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Kinematic Displacement Theory of Planar Structures

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Abstract

This paper presents a new curvature based kinematic displacement theory and a numerical method to calculate the planar displacement of structures from a geometrical viewpoint. The theory provides an opportunity to satisfy the kinematic equilibrium of a planar structure using a progressive numerical approach, in which the cross sections are assumed to remain plane, and the deflection curve was evaluated geometrically using the curvature values despite being solved using differential equations. The deflection curve is parameterized with the arc-length, and was taken as an assembly of the chains of circular arcs. Fast and accurate solutions of most complex deflections can be obtained with few inputs.

Keywords: Curvature, Deflection curve, Progressive collapse

1. Introduction

The equilibrium of internal and external forces can be satisfied only if the axial/in-plane stresses are defined precisely. Therefore, correct deflection modeling is essential. In addition, second-order theory must be considered with accurate deflection modeling for reliable results in post-buckling or in post-collapse. There are several methods in literature to define the deflection. Even if it is a numerical method, these methods employ models based on differential equations with assumptions. Even the smallest deviations from these assumptions can be dominant with respect to equilibrium when large rotations and large deflections are considered.

In this study, the curvature was used directly in deflection geometry. The deflection curve was obtained by arc length parameterization. The deflection curve of the structure can be modeled easily and determined using the curvature values, even if material or geometrical nonlinearities occur [1]. Furthermore, progressive collapse analysis can be obtained using a single numerical progressive procedure by curvature values or curvature function, regardless of the structure being in the elastic, plastic, or etc. Therefore, the relationship between the curvature and the deflection curve is kinematic.

Curvature has physical and geometrical meaning. The proposed theory prescribes how to parameterize the deflection curve with curvature values. This curve is represented by a sequence of arcs within a user-specified tolerance, and the relationship between arcs is established using proposed theory. Therefore, the most complex curves can be modeled without the assistance of polynomials or trigonometric series. The proposed theory allows the deflection to be removed geometrically without differential equations, thereby allowing fast and reliable deflection calculations. Equilibrium with external and internal forces is always available, even in post-collapse, until fracture occurs.

According to this assumption, even the stress distribution of the section is nonlinear; the strain distribution should be linear and proportional to the curvature. The range of this study is limited to the planar motion of structures for simplicity. The shear effect and torsion are not considered in the proposed theory. One–dimensional (1-D) structures, such as rods, beam columns, and stiffened plates can be included in this

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category.

A search of the publications recorded by the ‘‘ISI-Web of Science,” revealed, no paper on how to obtain the theory of deflection curves with the geometrical use of curvature values where the deflection curve is taken as an assembly of chains of circular arcs.

The proposed theory was adapted to a second-order theory-based numerical method for a one-dimensional (1-D) structure [1]. The results of the proposed method were used in Smith’s method to perform progressive collapse analysis of a Very Large Crude Carrier “Energy Consideration” [1].

The main motivation of this approach was to reduce the computational and modeling times, and obtain more accurate solutions. The theory and concept of the proposed method are presented in the next two sections with a basic mathematical definition of a 1-D structure. The method is presented in Section 4 with a simple application in Section 5. Section 6 is devoted to a discussion and conclusion.

2. Structure

All structures are physically three dimensional, but can be reduced to 2-D or 1-D representations with certain assumptions. The beams, beam-columns, rods, and stiffened plates can be classified as 1-D structures. This study assumed that there is a plane in which the forces act, called the “plane of loads.” In addition, it was assumed that the plane of loads passed through a point (the shear center) in the cross section such that there is no twisting (torsion) of the structure. This means that the resulting forces that act on any cross section of the structure consist of only bending moments, normal forces, and shear forces [2]. The cross section of the structure with respect to the y-z plane was assumed to be unchanging. Therefore, the compatibility effects are neglected.

