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A NOTE ON CONTINUED FRACTIONS WITH SEQUENCES OF PARTIAL QUOTIENTS OVER THE FIELD OF FORMAL POWER SERIES

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Abstract. Let $\mathbb{F}_q$ be a finite field with q elements and $\mathbb{F}_q((X^{-1}))$ be the field of all formal Laurent series with coefficients lying in $\mathbb{F}_q$. This paper concerns with the size of the set of points $x \in \mathbb{F}_q((X^{-1}))$ with their partial quotients $A_n(x)$ both lying in a given subset $\mathbb{B}$ of polynomials in $\mathbb{F}_q[X]$ ($\mathbb{F}_q[X]$ denotes the ring of polynomials with coefficients in $\mathbb{F}_q$) and $\deg A_n(x)$ tends to infinity at least with some given speed. Write $E_{\mathbb{B}} = \{x : A_n(x) \in \mathbb{B}, \deg A_n(x) \to \infty \text{ as } n \to \infty\}$. It was shown in [8] that the Hausdorff dimension of $E_{\mathbb{B}}$ is $\inf\{s : \sum_{b\in\mathbb{B}}(q^{-2\deg b})^s < \infty\}$. In this note, we will show that the above result is sharp. Moreover, we also attempt to give conditions under which the above dimensional formula still valid if we require the given speed of $\deg A_n(x)$ tends to infinity.

1. Introduction

Given a real number $x \in [0, 1)$, let

$$x = \cfrac{1}{a_1(x) + \cfrac{1}{a_2(x) + \cfrac{1}{a_3(x) + \cdots}}} := [a_1(x), a_2(x), a_3(x), \ldots],$$

be its continued fraction expansion, and $a_n(x) \in \mathbb{N}$ are called the partial quotients of x.

The theory of continued fractions has close connections with dynamical system, ergodic theory, probability theory, Diophantine approximation and so on. Some important results in above areas had obtained by means of continued fractions. Among them, Pollington and Velani [15] studied famous Littlewood's conjecture by taking advantage of continued fractions; Kim [10] used continued fractions to discuss shrinking target property of irrational rotations and so on.

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The investigation on the dimension theory of continued fractions with some restrictions on their partial quotients has a long history, which can be traced back to Jarnik [9]. Among them, Good [5] showed that

$$\{x \in [0,1): \ a_n(x) \to \infty \text{ as } n \to \infty\}$$

is of Hausdorff dimension $\frac{1}{2}$ and he also obtained that the general set

$$\{x \in [0,1): \ a_n(x) \geq f(n) \text{ for all } n \geq 1\}$$

enjoys the unchanged dimension one-half, when $f : \mathbb{N} \to \mathbb{N}$ tends to infinity at a moderate growth rate.

For a further investigation, Hirst [6] introduced the set with $a_n(x)$ being further restricted to some given sequence of natural numbers. More precisely, let Λ be an infinite sequence of positive integers $b_1 < b_2 < \cdots$. Set

$$E_\Lambda = \{x \in [0,1): \ a_n(x) \in \Lambda \ (n \geq 1) \text{ and } a_n(x) \to \infty \text{ as } n \to \infty\}.$$

The size of E_Λ was studied by Hirst [6], Cusick [3] and was determined completely by Wang and Wu [17], who showed that ($\dim_H$ denotes the Hausdorff dimension of a set)

$$\dim_H E_\Lambda = \frac{\tau(\Lambda)}{2}, \quad \text{where} \ \ \tau(\Lambda) = \limsup_{n\to\infty} \frac{\log n}{\log b_n}. \tag{1}$$

At the same time, in [6], Hirst also conjectured that the size of the set

$$E_\Lambda(f) = \{x \in [0,1): \ a_n(x) \in \Lambda \text{ and } a_n(x) \geq f(n), \ \forall n \geq 1\} \tag{2}$$

should be independent with f, where $f : \mathbb{N} \to \mathbb{N}$ tends to infinity as $n \to \infty$.

In contrast with Good's result, it should be possible that, when f grows slowly enough, $\dim_H E_\Lambda(f)$ should be unchanged. Nevertheless, Wu [20] recently showed that no matter how slow f grows, there exists some infinite subset $\Lambda \in \mathbb{N}$ such that the formula (1) does not hold any more.

