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제조업의 주기성 시계열분석에서 힐버트 황 변환의 효용성 평가

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Evaluating Efficacy of Hilbert-Huang Transform in Analyzing Manufacturing Time Series Data with Periodic Components

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Real-life time series characteristic data has significant amount of non-stationary components, especially periodic components in nature. Extracting such components has required many ad-hoc techniques with external parameters set by users in case-by-case manner. In our study, we evaluate whether Hilbert-Huang Transform, a new tool of time-series analysis can be used for effective analysis of such data. It is divided into two points : 1) how effective it is in finding periodic components, 2) whether we can use its results directly in detecting values outside control limits, for which a traditional method such as ARIMA had been used. We use glass furnace temperature data to illustrate the method.

Keywords : Hilbert-Huang Transformation, Time-Series Data, Control Limits

1. Overview

Real-life time-series characteristic data tend to contain many levels of dynamics operating on different time-scales. That is, time-series can be an aggregation of components, whose rates of change can range from “very fast” to “very slow.” Slow changing ones tend to create a significant level of long-distance autocorrelation. A rather simple example is time-series data showing multiple periods of varying lengths. We need more than the ARIMA model to help address this problem. There have been many researches which attempted to address the presence of periodic components in time-series. They showed some limited success, but a more effective approach is needed.

2. Previous Research on Periodicity in Time-Series Data

Seasonality [1] is a periodic trend or fluctuation which frequently appears in time series. They could be well-defined and precise or merely semi-regular. It frequently shows up in economic data as a regular seasonal variation, hence the name ‘seasonality.’ However, such periodic fluctuations are common in data from various fields. The typical method to identify seasonality is as follows : (a) Run sequence plot (b) Seasonal subseries plot (c) Multiple box plots (d) Auto-correlation plot (e) Seasonal index measures. These techniques can be useful but they assume that periods of seasonal components are already known to us beforehand. It is not

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an effective way to find out periods we are not aware of in advance. Besides, it is not systematic and tends to depend on visual inspection by humans.

Least-squares Spectral Analysis (LSSA) known as Vanicek Method, employs an iterative algorithm to find the best fit of sinusoids to time series data, using least squares method [15]. It goes as follows: Find a frequency from a preordered list of frequencies, which produces the best fit sinusoid for the data using least squares method. Then subtract the best fit sinusoid from the time series data, and make the new time series data. Repeat the same on this new data. Keep doing this until appropriate termination condition is met. LSSA addresses the shortcoming of FFT which tends to exaggerate long-period noises in long-gapped times series data. Lomb and Scargle developed an improved version named "Lomb-Scargle method" [9, 13].

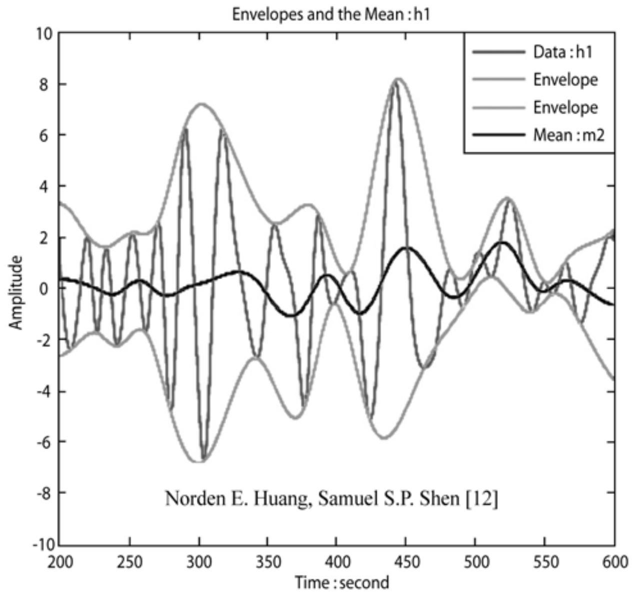
FFT and Wavelet analyses [2, 4, 6, 7] are the well-known techniques. They produce frequency amplitude chart from a given time series data. While FFT assumes that the major periodic components of time series data does not change in amplitude over the entire span of time series data, Wavelet assumes that such periodic components can change over time as can be observed in human voice spectrogram. So Wavelet analysis provides, 3-dimensional plot, made up of time, frequency and amplitude. Wavelet resolved many difficulties arising from windowed FFT or other variation of FFT to capture slow change in major period (frequency) components, that is, a spectral spectrum Wavelet algorithm is more efficient than FFT algorithm, but uses a crucial assumption that major frequencies tend to have exponentially sparse distribution, as we move to higher frequency region of overall frequency spectrum. It is not a significant drawback in reality, because such distribution is commonplace among natural and man-made phenomena. Wavelet Transform is superior to any other methods described above including FFT. Nevertheless, Wavelet Transform requires threshold parameters for filters. In many cases, it is a crucial step to produce desirable Wavelet charts. Still, finding threshold values are mostly of ad-hoc nature, lacking any systematic method. Furthermore, frequency distribution is preset (in an exponentially sparse fashion) and amplitudes are calculated on those frequencies. In technical term, such a frequency distribution is called a priori basis. It does not have a basis which adapts itself to a given time-series. Wavelet Transform can deal with non-stationary data but not non-linear data.

