8.777

Improved Vibration Vector Intensity Field for FEM and Experimental Vibrating Plate Using Streamlines Visualization

유선 가시화를 이용한 FEM과 실험에 의한 진동판에 대한 개선된 진동 벡터 인텐시티장

Noor Fawazi*, Jae-Eun Jeong* and Jae-Eung Oh†

누룰 파와지·정 재 은·오 재 응

(Received May 30, 2012 ; Accepted June 18, 2012)

Key Words : Experimental Vibration Intensity(실험적 진동 인텐시티), Vector Intensity Flow(벡터 인텐시티 흐름), Streamline(유선), Finite Element Method(유한요소법)

ABSTRACT

Vibration intensity has been used to identify the location of a vibration source in a vibrating system. By using vectors representation, the source of the power flow and the vibration energy transmission paths can be revealed. However, due to the large surface area of a plate-like structure, clear transmission paths cannot be achieved using the vectors representation. Experimentally, for a large surface object, the number of measured points will also be increased. This requires a lot of time for measurement. In this study, streamlines representation is used to clearly indicate the power flow transmission paths at all surface plate for FEM and experiment. To clearly improve the vibration intensity transmission paths, streamlines representation from experimental works and FEM computations are compared. Improved transmission paths visualization for both FEM and experiment are shown in comparison to conventional vectors representation. These streamlines visualization is useful to clearly identify vibration source and detail energy transmission paths especially for large surface plate-like structures. Not only that, this visualization does not need many measured point either for experiment or FEM analysis.

요 약

진동 인텐시티는 진동시스템에서 진동원의 위치를 찾는데 사용되어 왔다. 벡터 표현법 사용에 의해 파워흐름의 원인과 진동에너지 전달경로가 밝혀질 수 있다. 그러나, 판과 같은 구조물의 넓은 면적으로 인해 벡터 가시화를 사용하여 명확한 전달경로를 알아낼 수 없었다. 실험적으로 큰 면적의 물체에서는 측정점의 수가 늘어나게 된다. 이것은 측정에 많은 시간이 요구된다. 이번 연구에서는 FEM과 실험에 의한 모든 면에서 파워흐름 전달경로를 분명하게 가리키기 위해

---

† Corresponding Author ; Member, School of Mechanical Engineering, Hanyang Uni.
E-mail : jeoh@hanyang.ac.kr
Tel : (02)2294-8294, Fax : (02)2299-3153

* Member, Graduate School of Mechanical Eng., Hanyang Uni.

유선 표현법이 사용되었다. 진동 인텐시티 전달경로를 분명하게 향상시키기 위해 실험과 FEM으로부터의 유선 표현을 비교하였다. 또한 FEM과 실험의 개선된 전달경로 가시화를 기존의 벡터 표현과 비교하였다. 이 유선 가시화는 큰 표면을 갖는 판과 같은 구조물에 대해 진동원과 상세한 에너지 전달경로를 확인하는데 유용하다. 그 뿐만 아니라, 이 가시화 방법은 실험과 FEM 해석에 대해 많은 측정점을 필요로 하지 않는다.

Appendix

B : Flexural(bending) stiffness
C_{12} : Cross spectrum of two signals
E : Elastic modulus
η : Displacement
$\ddot{\eta}$: Acceleration
h : Plate thickness
I_x, I_y, I_z : Vibration intensity component
l : Separated distance between 2 accelerometers
M_x, M_y, M_{xy} : Complex bending and twisting moment
m : Mass density
N_x, N_{xy}: Complex membrane forces
Q_x, Q_y : Complex transverse shear forces
μ : Poisson's ratio

1. Introduction

A great attention has to be paid for to the effects of dynamic loading on engineering structures. The vibration energy traveling along a structure is usually radiated as noise. In order to control vibration and structure-borne problems, detail understanding of transmitted vibration power flow is required. The vibration intensity technique is a convenient way to describe this phenomenon.

Vibration intensity is a useful quantity for identifying vibration sources and power flow propagation paths. Vibration intensity shows the magnitude and the direction of vibration energy flow. To identify the changing of vibration power flow, it is important to clearly understand the vibration propagation paths. The vectors of the vibration intensity indicate the energy flow by elastic vibrations in magnitude and direction. Besides identifying the location of source, vibration intensity using vectors flow has been used for structural diagnostic such as crack and mounted stiffness identification. The changes of the vibration intensity vectors flow help us to identify any regions of vibrating surface that are cracked or mounted with stiffeners.

