Performance Analysis of Amplify-and-Forward Two-Way Relaying with Antenna Correlation

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Abstract

This paper investigates the performance of an amplify-and-forward (AF) two-way relaying system with antenna correlation. The system consists of two multiple-antenna sources, which exchange information via the aid of a single-antenna relay. In particular, we derive the exact outage probability expression. Furthermore, we provide a simple, tight closed-form lower bound for the outage probability. Based on the lower bound, we obtain the closed-form asymptotic outage probability and the average symbol error rate expressions at high signal-to-noise ratio (SNR), which reveal the system's diversity order and coding gain with antenna correlation. To investigate the system's throughput performance with antenna correlation, we also derive a closed-form lower bound for the average sum-rate, which is quite tight from medium to high SNR regime. The analytical results readily enable us to obtain insight into the effect of antenna correlation on the system's performance. Extensive Monte Carlo simulations are conducted to verify the analytical results.

Keywords: Two-way relaying, antenna correlation, outage probability, average symbol error rate, average sum-rate

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1. Introduction

In recent years, two-way relaying has attracted enormous research interest, due to its ability to provide high spectral efficiency [1][2]. In a two-way relaying system, two sources exchange information with the aid of an intermediate relay, and one round of information exchange can be divided into two orthogonal phases [3]. In the first phase, two sources transmit signals simultaneously to the relay. In the second phase, the relay processes manipulations on the signals received and broadcasts the processed signals to both sources. According to the processing method at the relay, the relaying protocols can be categorized into two protocols: decode-and-forward (DF) protocol and amplify-and-forward (AF) protocol [1]. In the DF scheme, also called physical layer network coding (PLNC), the relay decodes the signals it has received, manipulates XOR operation in bit-level, and broadcasts to both sources [4]. In the AF scheme, also called analog network coding (ANC), the relay simply amplifies the superimposed signals it has received, and broadcasts the amplified signals to both sources [5]. AF protocol thus has lower implementation complexity than DF protocol; therefore we focus on AF protocol in this paper.

The performance of AF two-way relaying has been analyzed for single-antenna systems in [6][7][8][9]. For the single-relay system, the generalized outage probability expression was derived in [6], while the tight closed-form average symbol error rate (SER) expression was presented in [7]. For the multiple-relay system, the diversity orders of various relay selection schemes were analyzed in [8], and the asymptotic average SER expression for the max-min relay selection scheme was derived in [9].

When the multiple-antenna technique is incorporated into two-way relaying, the system's throughput and transmission reliability can be enhanced [10][11]. For a system in which all nodes were mounted with multiple antennas, the upper and lower bounds of outage probability were derived for AF two-way relaying [11], and the achievable rate was investigated in [12]. In case the sources were not able to mount multiple antennas and only the relay was mounted with multiple antennas, the capacity region was analyzed in [13] for the AF two-way relaying system, with linear processing at the relay. Another scenario occurs when both sources are mounted with multiple antennas while the relay is mounted with a single antenna [14][15]. In such a case, the average sum rate of two-way relaying was analyzed [14] over Nakagami-m fading channels. However, the above mentioned works did not consider antenna correlation. Antenna correlation occurs in many practical situations due to insufficient antenna spacing at the terminals. This is the motivation for our investigation of the effect of antenna correlation on the performance of a two-way relaying system, presented in this paper.

For completeness, it is worth noting that the effect of antenna correlation has been analyzed for the one-way relaying system in [16][17][18][19]. The outage probability and average SER expressions have been derived in [16] and [17] over Rayleigh fading channels for variable and fixed gain relaying, respectively. Furthermore, the authors of [18] have provided unified performance analysis for an AF one-way relaying system with antenna correlation. The authors in [19] proposed a new system model that was different than the system model in [16][17][18], namely the source and relay transmit using a selected antenna, while the relay and destination receive using multiple antennas. Taking into account the antenna correlation at the receivers, the authors in [19] have analyzed the outage probability, average SER and ergodic capacity. In this paper, we extend the above mentioned works from one-way relaying to two-way relaying.