In view that the structure of continued fractions is more regular in the field of formal Laurent series than that in real field, one would like to suspect what will happen in the field of formal Laurent series. In fact, the first author and Wu [8] had obtained an analogous result as (1) by proving that:

Theorem 1.1 ([8]). *Let $\mathbb{B}$ be an infinite subset of polynomials with positive degree in $\mathbb{F}_q[X]$, and set*

$$E(\mathbb{B}) = \{x \in I: \ A_n(x) \in \mathbb{B}, \ \forall\ n \geq 1 \textit{ and } \deg A_n(x) \to \infty \textit{ as } n \to \infty\}.$$

We have

$$\begin{aligned}\dim_{\mathrm{H}} E(\mathbb{B}) = \alpha_{\mathbb{B}} :&= \inf\{s : \sum_{b\in\mathbb{B}} (q^{-2\deg b})^s < \infty\} \\ &= \limsup_{n\to\infty} \frac{\log \sharp\{b \in \mathbb{B} : \deg b = n\}}{2n \log q}.\end{aligned}$$

Similar to the real case, it arises naturally to ask whether the above formula still holds in the field of formal Laurent series, when $\deg A_n(x)$ is assumed with given asymptotic growth orders? More precisely, let $f : \mathbb{N} \to \mathbb{N}$ with $f(n) \to \infty$ as $n \to \infty$ and set

$$E(\mathbb{B}, f) = \{x \in I : \ A_n(x) \in \mathbb{B} \ \text{ and } \ \deg A_n(x) \geq f(n) \text{ for all } n \geq 1\}.$$

Will we still have

$$\dim_H E(\mathbb{B}, f) = \alpha_{\mathbb{B}}? \tag{3}$$

If not, under what conditions it will be the case. In this note, we will answer these two questions by showing that:

Theorem 1.2. *Let $f : \mathbb{N} \to \mathbb{N}$ with $f(n) \to \infty$ as $n \to \infty$. Then there exists $\mathbb{B} \subset \mathbb{F}_q[X]$ such that*

$$\alpha_{\mathbb{B}} = \frac{1}{2}, \ \textit{but} \ \ \dim_{\mathrm{H}} E(\mathbb{B}, f) = 0.$$

Theorem 1.3. *Let $\mathbb{B}$ be an infinite set of polynomials with positive degree in $\mathbb{F}_q[X]$. Assume that the limit*

$$\lim_{n\to\infty} \frac{\log \sharp\{b \in \mathbb{B} : \ \deg b = n\}}{2n \log q} \tag{4}$$

exists (*denote by* $\alpha_{\mathbb{B}}$) *and $f : \mathbb{N} \to \mathbb{N}$ satisfying*

$$\lim_{n\to\infty} f(n) = \infty, \ \limsup_{n\to\infty} \frac{\log f(n)}{n} = 0. \tag{5}$$

Then we have

$$\dim_H E(\mathbb{B}, f) = \alpha_{\mathbb{B}}.$$

Remark 1.4. We remark that the restrictions in Theorem 1.3 is optimal, to some extent, in the sense that:

(I) Theorem 1.2 implies that the formula (3) can't be held in general with no restrictions on asymptotic behaviors on $\sharp\{b \in \mathbb{B} : \deg b = n\}$;

(II) If we take $\mathbb{B} = \mathbb{F}_q[X]$. In [7], we showed that

$$\dim_H\{x \in I : \deg A_n(x) \geq a^n, \ \forall n \geq 1\} = \frac{1}{a+1} \neq \alpha_{\mathbb{B}},$$

which implies that, to make formula (3) still functions in general, some restrictions should also be assumed on the growth order of f.

2. Preliminaries

In this section, we give a briefly review on the continued fraction expansions over the field of formal Laurent series.

Let $\mathbb{F}_q$ be a finite field of q elements and $\mathbb{F}_q\left((X^{-1})\right)$ denote the field of all formal Laurent series with coefficients in $\mathbb{F}_q$, and $\mathbb{F}_q[X]$ denotes the ring of polynomials with coefficients in $\mathbb{F}_q$, i.e.,

$$\mathbb{F}_q\left((X^{-1})\right) = \Big\{x = \sum_{n=n_0}^{\infty} c_n X^{-n} : n_0 \in \mathbb{Z}, c_n \in \mathbb{F}_q\Big\},$$

$$\mathbb{F}_q[X] = \Big\{[x] = \sum_{n=n_0}^{0} c_n X^{-n} : x = \sum_{n=n_0}^{\infty} c_n X^{-n} \in \mathbb{F}_q\left((X^{-1})\right)\Big\}.$$

For each $x \in \mathbb{F}_q\left((X^{-1})\right)$, the order of x is defined as $v(x) = -\deg x = \inf\{n \in \mathbb{Z} : c_n \neq 0\}$ and with the convention $v(0) = +\infty$. Define a non-Archimedean valuation on $\mathbb{F}_q\left((X^{-1})\right)$ as

$$\|x\| = q^{-v(x)} \text{ for all } x \in \mathbb{F}_q\left((X^{-1})\right).$$

The field $\mathbb{F}_q\left((X^{-1})\right)$ is locally compact and complete under the metric $\rho(x, x') = \|x - x'\|$.