Vibration intensity can be defined as the vibrational power flow per unit cross-sectional area of a dynamically loaded elastic body(1,2). This is analogous to acoustic intensity in a fluid medium. Vibration intensity computation has become a great interest for practical reasons. This computation indicates the magnitude and direction of vibrational energy flow at any point of structure and energy transmission paths. Not only that, the position of vibration source of mechanical energy can also be revealed.

However, due to the large surface area, clear transmission path cannot be achieved using vibration intensity vectors flow. Due to small magnitude of vectors, clear transmission path cannot be achieved. It is therefore, accurate transmission paths cannot be obtained.

In this study, the idea of field lines is used to overcome such problem. Streamlines in a vectors field is drawn so that the direction at any point is the same as the direction of the field at that point. This streamline visualization helps us to understand and obtain clear information on vibrational energy flow especially for complex structure such as stiffened plate which is not yet considered in any previous work. With the changes of streamlines, not only the source can be identified, the areas that are mounted with stiffener can also be identified. Comparisons between FEM computations to experimental works are also shown. Previous research has never highlighted the use of streamlines in vibration intensity especially for experimental work which can reduce the number of measured points.

2. Theoretical Background

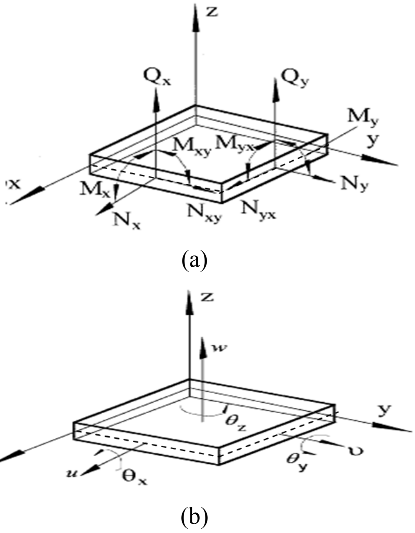
2.1 Vibration Intensity Formulation

The instantaneous vibration intensity component in the time domain can be defined as$^{(1)}$

$$i_k(t) = -\sigma_{kl}(t) v_l(t), \tag{1}$$

where $\sigma_{kl}(t)$ and $v_l(t)$ are the time history of stress and velocity in the l-th direction.

The vibration intensity can be expressed in the form of the net energy flow per-unit width for shells and plates. The energy flow lies in the plane tangential to the midsurface of the structure.



(a)

(b)

Fig. 1 Stress resultants, translational and angular displacements (a) moment and forces (b) displacements

Displacements of any point of a thin-walled structure can be expressed by translational and angular displacements of the midsurface as shown in Fig. 1. The two components of the vibration intensity for a flat thin plate are as follows$^{(1)}$

$$I_x = -\frac{\omega}{2}\mathrm{Im}\left[N_x u^* + N_{xy} v^* + Q_x w^* + M_x \theta_y^* - M_{xy}\theta_x^*\right] \tag{2}$$

$$I_y = -\frac{\omega}{2}\mathrm{Im}\left[N_y u^* + N_{yx} v^* + Q_y w^* + M_y \theta_x^* - M_{yx}\theta_y^*\right] \tag{3}$$

2.2 Vibration Intensity Computation using FEM

By using finite element analysis, all required inputs for vibration intensity computation can be obtained.

In this study, full method for harmonic response solution was used$^{(1,6,7)}$ to compute vibration energy flow in an elastic structure. The computation of vibration intensity was carried out on a fully supported steel plate which is 0.5 m long and 0.4 m wide with a thickness of 1 mm. The material properties used for the plate are as follows: the Young's modulus 200 GPa, the Poisson ratio is 0.3 and the density is 7800 kg/m^3. The plate was modeled using 20 eight-noded isoparametric shell element. A constant damping ratio of 0.005 was used and a point excitation force with a magnitude of 1 N with three different frequencies(46 Hz, 119 Hz and 154 Hz) was applied at the center of the plate. These excitation frequencies are similar to the fundamental frequencies of the vibrating plate.

2.3 Experimental Vibration Intensity

For experiment, two accelerometers can be used to measure vibration power flow. As stated in reference$^{(3)}$, only flexural waves are considered.