First, a 1-D structure cross section *A* is defined as a region bounded by a general closed major principal plane curve, and *C* is the centroid of cross section, as shown in Fig. 1 [3]. Rectangular Cartesian coordinates and curvilinear coordinate *s* are used. *s* is the arc length measured along a cross section from one end of the structure to *C*. x, y, and z are rectangular Cartesian coordinates in the cross section through *C*, with the origin at *C*. The axes are the directions of the normal vectors of the principal planes. $\boldsymbol{\alpha}$ is a general regular skew curve called a “deflection curve.” The structure is generated by a cross section when *C* moves along α with the plane of the cross section normal to $\boldsymbol{\alpha}$, as shown in Fig. 1. When a model of the structure was defined in this manner, it has a uniform or nonuniform normal cross section and a regular curve of centroids $\boldsymbol{\alpha}$. Discontinuity between the cross sections can be neglected or smoothed by the appropriate method to obtain a regular curve.

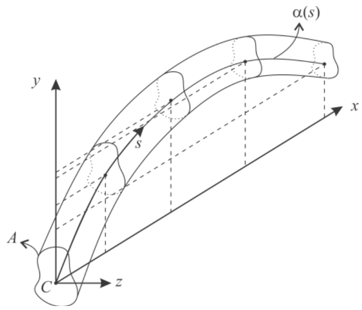

Fig. 1. Deflection curve of a 1-D structure.

Let I be an interval and a continuous function α: $I \to \mathbb{R}^3$ is an arc length parameterized curve defined by a position vector $\boldsymbol{\alpha}(s)=x(s)\boldsymbol{i}+y(s)\boldsymbol{j}+0\boldsymbol{k}$. Each component is differentiable and $s \in I$. A curve is regular if the vector $\boldsymbol{\alpha}'(s) \neq 0$; that is, $\|\boldsymbol{\alpha}'(s)\| \neq 0$ [4]. At each point *C* on **α** there is an orthogonal triad associated with the coordinate axes.

3. Kinematic Relation

A rotation vector, κ, specifying the rotation of this triad as *C* moves along **α** was used to describe the configuration of $\boldsymbol{\alpha}$. $\kappa\boldsymbol{N}$ is then the rotation per unit length about the direction of the principal unit normal vector $\boldsymbol{N}$. κ is referred to as the curvature on the osculating plane on a normal plane of the model [3].

The function $\boldsymbol{\alpha}$ is differentiable at a point *s* if both the right and left derivatives exist and are equal at that point. Hence, the following expression can be obtained:

$$\boldsymbol{\alpha}'(s) = \boldsymbol{\alpha}'(s^-) = \boldsymbol{\alpha}'(s^+) \tag{1}$$

The first derivative of the deflection curve is a tangent vector and is composed of a magnitude and direction vector given by Eq. (2) [5]. The direction vector is called the unit tangent vector $\boldsymbol{T}$.

$$\boldsymbol{\alpha}'(s) = \|\boldsymbol{\alpha}'(s)\|\, \boldsymbol{T}(s) \tag{2}$$

Using the relations given in Eqs. (1) and (2), the unit tangent vector can be denoted as follows: This means that every point on the curve has only a single unit tangent vector [6]. Thus,

$$\boldsymbol{T}(s) = \boldsymbol{T}(s^-) = \boldsymbol{T}(s^+)$$

$$\boldsymbol{T}(s) = \boldsymbol{\alpha}'(s)/\|\boldsymbol{\alpha}'(s)\|$$

The derivative of the unit tangent vector is also a vector with a magnitude and direction called a principal unit normal vector ***N***, as given by Eq. (3) [5]. A principal normal unit vector is perpendicular to a unit tangent vector.

$$\boldsymbol{T}'(s) = \|\boldsymbol{T}'(s)\|\, \boldsymbol{N}(s) \tag{3}$$

Eq. (3) can be expressed simply as Eq. (4), in which *κ(s)* is called curvature.

$$\boldsymbol{T}'(s) = \kappa(s)\boldsymbol{N}(s \tag{4}$$

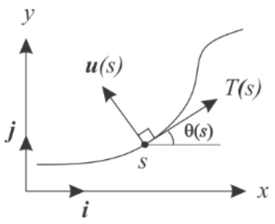

Fig. 2. Unit tangent vector of a deflection curve at a point.