3. Hilbert-Huang Transform

The Hilbert-Huang transform (HHT) [5, 9, 10, 11, 14] is a method which can analyze time series data and split it into multiple oscillatory components, called Intrinsic Mode Functions (IMF). The result is superior to any comparable techniques, especially for non-stationary and nonlinear time series. Oscillations in time series frequently exhibit intra-wave frequency modulation, which does not conform to typical sinusoidal shape. This is typical of oscillatory behavior from highly non-linear systems. Fourier transform (or wavelet transform) tends to destroy essential feature of such pattern, while Hilbert-Huang transform can capture it rather effectively. On the other hand, unlike Fourier analysis, Hilbert-Huang transform was not derived from pre-existing mathematical theory. It is more of an empirically conceived algorithm, whose effectiveness has been shown by applying it to time series data from various fields. It makes use of iterative procedure in which the component (IMF) of highest frequency was computed and subtracted out and proceed to isolate the component (IMF) of lower and lower frequency. The procedure is called Empirical Mode Decomposition (EMD) method. The rough description of Hilbert-Huang Transform is in the below [9].

(1) Find all local maxima in the entire data set, and compute their envelope curve using cubic-spline lines.
(2) Do the same for local minima.
(3) Find the mean curve $m_t^{(1)}$ from the two envelope curves.
(4) Define $h_t^{(1)} = X_t - m_t^{(1)}$
(5) For two successive zero crossings, do steps (1) and (2)
(6) do (3) and get $m_t^{(2)}$
(7) Define $h_t^{(2)} = h_t^{(1)} - m_t^{(2)}$
(8) Continue steps (5)~(7) until $m_t^{(k)}$ becomes nearly a constant value.
(9) Then $c_1(t) = h_t^{(k+1)}$ is the first IMF.
(10) $r_1(t) = X(t) - c_1(t)$
(11) Take $r_1(t)$ in place of $X(t)$ and repeat steps (1) ~ (10) to generate $c_2(t)$ and $r_2(t) = r_1(t) - c_2(t)$.
(12) repeat it until $r_n(t)$ becomes a monotone function which cannot generate IMF any further.

$$\text{Then } X_t = \left(\sum_{k=1}^{n} c_k\right) + r_k.$$



<Figure 1> Empirical Decomposition Process

X_t is now decomposed into n empirical modes. <Figure 1> shows how steps (1)~(4) are done. The sample result of EMD is shown in <Figure 2>~<Figure 3>.

<Figure 2> shows a sample curve(top) and its power spectrum(bottom). EMD produces IMF's on the left side in <Figure 3> The right-side of <Figure 3> shows power spectrum of each IMF. Each IMF has a power spectrum concentrated around a single major frequency unlike the original curve with more spread-out power spectrum. In short, EMD decomposes the original curve into curves in different frequency bands. This process is adaptive in that the decomposition interval is not defined a priori as in FFT and Wavelet Theory. It can produce better results in that sense. It should be noted that power spectrum of each IMF could be complicated. Some has no discernible major frequencies, while others have a few. Some peaks are sharp, while others are more spread-out and look more ambiguous.