As shown in Fig. 2, the intensity the in x-direction passing through a small cross section of a

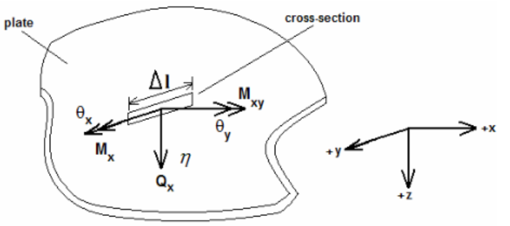


Fig. 2 Force and moments applied on the cross section of plate

plate is transported by the bending M_x and twisting moments M_{xy} and the shear force Q_x. By multiplying these inputs by the corresponding translational velocity and the angular velocity, vibration intensity can be calculated as follows:

$$I_x(x,t) = \frac{Q_x}{\Delta l}\cdot\frac{\partial\eta}{\partial t} + \frac{M_{xy}}{\Delta l}\cdot\dot{\theta}_y + \frac{M_x}{\Delta l}\cdot\dot{\theta}_x \qquad (4)$$

where η represents the displacement of the plate in normal direction(+z) and the

$$\dot{\theta}_x = \frac{\partial^2\eta}{\partial t\cdot\partial x} \qquad (5)$$

$$\dot{\theta}_y = \frac{\partial^2\eta}{\partial t\cdot\partial y} \qquad (6)$$

The force and the moments can be expressed in kinematic quantities as,

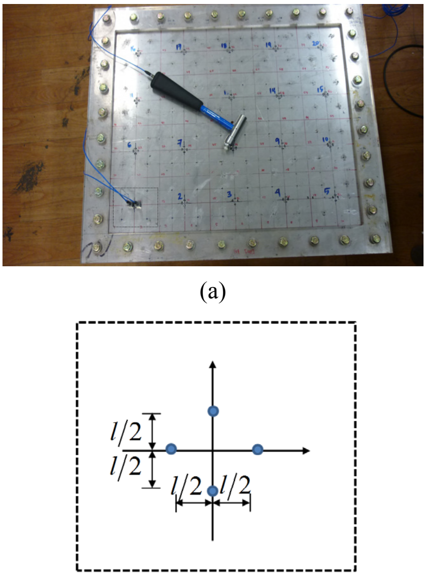
$$Q_x = B\cdot\Delta l\frac{\partial^2}{\partial x}\left(\nabla^2\eta\right) \qquad (7)$$

$$M_x = -B\cdot\Delta l\left(\frac{\partial^2\eta}{\partial x^2} + \mu\frac{\partial^2\eta}{\partial y^2}\right) \qquad (8)$$

$$M_{xy} = -B\cdot\Delta l\left(1-\mu\right)\cdot\frac{\delta^2\eta}{\partial x\partial y} \qquad (9)$$

where μ is the Poisson's ratio of the plate, ∇^2 is the two-dimensional Laplace operator and B is the flexural(bending) stiffness per unit width of the plate and expressed as(4)

$$B = \frac{Eh^3}{12\cdot(1-\mu^2)} \qquad (10)$$



(a)

(b)

Fig. 3 (a) Experimental fully support plate for vibration intensity measurement (b) four closely located points for vibration intensity measurement using two accelerometers

In this equation, E represents the modulus of elasticity and h is the height of the cross-section of the plate shown in Fig. 2. Above equations are in the time-domain formulation for structural intensity measurements on a plate's surface. Even though the time-domain formulation can be directly used in measurements, a frequency domain formulation is obviously needed for quick and easy measurement of vibration intensity. For simplification, detail derivation is not shown in this paper. Frequency domain formulation of vibration intensity can be defined as

$$I_x(f) = \frac{2\sqrt{Bm}}{\Delta l\omega^2}E\left[\mathrm{Im}\,C_{12}\right] \qquad (11)$$

where $E[\ ]$ represents an expected value operation and ImC_{12} is the imaginary value of cross spectrum(3) of acceleration signals as follows(3) :

$$C_{1,2} = 2\cdot E\left[\ddot{\eta}_1(f)\cdot\ddot{\eta}_2(f)\right] \qquad (12)$$

In this study, the experimental model was a fully supported steel plate 0.5 m × 0.4 m and 0.001 m thickness as shown in Fig. 3. To ensure the whole plate was fully clamped, all bolts were uniformly tightened using torque meter. Similar to experimental modal testing, impact hammer(B&K type 4810) was used to apply load at the central plate and two accelerometers(B&K4708) with approximately 10 mm separated distance were used to measure vibration intensity at four closely located points. To ensure the distance between the two accelerometers were constant throughout the experiment, all divided 20 measured points were marked on the plate. Unlike reference$^{(8)}$ magnet or built-in vibration intensity probe were purposely not used to decrease the mass effects of the two accelerometers. The vibration intensity field was measured at 20 different measured points for each I_x and I_y. All measured vibration intensities were then plotted as vector field and streamlines using MATLAB.