The unit tangent vector can be written in the form of an angle between the x-axis, as given in Eq. (5), from scalar multiplication with unit vectors, where ***i***, and ***j*** are the unit vectors of the x and y-axis, respectively, as shown in Fig. (2). The normal of the ***T*** is differentiable and is called the unit normal vector ***u*** by rotating the tangent vector through a right angle counterclockwise (Fig. (2)), and is given by Eqs. (6) and (7).

$$\boldsymbol{T}(s) = cos\big(\theta(s)\big)\,\boldsymbol{i} + sin\big(\theta(s)\big)\,\boldsymbol{j} \tag{5}$$

$$\boldsymbol{u}(s) = cos\left(\theta(s) + \tfrac{\pi}{2}\right)\boldsymbol{i} + sin\left(\theta(s) + \tfrac{\pi}{2}\right)\boldsymbol{j} \tag{6}$$

$$\boldsymbol{u}(\boldsymbol{s}) = \boldsymbol{u}(\boldsymbol{s}^-) = \boldsymbol{u}(s) \tag{7}$$

The derivative of the unit tangent vector is obtained from Eq. (5), and can be expressed as Eq. (8).

$$\begin{aligned}\boldsymbol{T}'(s) &= -\sin\big(\theta(s)\big)\theta'\,\boldsymbol{i} + \cos\big(\theta(s)\big)\theta'(s)\,\boldsymbol{j}\\ &= \theta'(s)\big(-\sin\big(\theta(s)\big)\,\boldsymbol{i} + \cos\big(\theta(s)\big)\,\boldsymbol{j}\big)\\ &= \theta'(s)\left[\cos\left(\theta(s) + \tfrac{\pi}{2}\right)\boldsymbol{i} + \sin\left(\theta(s) + \tfrac{\pi}{2}\right)j\right]\end{aligned}$$

$$\boldsymbol{T}'(s) = \theta'(s)\boldsymbol{u}(s) \tag{8}$$

The norm of the second derivative $\|T'(s)\|$ measures the rate of change of the angle that the neighboring tangents make with the tangent at *s*. By substituting Eq. (8) into Eq. (4), the principal unit normal vector is non-differentiable and can be expressed simply as follows:

$$\begin{aligned}\boldsymbol{N}(s) &= \boldsymbol{T}'(s)/\|\boldsymbol{T}'(s)\| = \theta'(s)/|\theta'(s)|\,\boldsymbol{u}(s)\\ &= \mathrm{Sgn}(d\theta(s)/ds)\boldsymbol{u}(s)\end{aligned} \tag{9}$$

From Eqs. (4), (6), and (9), the curvature can be written in the form of Eq. (10) as an equation for the rate of change of the unit tangent with respect to the arc length [7] :

$$\kappa(s) = \|\boldsymbol{T}'(s)\| = \|\theta'(s)\| = d\theta(s)/ds \tag{10}$$

Here, for increasing and decreasing tangent values, the curvature will have a positive and negative sign, respectively. The osculating circle at ***α(s)*** is a circle with a radius *r(s)* = |*1/κ(s)*| whose center lies along a line collinear with the principal unit normal vector at ***α(s)***, a distance *r(s)* from ***α(s)***[7]. Normally, the unit normal vector shows the direction at a right angle to the unit tangent vector, but a principle unit normal vector shows the direction of the center of curvature as expressed in the following equation:

$$\boldsymbol{N}(s) = \mathrm{Sgn}\big(\kappa(s)\big)\boldsymbol{u}(s) \tag{11}$$

Hence, the vector starts from the curve to the corresponding center of the osculating circle Fig. (3). The ***R(s)*** vector can be given by Eq. (12) in terms of the principal unit normal vector given in Eq. (11):

$$\boldsymbol{R}(s) = \boldsymbol{N}(s)/|\kappa(s)| \tag{12}$$

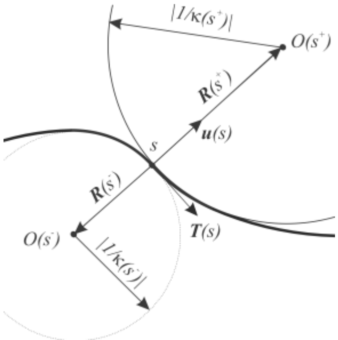

Fig. 3. Osculating circles of a point.