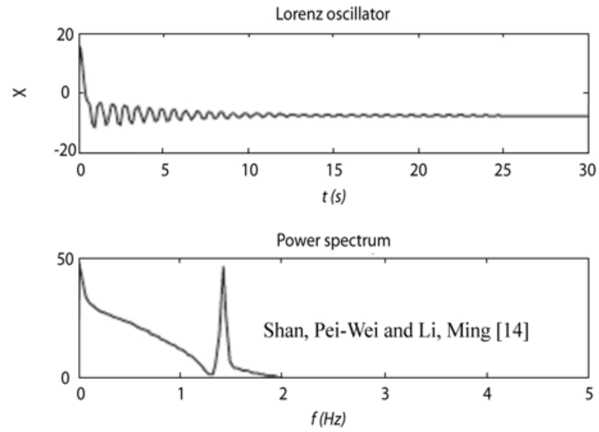
Given a signal function $x(t)$ is complex conjugate $y(t)$ can be computed by Hilbert Transform such that

$$y(t) = H[x(t)] = 1/\pi PV \int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau,$$

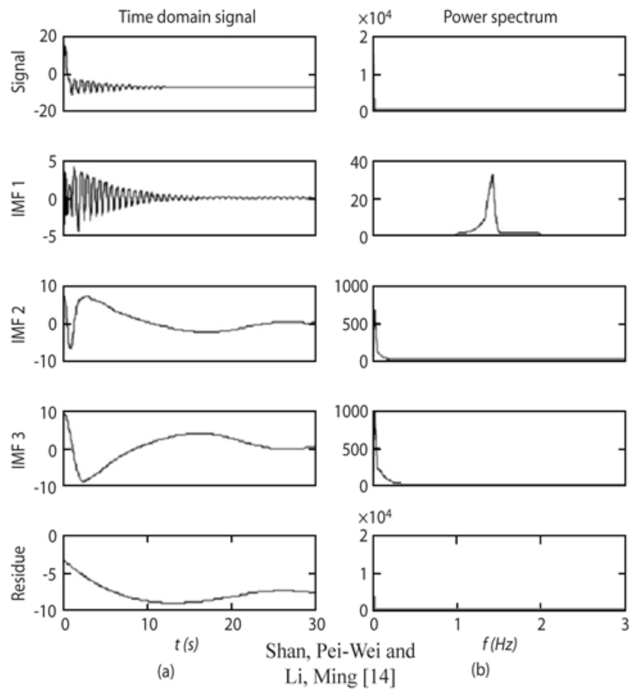
where PV is a principal value of singular integral. Then analytic signal is defined as :