In light of the aforementioned researches, in this paper we analyze the performance of an AF two-way relaying system with antenna correlation, aiming to gain insight into the effect of antenna correlation on system performance and to acquire the theoretical guidelines to design system parameters in practice (for example, determining the number of antennas, antenna spacing, relay location, etc.). Here, we consider a system that consists of two multiple-antenna sources and one single-antenna relay. Such systems occur in many practical scenarios. For example, two base stations mounted with multiple-antennas communicate with each other via the aid of a relay node, which, due to size and complexity constraints, is mounted with a single antenna [16]. For the purpose of improving transmission reliability, maximum ratio transmission (MRT) and maximum ratio combining (MRC) techniques are employed at the transmitter and receiver, respectively. We first derive the exact outage probability expression that is applicable for the entire signal-to-noise ratio (SNR) regime. To get better insight, we provide a simple, tight closed-form lower bound for the outage probability. From this lower bound, the asymptotic outage probability and average SER expressions are derived in closed form, which enable us to analyze the diversity order and coding gain. To investigate the system's throughput with antenna correlation, we also derive a tight, closed-form lower bound for the average sum-rate. Numerical results show that our derived exact outage probability expression is in excellent agreement with the results generated via Monte Carlo simulations. The lower bounds and asymptotic expressions are also shown to match the simulation results quite well. These analytical results readily enable us to evaluate the effect of antenna correlation and design system in practice. Both the asymptotic analysis and simulation results reveal that the system can achieve full spatial diversity even with antenna correlation. However, the coding gain and average sum-rate are degraded by antenna correlation.

The rest of this paper is organized as follows. The system model is introduced in section 2. Section 3 presents the performance analysis. In section 4, simulation results and discussions are presented. Finally, section 5 concludes this paper.

Notations: Throughout this paper, all the vectors are denoted in boldface. $(\cdot)^H$ and $\|\bullet\|_F^2$ denote the conjugate transpose and Frobenius norm, respectively. $E(\bullet)$ denotes the expectation operation and $\Pr(\bullet)$ represents the probability. $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. We represent the function $g(x)$ as $o(x^k)$ if $\lim_{x\to 0} g(x)/x^k = 0$.

2. System Model

Consider an AF two-way relaying system, as depicted in **Fig. 1**. Two sources $T1$ and $T2$, which are mounted with N_1 and N_2 antennas respectively, exchange information with the aid of a single-antenna relay R [1]. We assume that there is no direct link between $T1$ and $T2$

[1] This system has been extensively investigated in [14][15][16][17][18][20][21] and occurs in many practical scenarios. For example, as stated in [16], two multiple-antenna base stations exchange information via a single-antenna relay. In military communications, two multiple-antenna nodes communicate with each other relying on a single-antenna relay at a critical moment. It should be pointed out that the model we consider here includes the system model that has been investigated in [22][23] as a special case. To be specific, when either N_1 or N_2 reduces to one, our model corresponds to the scenario of a multiple-antenna base station, communicating

due to high shadowing, and all nodes operate in a half-duplex mode. Channels for both hops, namely the channels between $Ti, i=1,2$ and R, are assumed to undergo independent, quasi-static and flat Rayleigh fading. Assuming time-division duplex (TDD) is adopted, the channels from Ti to R are the same as those from R to Ti within one round of information exchange, $i=1,2$. We also assume that all nodes have accurate channel state information (CSI) for both hops.

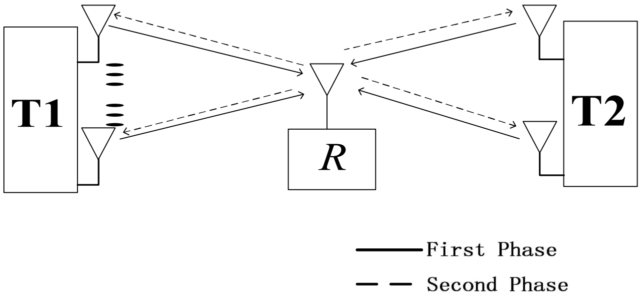

Fig. 1. System model of an AF two-way relaying system

One round of information exchange consists of two orthogonal phases. During the first phase, $T1$ and $T2$ perform MRT operation for the purpose of improving transmission reliability, and transmit simultaneously to R. The signal received at R can be represented as

$$y_r = \sqrt{p_s}\mathbf{h}_1^H\mathbf{w}_1 s_1 + \sqrt{p_s}\mathbf{h}_2^H\mathbf{w}_2 s_2 + n_r \quad (1)$$