From Eqs. (7), and (11), the left and right hand side values of the $\boldsymbol{R}$(s) vector can be expressed in the following equation Fig. (3):

$$\boldsymbol{R}(s^-) = \boldsymbol{u}(s)/\kappa(s^-) \tag{13}$$

$$\boldsymbol{R}(s^+) = \boldsymbol{u}(s)/\kappa(s^+) \tag{14}$$

The osculating circles have the same unit normal vector at a common point of the deflection curve, which means a line connecting the center of the adjacent osculating circles always passes through the common point, as shown in Fig. (3). Therefore, it is possible to calculate the center position of the adjacent osculating circle. This theory provides an opportunity to form the most complex deflection shapes numerically with little input, without of the need to solve the differential equations or polynomials of a trigonometric series, as shown in Fig. (4).

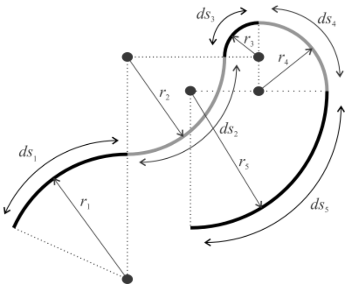

Fig. 4. Deflection curve modeling with five segments with different curvatures and lengths.

4. Displacement Determinations

Three main geometric assumptions are using to calculate the displacements of a deflection curve: linear, nonlinear and large deflections. Consider a sufficiently small segment between points 1 and 2 on the deflection curve: the length between these points is ds, the radius of curvature on the segment is equal to r and the chord length between these points is dc. The tangent angles at points 1 and 2 are θ_1 and θ_2, respectively. The central angle of the arc is the difference between the tangent angles $d\theta$ (Fig. (5)).

In linear displacement calculations, the kinematic assumptions are [7]:

$$ds \approx dx$$

$$\theta_1 \approx \tan\theta_1 = (dy + \xi)/dx \approx dy/dx.$$

In nonlinear displacement calculations, the kinematic assumptions are [8]:

$$ds \approx dc = \sqrt{dx^2 + dy^2}$$

$$\theta_1 = \arctan[(dy + \xi)/dx] \approx \arctan dy/dx$$

In large deflection displacement calculations, the assumptions are [8]:

$$\sin\theta_1 = (dy + \xi)/\sqrt{(dx^2 + dy^2 + \xi^2)} \approx dy/ds$$

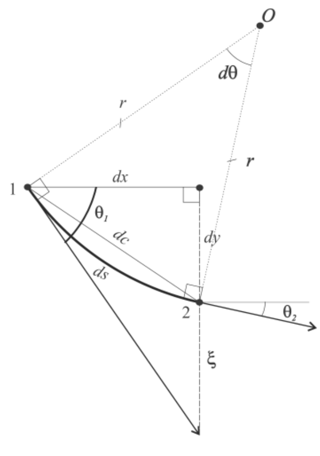

Fig. 5. Displacement of an osculating circle.

The conventional way to determine the displacements is to substitute the assumptions into Eq. (10) and solve analytically or numerically the differential equation obtained. As shown in Fig. (5), the value of ξ is neglected and the slope angle of the chord is taken as the tangent angle of point A in the conventional methods. There is no problem if ξ is relatively smaller. On the other hand, considering large deflections and large rotations, it can cause significant errors in analytical methods or increase the required element number and solution time in numerical methods.

A progressive numerical method can be developed from the proposed theory. Suppose that the internal forces acting on an initial deflection curve are known, it is possible to divide the deflection curve into segments, and calculate the curvature values of each segment according to the corresponding internal loads. Each segment might have a different length and different curvature value. The inputs for the kinematic relation are the length of the segments and curvature values. A more geometric description is that the curve is composed of circle segments or arcs whose radii and lengths are known. The end point of an arc is common with the starting point of the adjacent arc.

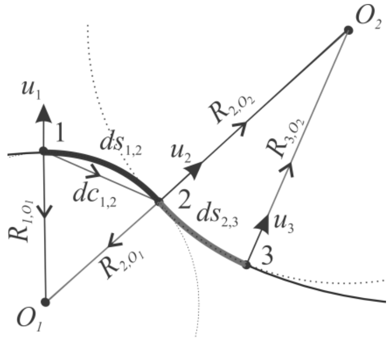

Fig. 6. Radius vectors of two adjacent osculating circles.