Remark 2.1. Since the valuation $\|\cdot\|$ is non-Archimedean, it follows that if two discs intersect, then one contains the other.

Let I denote the valuation ideal of $\mathbb{F}_q\left((X^{-1})\right)$, i.e.,

$$I = \Big\{x \in \mathbb{F}_q\left((X^{-1})\right) : \|x\| < 1\Big\} = \Big\{x = \sum_{n=1}^{\infty} c_n X^{-n} : c_n \in \mathbb{F}_q\Big\}.$$

Let P denote the Haar measure on $\mathbb{F}_q\left((X^{-1})\right)$ normalized to 1 on I.

Consider the following transformation from I to I defined by

$$Tx := \frac{1}{x} - \left[\frac{1}{x}\right], \quad T0 := 0.$$

This map describes the regular continued fraction over the field of Laurent series and has been introduced by Artin [1]. As in the classical theory, every $x \in I$ has the following continued fraction expansion:

$$x = \cfrac{1}{A_1(x) + \cfrac{1}{A_2(x) + \cfrac{1}{A_3(x) + \cdots}}} := [A_1(x), A_2(x), A_3(x), \ldots],$$

where the digits $A_i(x)$ are polynomials of strictly positive degree and are defined by $A_n(x) = \left[\frac{1}{T^{n-1}(x)}\right]$, when $T^{n-1}(x) \neq 0$ for all $n \geq 1$.

The metric properties and the Diophantine approximation of elements of $\mathbb{F}_q\left((X^{-1})\right)$ have been studied extensively (see, e.g. the survey papers by Schmidt [16], Berthé and Nakada [2], Lasjaunias [12]). For results on the dimension theory over the field of formal Laurent series, one refers to Niederreiter and Vielhaber [13, 14], Kristensen [11], Wu [7, 18, 19].

At the end of this section, we recall the following result, which will be used frequently. For details, we refer to Niederreiter [13] or Berthe and Nakada [2].

Lemma 2.2 ([2, 13]). *For any $A_1, A_2, \dots, A_n \in F_q[X]$ with strictly positive degree, call*

$$I(A_1, \dots, A_n) = \{x \in I : A_1(x) = A_1, \dots, A_n(x) = A_n\}$$

an nth order fundamental cylinder. Each nth order fundamental cylinder $I(A_1, A_2, \dots, A_n)$ is a disc with diameter

$$|I(A_1, A_2, \dots, A_n)| = q^{-2\sum_{k=1}^{n} \deg A_k - 1}$$

and

$$\mathbf{P}(I(A_1, A_2, \dots, A_n)) = q^{-2\sum_{k=1}^{n} \deg A_k},$$

where $\mathbf{P}$ is the Haar measure on I.

3. Proof of results

Proof of Theorem 1.2. Let f be a given function defined on $\mathbb{N}$ such that $f(n) \to \infty$ as $n \to \infty$. We will construct a set $\mathbb{B} \subset \mathbb{F}_q[X]$ such that $\alpha_{\mathbb{B}} = \frac{1}{2}$, but $\dim_{\mathrm{H}} E(\mathbb{B}, f) = 0$.

At first, choose an integer sequence $\{n_k, k \geq 1\} \subset \mathbb{N}$ by induction. Let $n_1 = \min\{n \in \mathbb{N} : f(n) > 2\}, n_2 = n_1 + 1$, and then choose $\{n_k, k \geq 3\}$ satisfying

$$(k-1)^{n_{k-1}} + 1 < f(n_k) \text{ for all } k \geq 3.$$

Let $\mathbb{B}$ be all polynomials with positive degree and coefficients lying in $\mathbb{F}_q$ whose degree equal to $2^{n_2}, 3^{n_3}, \dots, (k-1)^{n_{k-1}}, \dots$, that is

$$\mathbb{B} = \{b \in \mathbb{F}_q[X] : \deg b = 2^{n_2}, 3^{n_3}, \dots, (k-1)^{n_{k-1}}, k^{n_k}, \dots\}.$$

By the results of Theorem 1.1, we have

$$\alpha_{\mathbb{B}} = \limsup_{n\to\infty} \frac{\log \sharp\{b \in \mathbb{B} : \ \deg b = n\}}{2n \log q} \geq \limsup_{k\to\infty} \frac{\log(q-1)q^{k^{n_k}}}{2 \log q^{k^{n_k}}} = \frac{1}{2}.$$

On the other hand, we note that $\sharp\{b \in \mathbb{B} : \ \deg b = n\} \leq (q-1)q^n$ for all $n \geq 1$, this leads to $\alpha_{\mathbb{B}} \leq \frac{1}{2}$.