$$z(t) = x(t) + iy(t) = a(t)e^{i\theta(t)}$$

where $a(t) = \sqrt{x^2 + y^2}$ and $\theta(t) = \tan^{-1}\frac{y}{x}$



<Figure 2> Lorenz Oscillator Time-series Data and its Power Spectrum

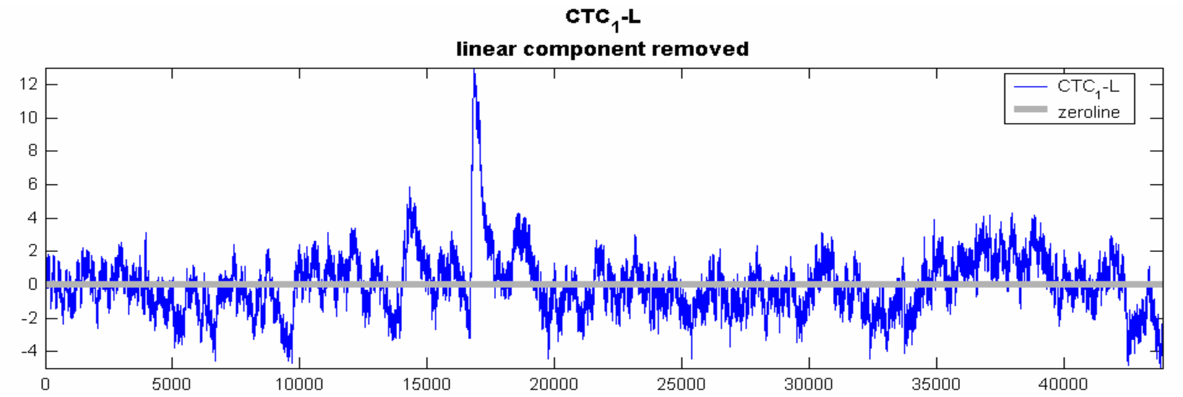


<Figure 3> IMF's of the Above Time-series Data and Corresponding Power Spectrums

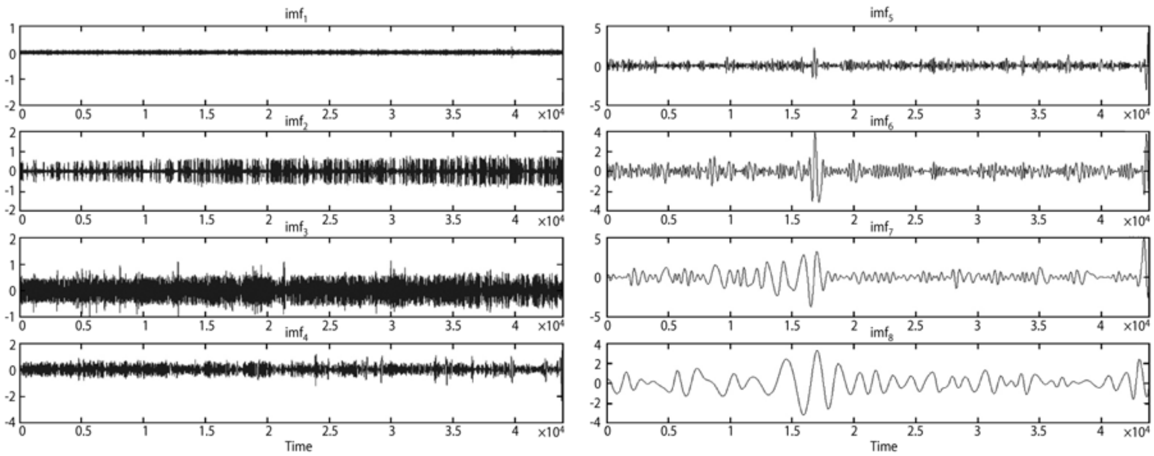
Here, $a(t)$ is the instantaneous amplitude, and $\omega(t) = d\theta/dx$ is the instantaneous frequency. Let $\wp$ be the power spectrum of $x(t)$. For each IMF $x(t)$, we have, $\wp, a(t), \theta(t)$. Each shows important aspect of IMF $x(t)$.

4. Evaluation of HHT's Efficacy

<Figure 4> shows a sample time-series of temperature readings from a manufacturing process [3]. It was collected



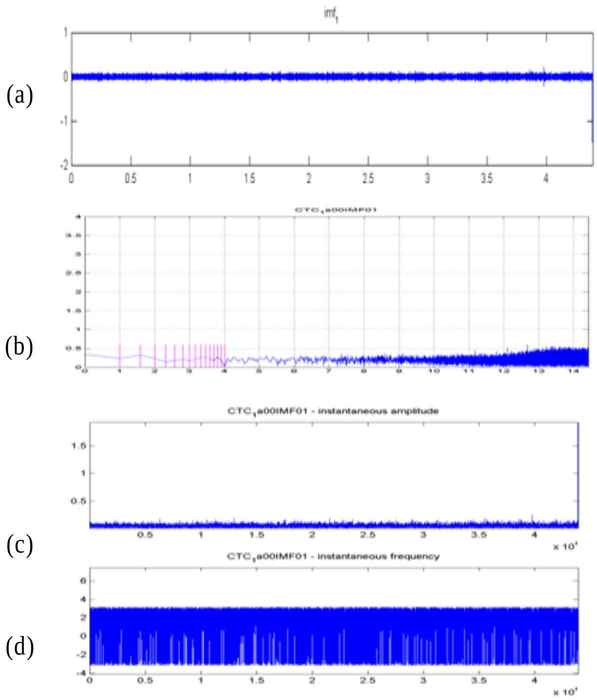
<Figure 4> Time-series Data of Temperature from a Manufacturing Process



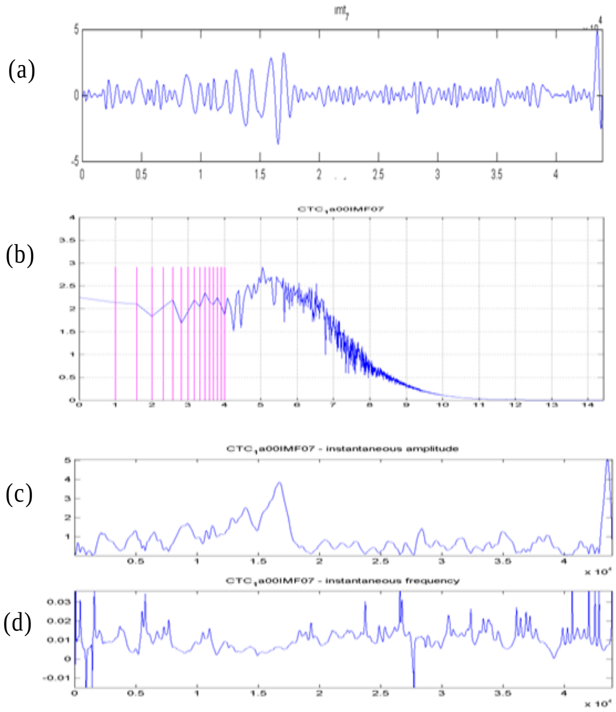
<Figure 5> Left : IMF1~IMF4(top to bottom), Right : IMF5~IMF8(top to bottom)