The curvature $\kappa_{1,2}$ on $ds_{1,2}$ length segment $S_{1,2}$ between point 1 and 2 is assumed to be constant. Therefore, these points are on the same osculating circle. The osculating circle on $S_{1,2}$ has two different $\boldsymbol{R}$ vectors $\boldsymbol{R}_{1,O1}$ and $\boldsymbol{R}_{2,O1}$, at points 1 and 2, respectively, with same magnitudes and different directions through the center of the osculating circle O_1 Fig. (6). The displacement vector $\boldsymbol{dc}_{1,2}$ from point 1 to 2 can be defined from the vectorial summation of $\boldsymbol{R}$ vectors as follows by Eqs. (13) and (14) Fig. (6):

$$\boldsymbol{dc}_{1,2} = \boldsymbol{R}_{1,O_1} - \boldsymbol{R}_{2,O_1} = \left(1/\kappa_{1,2}\right)\left[\,\boldsymbol{u}_1 - \boldsymbol{u}_2\right] \qquad (15)$$

Consider point 3, which is a $ds_{2,3}$ length away from point 2 Fig. (6). As can be seen, there are one unit tangent vector and two different $\boldsymbol{R}$ vectors on point 2: one is through the curvature center of the $S_{1,2}$ segment and the other is through the curvature center of $S_{2,3}$. The displacement vector between points 2 and 3 can be expressed by Eqs. (13) and (14) Fig (6) as follows:

$$\boldsymbol{dc}_{2,3} = \boldsymbol{R}_{2,O_2} - \boldsymbol{R}_{3,O_2} = \left(1/\kappa_{2,3}\right)\left[\,\boldsymbol{u}_2 - \boldsymbol{u}_3\right] \qquad (16)$$

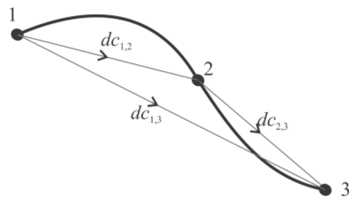

Fig. 7. Displacement vectors of two adjacent segment.

The total displacement between points 1 and 3 can be expressed by the vectorial summation of the two displacement vectors in Eqs. (15) and (16) as follows (Fig. (7)):

$$\boldsymbol{dc}_{1,3} = \boldsymbol{dc}_{1,2} + \boldsymbol{dc}_{2,3}$$

Therefore, the general displacement vector between the starting point and any point of the deflection curve can be obtained by a simple vectorial summation of all previous segments displacement vector:

$$\boldsymbol{dc}_{1,n} = \boldsymbol{dc}_{1,2} + \boldsymbol{dc}_{2,3} + \boldsymbol{dc}_{3,4} + \cdots + \boldsymbol{dc}_{n-1,n}$$

$$\boldsymbol{dc}_{1,n} = \boldsymbol{\alpha}_n = \textstyle\sum_{m=1}^{n}\left(1/\kappa_{m,m+1}\right)\left[\,\boldsymbol{u}_m - \boldsymbol{u}_{m+1}\right] \quad (17)$$

Eq. (17) can be expressed in terms of the curvature and length. The unit normal vector is expressed in terms of the tangent angles, as in Eq. (6). Therefore, the tangent angle needs to be expressed by the curvature values. The curvature value is constant between points 1 and 2. By integrating Eq. (10), the tangent angle at point 2 can be defined as follows:

$$\int_1^2 \mathrm{d}\theta = k_{1,2}\int_1^2 \mathrm{d}s = d\theta_1 = \theta_2 - \theta_1 = k_{1,2}ds_{1,2}$$

$$\theta_2 = k_{1,2}ds_{1,2} + \theta_1$$

Using the same procedure the adjacent tangent angle can be obtained as follows:

$$\int_2^3 \mathrm{d}\theta = k_{2,3}\int_2^3 \mathrm{d}s = \theta_3 - \theta_2 = k_{2,3}ds_{2,3}$$

$\theta_3 = k_{2,3}ds_{2,3} + \theta_2$

Therefore, general formulation of the tangent angles at point m can be expressed as follows:

$$\theta_m = \theta_1 + \sum_{a=1}^{m-1} \kappa_{a,a+1} ds_{a,a+1} \tag{18}$$

The unit normal at point m was obtained from Eqs. (6) and (18) as follows:

$$\boldsymbol{u}_m = -\sin(\theta_m)\,\boldsymbol{i} + \cos(\theta_m)\,\boldsymbol{j}$$

$$\boldsymbol{u}_m = -\sin\left(\theta_1 + \sum_{a=1}^{m-1} \kappa_{a,a+1} ds_{a,a+1}\right)\boldsymbol{i} + cos\left(\theta_1 + \sum_{a=1}^{m-1} \kappa_{a,a+1} ds_{a,a+1}\right)\boldsymbol{j} \tag{19}$$