Next, we will show that the Hausdorff dimension of $E(\mathbb{B}, f)$ is zero for the above $\mathbb{B}$. For any $x \in E(\mathbb{B}, f)$, we have $A_n(x) \in \mathbb{B}$ and $\deg A_n(x) \geq f(n), \forall\, n \geq 1$. By the construction of the set $\mathbb{B}$ and the sequence n_k, for any $k \geq 1$, we get

$$\deg A_{n_k}(x) \geq f(n_k) > (k-1)^{n_{k-1}} + 1 \Longrightarrow \deg A_{n_k}(x) \geq k^{n_k}.$$

Therefore, for any $M > 1$,

$$E(\mathbb{B}, f) \subset \{x \in I : \deg A_n(x) \geq M^n, \text{ i.o. } n\}.$$

By [7, Theorem 2.2], $\dim_H E(\mathbb{B}, f) \leq \frac{1}{M+1}$. Since $M > 1$ is arbitrary, we have

$$\dim_H E(\mathbb{B}, f) = 0.$$

This completes the proof. □

Next we will apply the following lemma, see [4, Proposition 2.3], to give a lower bound of Hausdorff dimension of the set in question.

Lemma 3.1. *Suppose $E \subset I$, and μ is a finite measure with $\mu(E) > 0$. If*

$$\underline{\dim}_{loc}\mu(x) = \liminf_{r\to 0} \frac{\log \mu(B(x,r))}{\log r} \geq s \tag{6}$$

for all $x \in E$, then

$$\dim_H E \geq s.$$

Proof of Theorem 1.3. For the upper bound, we note that

$$\begin{aligned} E(\mathbb{B}, f) &= \{x \in I : \ A_n(x) \in \mathbb{B} \ \text{ and } \deg A_n(x) \geq f(n) \text{ for all } n \geq 1\} \\ &\subset E(\mathbb{B}) = \{x : \ A_n(x) \in \mathbb{B} \text{ for all } n \geq 1 \text{ and } \deg A_n(x) \to \infty\}, \end{aligned}$$

and by Theorem 1.1, we get $\dim_H E(\mathbb{B}, f) \leq \alpha_{\mathbb{B}}$ immediately.

And for the lower bound, we first note that $\alpha_{\mathbb{B}} > 0$, and then the assumptions (4) and (5) imply that for any $\epsilon > 0$, there exists some $N(\epsilon) \in \mathbb{N}$ such that when $n \geq N(\epsilon)$, we have

$$f(n) \leq (1+\epsilon)^n \text{ and } \sharp\{b \in \mathbb{B} : \deg b = n\} \geq q^{2n(\alpha_{\mathbb{B}}-\epsilon)}. \tag{7}$$

Write $\mathbb{B}_n = \{b \in \mathbb{B} : \deg b = n\}$. Next we define the sets $\mathcal{L}_n$:

For $1 \leq n < N(\epsilon)$, let $\mathcal{L}_n$ be a singleton with its element $b_n \in \mathbb{B}$ and $\deg b_n \geq f(n)$;

For $n \geq N(\epsilon)$, in the light of (7), we can choose $\mathcal{L}_n$ a subset of $\mathbb{B}_n$ with

$$\sharp\mathcal{L}_n = \left\lfloor q^{2n(\alpha_{\mathbb{B}}-\epsilon)} \right\rfloor + 1. \tag{8}$$

Set

$$F(\mathbb{B}, \epsilon) := \Big\{x \in I : A_n(x) \in \mathcal{L}_{\lfloor (1+\epsilon)^n \rfloor + 1} \text{ for all } n \geq 1\Big\}.$$

It is evident that $F(\mathbb{B}, \epsilon) \subset E(\mathbb{B}, f)$ by the fact shown by (7).