2.4 Streamlines

The streamlines representation shows the flow as lines at any location parallel to the velocity field. The relative spacing of the lines indicates the speed of the flow. The vibration intensity streamline can be defined as

$$d\vec{r} \times \vec{I}(\vec{r},t) = 0 \tag{13}$$

Where $\vec{r}$ is the energy flow particle position. For the steady state energy flows the cross product can be written as

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ I_x & I_y & I_z \\ dx & dy & dz \end{vmatrix} = 0 \tag{14}$$

Thus, for 2-dimensional plate structures, the differential equation describing a streamline is

$$\frac{dx}{I_x} = \frac{dy}{I_y} \tag{15}$$

3. FEM Computation and Experimental Results

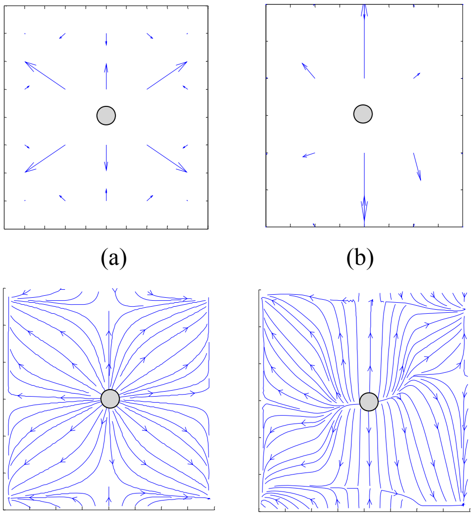
FEM computation and experimental results are shown in Fig. 4 to Fig. 6 at each respected resonance frequency. For each figure, computation and experimental vibration intensity visualizations using vectors and streamlines are compared.

Both finite element computation and experimental vector flow reveal the region where the source is applied It can be noticed in all results(see Figs. 4~6), the vibration energy emerges at the central plate where the force is applied. Clear transmission path cannot be obtained from the vectors visualization since the small number of measured points. To solve this problem, streamlines representations are shown. The streamlines visualization clearly describes the transmission path of vibration energy at all surface. This visualization is better than the vectors representation which not only indicates the location where the source is emerged, but also the significant pattern of vibration power flow.

From the streamlines visualization, not only the location source is identified, but also the detail vibration power transmission paths can be identified. Comparison between the FEM and experimental results show the streamlines visualization can be a good alternative for experimental vibration intensity. This is because, by using streamlines, tedious measurement and increasing number of measured points can be avoided.

From the FEM computation results, symmetrical vibration power flow pattern is observed. A slight different flow pattern in the experimental work was probably affected by the masses of the accelerometers.

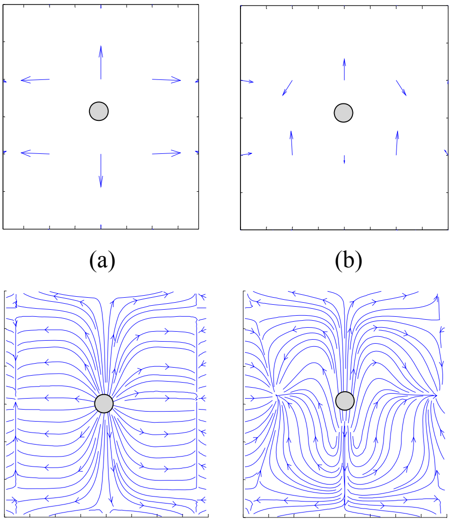
The reflected wave from the boundary condition is also considered as the factor to the slight different between experimental and FEM results.