Finally by substituting Eq. (19) into Eq. (17) the displacement vector can be expressed as follows:

$$\boldsymbol{dc}_{1,n+1} = \boldsymbol{\alpha}_{n+1} = \sum_{m=1}^{n} \frac{1}{\kappa_{m,m+1}} \left[sin\left(\sum_{a=1}^{m} \kappa_{a,a+1} ds_{a,a+1} + \theta_1\right) - sin\left(\sum_{a=1}^{m-1} \kappa_{a,a+1} ds_{a,a+1} + \theta_1\right)\right]\boldsymbol{i}$$
$$\sum_{m=1}^{n} \frac{1}{\kappa_{m,m+1}} \left[cos\left(\sum_{a=1}^{m-1} \kappa_{a,a+1} ds_{a,a+1} + \theta_1\right) - cos\left(\sum_{a=1}^{m} \kappa_{a,a+1} ds_{a,a+1} + \theta_1\right)\right]\boldsymbol{j} \tag{20}$$

The origin point's tangent angle is necessary to start and obtain this progressive solution. The points at the clamped ends or the point where the moment value is at a maximum has zeroed a tangent angle. Therefore, these points are suitable places to begin the procedure. This method makes only a kinematic relation, the curvature values need to be obtained from the moment values of the structure using the equilibrium equations and material model of the structure. The length of the segments can be defined by the user tolerance, and curvature values can be determined from the equilibrium equations depending on the internal load and material properties.

5. Application

A small application can be used to determine how simple the method is. Suppose an elastic tapered cantilever beam under uniform moment, *M,* and total length of the beam is *L*. Material of the beam is elastic and the flexural rigidity is *EI* where *E* is the elastic modulus and *I* is the moment of inertia. The width of the rectangular beam cross-section is *B* and height of the beam cross-section is *H* in the clamped edge and H_{min} in the free end (Fig. (8)). The material of the beam is assumed to be elastic. The dimension and properties are *L*=800 mm, Elasticity modulus = 200000 N/mm2, B = 10mm, H_{max} = 12 mm and H_{min} = 2 mm, and *M* is up to 266667 Nmm. Therefore, the relation of the curvature can be given by following

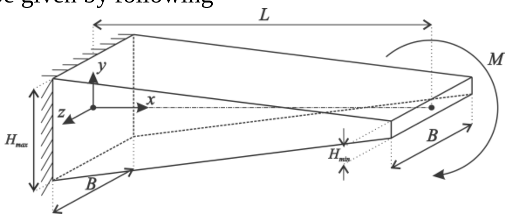

Fig. 8. Cantilever elastic tapered beam.

First, structure is divided into the total *n* equal length segments where *ds* is expressed as follows:

$$ds_{m,m+1} = ds = L/n \tag{21}$$

To use Eq. (20), the curvature of the each segment needs to be defined. The segment was assumed to have a constant curvature that is equal to the midpoint curvature of the segment. The curvature equation for the elastic rectangular beam cross-section is given as follows [7]:

$$\kappa_{m,m+1} = M/EI_{m,m+1} \tag{22}$$

The moment of inertia of the segments midpoint can be expressed as follows where $h_{m,m+1}$ is the mean height of the beam segment $S_{m,m+1}$.

$$I_{m,m+1} = Bh_{m,m+1}{}^{3}/12 \tag{23}$$

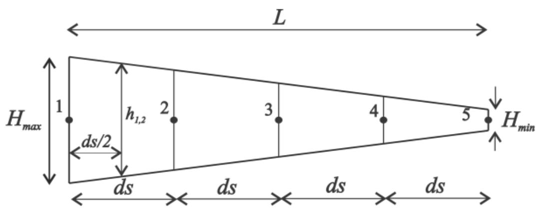

Fig. 9. Tapered beam divided into 4 segment.