where s_1 and s_2 denote the transmitted signal with average unit energy, i.e., $E\left[|s_i|^2\right]=1, i=1,2$. p_s denotes the transmitting power of $Ti, i=1,2$ and n_r represents the zero mean additive white Gaussian noise (AWGN) with variance N_0 at R. $\mathbf{h}_1$ denotes the $N_1 \times 1$ channel vector and $\mathbf{h}_2$ denotes the $N_2 \times 1$ channel vector. All the elements in $\mathbf{h}_1$ and $\mathbf{h}_2$ are modeled as zero mean complex Gaussian variables. Due to antenna correlation at Ti, the elements in $\mathbf{h}_i$ are mutually correlated, $i=1,2$. According to the analysis in [24], the correlation and gain imbalances between the elements of $\mathbf{h}_i$ are entirely characterized by the covariance matrix $\mathbf{\Phi}_i = E\left[\mathbf{h}_i\mathbf{h}_i^H\right]$, $i=1,2$. In other words, the channel vector $\mathbf{h}_i$ can be represented in the following form [25]

$$\mathbf{h}_i = \mathbf{\Phi}_i^{\frac{1}{2}}\bar{\mathbf{h}}_i \quad (2)$$

where all the elements in $\bar{\mathbf{h}}_i$ are modeled as zero mean and unit variance complex Gaussian variables, and are independent from each other. $\mathbf{w}_i$ in (1) denotes the transmitting weight vector, and is chosen as $\mathbf{w}_i = \mathbf{h}_i / \|\mathbf{h}_i\|_F$ according to the principle of maximum ratio transmission [26].

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with a single-antenna mobile user, via a single-antenna relay [22][23].

During the second phase, the signal received at R is multiplied by a factor G and forwarded to both $T1$ and $T2$. In this paper, we consider the varying gain relaying [27], and thus $G=\sqrt{1/\left[p_s\|\mathbf{h}_1\|_F^2+p_s\|\mathbf{h}_2\|_F^2+N_0\right]}$. The signal received at $Ti, i=1,2$ can be written as

$$\begin{aligned}\mathbf{y}_i &= \sqrt{p_r}G\mathbf{h}_i y_r+\mathbf{n}_i \\ &= \sqrt{p_s p_r}G\mathbf{h}_i\mathbf{h}_1^H\mathbf{w}_1 s_1+\sqrt{p_s p_r}G\mathbf{h}_i\mathbf{h}_2^H\mathbf{w}_2 s_2+\sqrt{p_r}G\mathbf{h}_i n_r+\mathbf{n}_i\end{aligned} \tag{3}$$

where p_r is the transmitting power of R, and $\mathbf{n}_i$ is the AWGN with zero mean and covariance matrix $N_0\mathbf{I}_{N_i}$ at Ti. After self-interference cancellation, the signal received at $T1$ and $T2$ can then be respectively written as

$$\begin{aligned}\bar{\mathbf{y}}_1 &= \sqrt{p_s p_r}G\mathbf{h}_1\mathbf{h}_2^H\mathbf{w}_2 s_2+\sqrt{p_r}G\mathbf{h}_1 n_r+\mathbf{n}_1 \\ \bar{\mathbf{y}}_2 &= \sqrt{p_s p_r}G\mathbf{h}_2\mathbf{h}_1^H\mathbf{w}_1 s_1+\sqrt{p_r}G\mathbf{h}_2 n_r+\mathbf{n}_2\end{aligned} \tag{4}$$

To improve the system performance, maximum ratio combining is applied to (4) [28], and then the resulting instantaneous SNR at $T1$ and $T2$ can be represented as

$$\begin{aligned}\gamma_1 &= \frac{\bar{p}_s\bar{p}_r z_1 z_2}{(\bar{p}_s+\bar{p}_r)z_1+\bar{p}_r z_2+1} \\ \gamma_2 &= \frac{\bar{p}_s\bar{p}_r z_1 z_2}{(\bar{p}_s+\bar{p}_r)z_2+\bar{p}_r z_1+1}\end{aligned} \tag{5}$$

respectively, in which $z_1=\|\mathbf{h}_1\|_F^2$, $z_2=\|\mathbf{h}_2\|_F^2$, $\bar{p}_s=p_s/N_0$ and $\bar{p}_r=p_r/N_0$.