A symbolic space will be introduced to make the argument briefly. For any $n \geq 1$, write

$$D_n = \{(\sigma_1, \ldots, \sigma_n) : \sigma_k \in \mathcal{L}_{\lfloor (1+\epsilon)^k \rfloor + 1}, 1 \leq k \leq n\}.$$

For any $(\sigma_1, \ldots, \sigma_n) \in D_n$, call

$$I(\sigma_1, \ldots, \sigma_n) := \{x \in I : A_k(x) = \sigma_k, 1 \leq k \leq n\}$$

an admissible cylinder of order n (with respect to $F(\mathbb{B}, \epsilon)$). Then it gives

$$F(\mathbb{B}, \epsilon) = \bigcap_{n=1}^{\infty} \bigcup_{(\sigma_1, \ldots, \sigma_n) \in D_n} I(\sigma_1, \ldots, \sigma_n).$$

Now a set function μ will be constructed on admissible cylinders by: $\mu(I) = 1$ and for any $n \geq 1$ and $(\sigma_1, \ldots, \sigma_n) \in D_n$

$$\mu(I(\sigma_1, \ldots, \sigma_n)) = \prod_{j=1}^{n} \frac{1}{\sharp\mathcal{L}_{\lfloor (1+\epsilon)^j \rfloor + 1}}. \tag{9}$$

It is easy to check that the measure μ is well defined. Then by Kolmogorov extension theorem, the set function μ can be extended into a probability measure supported on $F(\mathbb{B}, \epsilon)$.

To apply the mass distribution principle (Lemma 3.1) to obtain a lower bound on $\dim_H F(\mathbb{B}, \epsilon)$, we are in the position to estimate the μ-measure of arbitrary ball $B(x, r)$ with $x \in F(\mathbb{B}, \epsilon)$.

For any $x \in F(\mathbb{B}, \epsilon)$, there exists a sequence $\{\sigma_n\}_{n=1}^{\infty}$ such that $(\sigma_1, \ldots, \sigma_n) \in D_n$ and $x \in I(\sigma_1, \ldots, \sigma_n)$ for all $n \geq 1$. For any $r > 0$ small enough, there exists $n \in \mathbb{N}$ such that

$$|I(\sigma_1, \ldots, \sigma_n, \sigma_{n+1})| < r \leq |I(\sigma_1, \ldots, \sigma_n)|.$$

Then by Remark 2.1, it follows

$$B(x, r) \subset I(\sigma_1, \ldots, \sigma_n).$$

Denote $T(\epsilon) = \min\{n : \lfloor (1+\epsilon)^n \rfloor + 1 \geq N(\epsilon)\}$, it is easy to see that $T(\epsilon) < \infty$ by the boundedness of $N(\epsilon)$. Thus by the construction of both the measure μ (see (9)) and the sets $\mathcal{L}_n$ (see (8)), the measure μ on the ball $B(x, r)$ can be estimated by

$$\mu(B(x, r)) \leq \mu(I(\sigma_1, \ldots, \sigma_n)) \leq q^{-2(\alpha_{\mathbb{B}} - \epsilon) \sum_{k=T(\epsilon)}^{n} (\lfloor (1+\epsilon)^k \rfloor + 1)}.$$

On the other hand, with the result of Lemma 2.2, the diameter of admissible cylinder $I(\sigma_1, \ldots, \sigma_n, \sigma_{n+1})$ is

$$|I(\sigma_1, \ldots, \sigma_n, \sigma_{n+1})| = q^{-2(\sum_{k=1}^{T(\epsilon)-1} \deg \sigma_k + \sum_{k=T(\epsilon)}^{n+1} (\lfloor (1+\epsilon)^k \rfloor + 1)) - 1}.$$

As a result, we have

$$\begin{aligned}
&\liminf_{r \to 0} \frac{\log \mu(B(x, r))}{\log r} \\
\geq\ &\liminf_{n \to \infty} \frac{\log \mu(I(\sigma_1, \ldots, \sigma_n))}{\log |I(\sigma_1, \ldots, \sigma_n, \sigma_{n+1})|} \\
\geq\ &\liminf_{n \to \infty} \frac{2(\alpha_{\mathbb{B}} - \epsilon) \sum_{k=T(\epsilon)}^{n} (\lfloor (1+\epsilon)^k \rfloor + 1)}{2(\sum_{k=1}^{T(\epsilon)-1} \deg \sigma_k + \sum_{k=T(\epsilon)}^{n+1} (\lfloor (1+\epsilon)^k \rfloor + 1)) + 1} \\
=\ &\frac{\alpha_{\mathbb{B}} - \epsilon}{(1+\epsilon)}.
\end{aligned}$$

Thus

$$\dim_H E(\mathbb{B}, f) \geq \dim_H F(\mathbb{B}, \epsilon) \geq \frac{\alpha_{\mathbb{B}} - \epsilon}{(1+\epsilon)}.$$

Letting $\epsilon \to 0$, we obtain the desired result. $\square$

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