at each minute and spans over an entire month. Such manufacturing time-series contain stochastic component. It has been frequently the practice that we treat the time-series as essentially stochastic and other deterministic components are merely an impediment to be factored out at a preprocessing stage in order to apply well-known stochastic analyses. However, in numerous time-series data, that is not the case. Deterministic component, especially periodic or pseudo-periodic oscillatory components, are essential part of the time-series providing us with insight into the nature of time-series we want to analyze. How to separate stochastic and deterministic components is a matter of important concern.

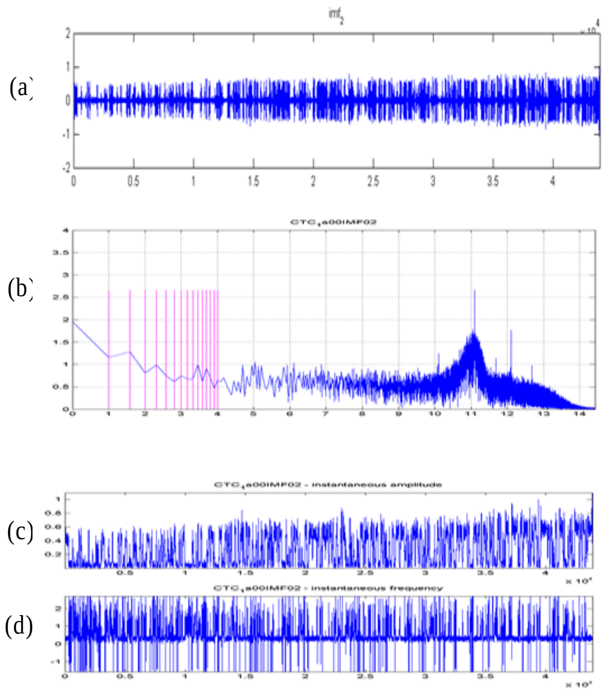
First we want to find the dominant periodic frequencies in time-series. Many of methods discussed in previous section can be used, but they all have certain limitations. Empirical Mode Decomposition (EMD) produces IMF's of different time-scales. In <Figure 5>, we can see the first 8 IMF's of the above time-series. Each IMF occupies a narrow band of frequency region. It would produce power spectrum peaking around a dominant frequency. These peak frequencies can represent dominant frequencies of original time-series. The typical example is seen in <Figure 2> and <Figure 3>. For each IMF, its corresponding power spectrum is shown on the right. It shows a peak around a certain frequency. A periodic component is made from this frequency. Such periodic components can be derived from all IMF's. <Figure 6>~<Figure 9> show IMF1, IMF2, IMF7, and IMF10 and their power spectrums (in log-log scale,) instantaneous amplitudes, and instantaneous frequencies (henceforth name IA and IF respectively). (a) shows an IMF and (b), its power spectrum. The power spectrum of IMF1 (<Figure 6>(b)) shows that IMF1 is random noise. <Figure 7>(b) shows that while IMF2 has random noise, it also has clear sharp peaks. The highest peak is the dominant frequency and other sharp peaks related companion frequencies, typically integer multiple (2, 3) or simple fraction(1/2, 1/3) of the highest peak. This peak corresponds to a pre-designed cyclic mechanical operation. That is why it is so sharp.