3. Performance Analysis

This section derives performance measures of the AF two-way relaying system by taking into account the antenna correlation at the source nodes. Section 3.1 presents the derivation of outage probability, section 3.2 derives the asymptotic average SER expression at high SNR and the lower bound of average sum-rate is presented in section 3.3.

To obtain these performance measures, we are required to obtain the probability density functions (PDF) of z_1 and z_2. According to the results in [24][29], the Laplace transform of the PDF of z_1 is derived as

$$\psi_{z_1}(s)=\frac{1}{\left(1+s\phi_{1,1}\right)^{u_{1,1}}}\frac{1}{\left(1+s\phi_{1,2}\right)^{u_{1,2}}}\cdots\frac{1}{\left(1+s\phi_{1,t_1}\right)^{u_{1,t_1}}} \tag{6}$$

where $\phi_{1,1},\phi_{1,2},\cdots\phi_{1,t_1}$ are the distinct real eigenvalues of correlation matrix $\mathbf{\Phi}_1$, with multiplicities $u_{1,1},u_{1,2},\cdots u_{1,t_1}$ such that $\sum_{i=1}^{t_1}u_{1,i}=N_1$. Expanding (6) into its partial fractions yields

$$\psi_{z_1}(s) = \sum_{i=1}^{t_1} \sum_{j=1}^{u_{1,i}} \frac{\omega_{1,i,j}}{\left(1+s\phi_{1,i}\right)^j} \tag{7}$$

where $\omega_{1,i,j}$ is given by [29]

$$\omega_{1,i,j} = \frac{1}{\left(u_{1,i}-j\right)!\phi_{1,i}^{u_{1,i}-j}} \frac{\partial^{u_{1,i}-j}}{\partial u^{u_{1,i}-j}} \left[\left(1+s\phi_{1,i}\right)^{u_{1,i}} \psi_{z_1}(s)\right]_{s=-1/\phi_{1,i}} \tag{8}$$

By using the inverse Laplace transform, the PDF of z_1 can then be written as

$$f_{z_1}(z_1) = \sum_{i=1}^{t_1} \sum_{j=1}^{u_{1,i}} \frac{\omega_{1,i,j}}{(j-1)!\phi_{1,i}^j} z_1^{j-1} e^{-\frac{z_1}{\phi_{1,i}}} \tag{9}$$

By using the same approach, the PDF of z_2 can be obtained as

$$f_{z_2}(z_2) = \sum_{l=1}^{t_2} \sum_{m=1}^{u_{2,l}} \frac{\omega_{2,l,m}}{(m-1)!\phi_{2,l}^m} z_2^{m-1} e^{-\frac{z_2}{\phi_{2,l}}} \tag{10}$$

where $\phi_{2,1},\phi_{2,2},\cdots\phi_{2,t_2}$ are the distinct real eigenvalues of correlation matrix $\mathbf{\Phi}_2$, with multiplicities $u_{2,1},u_{2,2},\cdots u_{2,t_2}$ such that $\sum_{l=1}^{t_2} u_{2,l} = N_2$. $\omega_{2,l,m}$ in (10) can be obtained following a similar way as (8).

3.1 Outage Probability

Outage probability is an important system performance measure. For two-way relaying systems, an outage event occurs when any end-to-end transmission is in outage, i.e., when either γ_1 or γ_2 falls below a predefined threshold γ_{th} [7][30]. In the following theorem, we present the exact outage probability expression.

Theorem 1: The exact outage probability expression of the AF two-way relaying system with antenna correlation is given by

$$P_{out}(\gamma_{th}) = \underbrace{\Pr\left[\gamma_1 < \gamma_{th}, \gamma_1 < \gamma_2\right]}_{P_1(\gamma_{th})} + \underbrace{\Pr\left[\gamma_2 < \gamma_{th}, \gamma_2 < \gamma_1\right]}_{P_2(\gamma_{th})} \tag{11}$$