(a) (b)

(c) (d)

Fig. 4 (a) VI vector flow using FEM-46 Hz (b) experimental VI vector Flow-45 Hz (c) VI streamlines using FEM-46 Hz (d) experimental VI strealines-45 Hz

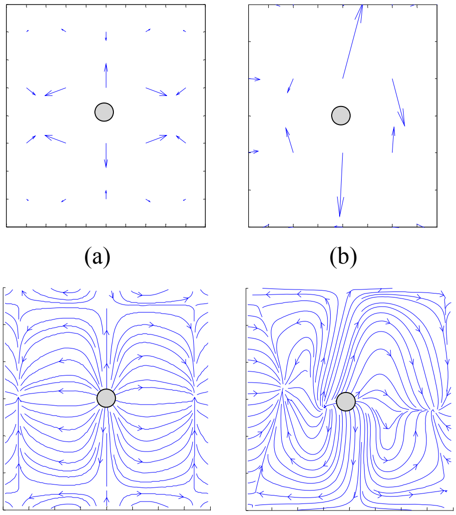


(a) (b)

(c) (d)

Fig. 5 (a) VI vector flow using FEM-119 Hz (b) experimental VI vector Flow-117 Hz (c) VI streamlines using FEM-119 Hz (d) experimental VI strealines-117 Hz

The pattern of vibration power flow using streamlines visualizations show the changing trend from the lower to higher resonance frequencies.



(a) (b)

(c) (d)

Fig. 6 (a) VI vector flow using FEM-154 Hz (b) experimental VI vector flow-149 Hz (c) VI streamlines using FEM-154 Hz (d) experimental VI strealines-149 Hz

The pattern of the vibration power flow is simple at lower frequency. With the increment of resonance frequencies, the pattern becomes more complex. The reflected streamlines can be clearly seen at higher resonance frequencies for both FEM and experimental results.

4. Conclusion

This study improves the visualization of vibration intensity field using streamlines representation. A clear vibration power flow transmission paths can be achieved using streamlines representation. Not only in the computation results, the streamlines representation calculated from the experimental work also shows good result to identify vibration source and transmission paths of vibration energy.

By using streamlines visualizations, the number of measured points for vibration intensity can be reduced. This is important for a large surface plate-like structure that requires many measurement points. Not only decreasing the number of measured points, detail vibration power transmission paths can also be obtained.

By using vectors representation, the direction of the transmission path is ambiguous at many other regions especially for large surface plate-like structure. Streamlines representation can be the best alternative since streamlines visualization provides better representation at all measured points than the vectors visualization in general.

By using streamlines visualization, the changes of the power flow transmission can be clearly observed in comparison to vector representations. Significant changes of vibration power flow transmission pattern can be observed at different respected resonance frequencies.

Although the streamline cannot indicate the relative magnitudes of vibration intensities, this representation provides a better means of visualizing the dominant power flow paths.

References

(1) Li, Y. J., 1999, Prediction of Surface Mobility of a Finite Plate with Uniform Force Excitation by Structural Intensity, Applied Acoustic, 60, pp. 371~383.

(2) Xu, X. D., 2004, The Energy Flow Analysis in Stiffened Plates of Marine Sturctures, Thin-Walled Structures, 42, pp. 979~994.

(3) Pavic, G., 1976, Measurement of Structure Borne Wave Intensity, Part 1 : Formulation of the Methods, Journal of Sound and Vibration, Vol. 49, No. 2, pp. 221~230.

(4) Cremer, L., Heckl, M. and Ungar, E. E., 1973, Structure Borne Sound, Springer-Verlag, Berlin.

(5) Verheij, J. W., 1980, Cross Spectral Density Mehods for Structure Borne Power Flow on Beams and Pipes, Journal of Sound and Vibration, Vol. 70, No. 1, pp. 133~139.

(6) Fawazi, N., Oh, J.-E., et al., 2011, Vibration Intensity Computation of A Single Plate with Uniform Force Excitation using FEM, Proceedings of the KSNVE Annual Autumn Conference, pp. 392~393.

(7) Fawazi, N., Oh, J.-E., et al., 2009, Identification of Exciting Source Location of a Vibrating Plate Using Vibration Intensity, Proceedings of the KSNVE Annual Spring Conference, pp. 222~223.

(8) Cho, J.-J., Oh, J.-E., 1995, Transmission Analysis of Vibration Energy of Plate by Finite Element Analysis, Transactions of the KSAE, Vol. 3, No. 4, pp. 97~104.