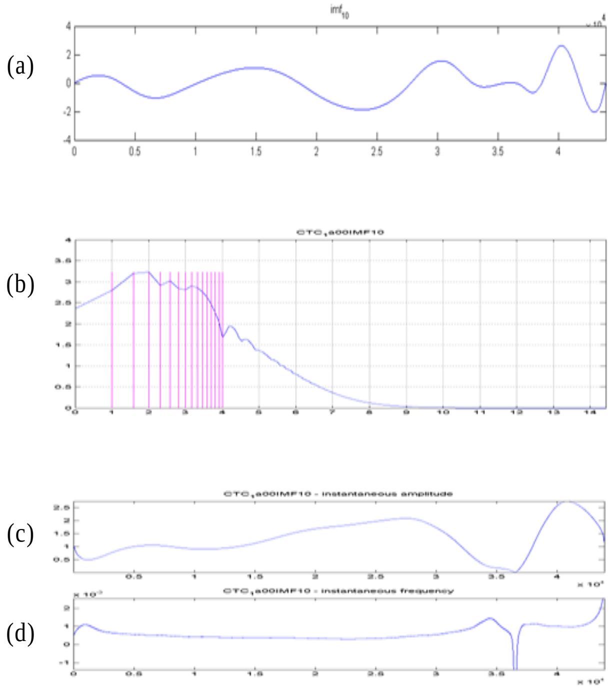
<Figure 6> (a) IMF : $c_{10}(t)$ (b) Power Spectrum : $\wp_1$
(c) Instantaneous Amplitude : $a_1(t)$
(d) Instantaneous Frequency : $\theta_1(t)$



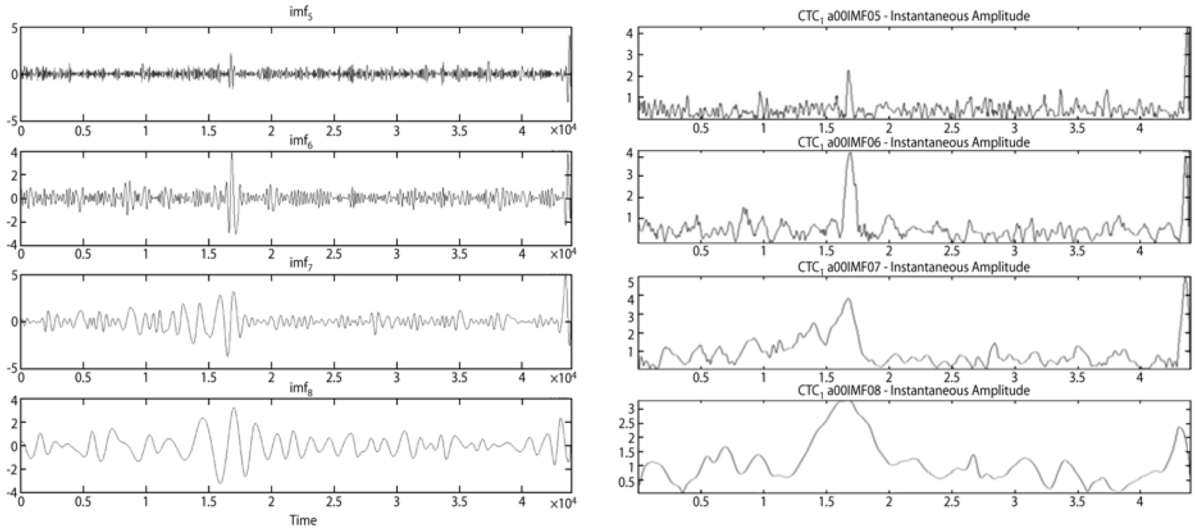
<Figure 8> (a) IMF: $c_7(t)$ (b) Power Spectrum : $\wp_7$
(c) Instantaneous Amplitude : $a_7(t)$
(d) Instantaneous Frequency : $\theta_7(t)$



<Figure 7> (a) IMF : $c_2(t)$ (b) Power Spectrum : $\wp_2$
(c) Instantaneous Amplitude : $a_2(t)$
(d) Instantaneous Frequency : $\theta_2(t)$



<Figure 9> (a) IMF : $c_{10}(t)$ (b) Power Spectrum : $\wp_{10}$
(c) Instantaneous Amplitude : $a_{10}(t)$
(d) Instantaneous Frequency : $\theta_{10}(t)$



<Figure 10> Left : IMF5~8 (Top to Bottom), Right : Corresponding Instantaneous Amplitude $a(t)$

<Figure 8>(a) shows IMF7 which corresponds to daily temperature cycle. Since daily temperature change is not a exact cyclic pattern, its power spectrum in <Figure 8>(b) shows less pronounced smooth peak. <Figure 9>(a) shows IMF10 which has long periods. However, since there are only small number of oscillations in the graph, less confidence may be given to its dominant frequency.