The first point, origin is selected as the clamped edge of the structure where tangent angle is zero. Therefore, the θ_1 angle in Eq. (20) is zero. The average height of the segments can be defined as follows (Fig. (9)) where $\Delta h = H_{max} - H_{min}$:

$$h_{m,m+1} = H_{max} - (m - 1/2)\Delta h \quad (24)$$

Therefore, the vertical and horizontal displacements can be expressed as follows by substituting Eqs. (24), (25), (22), and (21) into Eq. (20):

$$\boldsymbol{i}\,\boldsymbol{\alpha}_{n+1} = x_{n+1} = \sum_{m=1}^{n} \frac{EB(H_{max}-(m-1/2)\Delta h)^3}{12M} \left[\sin\left(\sum_{a=1}^{m} \frac{12M}{EB(H_{max}-(a-1/2)\Delta h)^3} ds\right) - \sin\left(\sum_{a=1}^{m-1} \frac{12M}{EB(H_{max}-(a-1/2)\Delta h)^3} ds\right)\right] \quad (25)$$

$$\boldsymbol{j}\,\boldsymbol{\alpha}_{n+1} = y_{n+1} = \sum_{m=1}^{n} \frac{EB(H_{max}-(m-1/2)\Delta h)^3}{12M} \left[\cos\left(\sum_{a=1}^{m-1} \frac{12M}{EB(H_{max}-(a-1/2)\Delta h)^3} ds\right) - \cos\left(\sum_{a=1}^{m} \frac{12M}{EB(H_{max}-(a-1/2)\Delta h)^3} ds\right)\right] \quad (26)$$

The solution of Eqs. (25), and (26) converges easily within the user specified tolerance. The author found that reliable results can be obtained if the segment number is greater than $4L/(\pi r_{min})$. Some results of the proposed method is given in Fig. (10).

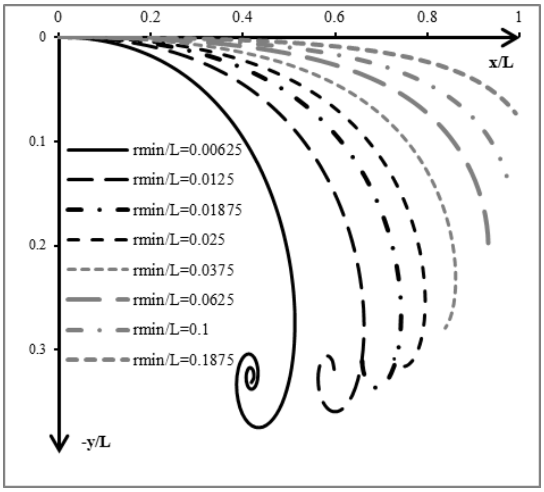

Fig. 10. Deflection shapes of the elastic tapered beam.

Large deflection Finite Element Method (FEM) solutions were calculated using ANSYS software. The maximum deviation between the converged FEM and proposed method result for the vertical displacement of the free end was ≤ 0.02 %, and the results are listed in Table 1.

Table 1. Comparison of the free end vertical displacements.

r_{min}/L	FEM $-y_{free}/L$	Proposed $-y_{free}/L$
0.00625	0.33120	0.33117
0.01250	0.31851	0.31847
0.01875	0.30058	0.30058
0.02500	0.30845	0.30842
0.03750	0.27903	0.27898
0.06250	0.20004	0.20000
0.10000	0.13326	0.13324
0.18750	0.07321	0.07320

6. Conclusions

The proposed method allows the most complex deflection shapes to be formed numerically with few inputs, without needing to solve differential equations or polynomials of trigonometric series. The initial imperfections can also be modeled using the same method for more successful calculations. The proposed solution was extremely fast and accurate.

When performing a displacement calculation, it is sufficient to obtain the moment curvature diagram only once. After calculating the moments, the curvature value can be read from the diagram even if it is in a plastic or post-buckling state. The exception occurs only when local plate buckling is considered. Therefore, the process can iterate with the effective width value. This is an advantage for the effective width calculation: The strain and stress at any point on the structure can be determined if the curvature value is known. Therefore, the effective width calculation can be obtained as a variable according to the deflections. This makes the solution fast because the equations are simple. The common assumptions in deflection modeling are not used. Hence the solutions are more realistic. Second-order theory can be used when the deflection of a structure under an external load is known. Therefore, more realistic solutions can be obtained with more iterations.

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