By using least-squares method, best-fit linear sum made up of such periodic components can be calculated and we can use it as periodic component of entire time-series. After the periodic component is subtracted from the original time-series, the remaining component can be regarded as purely stochastic and ARIMA analysis can be applied.

There is another method we can consider. Hilbert-Huang transform provides us with IMF's, each representing a component of different time-scale. While stochastic components could be spread into various IMF's, the one with shorter time-scale can contain most of stochastic components. It is not difficult to see that, as time-scale becomes longer, there tend to be higher long-distance autocorrelation. By its nature, random values go up and down at each time-step even in ARIMA. This leads to shorter time-scale and mostly captured in IMF's with very short time-scale. That is, we can note that the share of stochastic elements in IMF steadily decreases as IMF's time-scale gets longer. Hence, this enables us to use IMF's directly to detect aberrant data outside control limits. We can set up control limits of shorter time-scale IMF's in the same way we do with ARIMA model such as using 3-σ boundary. Since stochastic nature disappears as time-scale becomes longer, IMF's of longer time-scale may need different criteria for detecting values outside control limits. Tighter control limit could be necessary. A perfect sine function $f(x) = \sin x$ has all its values staying in 2-σ boundary. In many cases, values stay well within 2-σ boundary. It could be reasonable that control limits for longer time-scale IMF's should be 2~2.5-σ boundary(note: This is tentative suggestion which requires more empirical investigation.) Each IMF can now have its own control limit calculated from its values. This produces control chart for each IMF. Some IMF's values are all within their control limits while others are out of theirs. This provides an additional useful perspective on aberrant event in time-series. That is, the time-scale of event outside control limits. If it is a sharp burst in a single time step, it can be captured in IMF of shortest time-scale. If such outside-limit behavior persists over longer duration, it will show up in IMF's of longer time-scale as well. In <Figure 5>, aberrant event is detected clearly in IMF 5~6, and less so in IMF7. So we can give a proper time-scale for aberrant event which pushes values outside control limits: whether is a short-term or longer-term event. Additionally, we can see that aberrant event can be usually caught in multiple IMF's

There are two other charts derived from each IMF, $a(t)$, IA and $\theta(t)$, IF. $a(t)$ tracks amplitude of oscillation from moment-to-moment while $\theta(t)$ tracks its frequency. As for detecting aberrant values, <Figure 10> shows $a(t)$ can detect it more clearly than IMF's value itself, especially in the case of IMF8. Interaction between $a(t)$ and $\theta(t)$ can sometimes obscure amplitude of aberrant values.(as can be seen in the diagram) Hence, we can conclude that $a(t)$ of each IMF can be better at detecting aberrant event than IMF itself.

Finally, $\theta(t)$ can detect sudden change of frequency in an IMF. Sudden increase of $\theta(t)$ could mean increased volatility. Any sudden change can be viewed as a signal that there is a problem. In our case of temperature time-series, $\theta(t)$ does not add useful insight because we cannot detect any sustained change of frequency. Sudden burst of instantaneous frequency tends to be noise in our study.

5. Conclusion

We have evaluated how useful the results from HHT can be. Since EMD decomposes HHT into IMF's, which are essentially oscillatory curves in narrow frequency band, it helps identify dominant frequencies of original time-series by finding dominant frequency of each IMF and, if necessary, its companion frequencies. In finding aberrant values which are outside control limits, we examined how IMF's, IA's, and IF's can be used. It turns out that IA is better at detecting the aberrant values than IMF itself, while IF is not useful. Since there are multiple IMF's, we can set up control limit for each IMF, which represents different time-scale. This can determining whether out-of-control behavior is shorter-term or longer-term phenomena. Depending on the time-scale of such behavior, different factors can be at play, which can be more informative diagnostic tool.

Since an IMF become less stochastic and more deterministic as time-scale becomes longer, ones with shorter time-scale could be treated as stochastic signals, while longer ones are deterministic. Stochastic ones can have 3-σ boundary, while deterministic ones can have tighter boundary 2~2.5-σ. More research needs to be done on what new advantages IMF's, IA's, IF's, and power spectrums could provide. Overall, HHT can be a superior tool to any other ones we have mentioned above. It is better at handling not only non-stationary but also non-linear time-series data. Furthermore, it can determine the time-scale of aberrant behaviors which cannot be done in other methods.

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