where $P_1(\gamma_{th})$ and $P_2(\gamma_{th})$ are given by

$$\begin{aligned} P_1(\gamma_{th}) = 1 &- \sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1} \omega_{1,i,j}\omega_{2,l,m} \binom{n+j-1}{n} \frac{\phi_{1,i}^n \phi_{2,l}^j}{\left(\phi_{1,i}+\phi_{2,l}\right)^{n+j}} \\ &+ \sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1}\sum_{k=0}^{n+j-1} \left\{ \begin{array}{l} \omega_{1,i,j}\omega_{2,l,m} \dfrac{k!}{(j-1)!n!} \dbinom{n+j-1}{k} \\ \times \left(\dfrac{\phi_{1,i}\phi_{2,l}}{\phi_{1,i}+\phi_{2,l}}\right)^{k+1} \dfrac{1}{\phi_{1,i}^j \phi_{2,l}^n} \delta^{n+j-k-1} e^{-\frac{\phi_{1,i}\phi_{2,l}}{\phi_{1,i}+\phi_{2,l}}\delta} \end{array} \right\} \\ &- \sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1} \omega_{1,i,j}\omega_{2,l,m} \frac{1}{(j-1)!n!} \frac{1}{\phi_{1,i}^j \phi_{2,l}^n} \underbrace{\int_\delta^\infty x^{j-1} \left[\frac{\left(\overline{P}_s+\overline{P}_r\right)\gamma_{th}x+\gamma_{th}}{\overline{P}_s\overline{P}_r x - \overline{P}_s\gamma_{th}}\right]^n e^{-\frac{x}{\phi_{1,i}} - \frac{\left(\overline{P}_s+\overline{P}_r\right)\gamma_{th}x+\gamma_{th}}{\phi_{2,l}\left(\overline{P}_s\overline{P}_r x - \overline{P}_s\gamma_{th}\right)}} dx}_{\psi_1(x)} \end{aligned} \tag{12}$$

$$
\begin{aligned}
P_2(\gamma_{th}) = {} & 1-\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{k=0}^{j-1}\omega_{1,i,j}\omega_{2,l,m}\binom{k+m-1}{k}\frac{\phi_{2,l}^{k}\phi_{1,i}^{m}}{\left(\phi_{1,i}+\phi_{2,l}\right)^{k+m}} \\
& +\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{k=0}^{j-1}\sum_{n=0}^{k+m-1}\left\{\begin{array}{l}\omega_{1,i,j}\omega_{2,l,m}\dfrac{n!}{(m-1)!k!}\dbinom{k+m-1}{n} \\ \times\left(\dfrac{\phi_{1,i}\phi_{2,l}}{\phi_{1,i}+\phi_{2,l}}\right)^{n+1}\dfrac{1}{\phi_{2,l}^{m}\phi_{1,i}^{k}}\delta^{k+m-n-1}e^{-\frac{\phi_{1,i}\phi_{2,l}}{\phi_{1,i}+\phi_{2,l}}\delta}\end{array}\right\} \\
& -\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{k=0}^{j-1}\omega_{1,i,j}\omega_{2,l,m}\frac{1}{(m-1)!k!}\frac{1}{\phi_{2,l}^{m}\phi_{1,i}^{k}}\int_{\delta}^{\infty}\underbrace{x^{m-1}\left[\frac{(\bar{p}_s+\bar{p}_r)\gamma_{th}x+\gamma_{th}}{\bar{p}_s\bar{p}_r x-\bar{p}_s\gamma_{th}}\right]^{k}e^{-\frac{x}{\phi_{2,l}}-\frac{(\bar{p}_s+\bar{p}_r)\gamma_{th}x+\gamma_{th}}{\phi_{1,i}(\bar{p}_s\bar{p}_r x-\bar{p}_s\gamma_{th})}}}_{\psi_2(x)}dx
\end{aligned}
\tag{13}
$$

respectively, with $\delta=\left[(2\bar{p}_s+\bar{p}_r)\gamma_{th}+\sqrt{(2\bar{p}_s+\bar{p}_r)^2\gamma_{th}^2+4\bar{p}_s\bar{p}_r\gamma_{th}}\right]\Big/2\bar{p}_s\bar{p}_r$.

Proof: please see Appendix A.

Since the functions $\psi_1(x)$ and $\psi_2(x)$ in theorem 1 converge quickly to zero (as illustrated in **Fig. 2**), the integrations in (12) and (13) can be numerically evaluated by truncating the integration range in mathematical softwares such as MATLAB and MATHMATICA. As illustrated in section 4, (11) matches excellently with the simulated outage probability from low to high SNR regime. In summary, though the exact outage probability expression is not in closed form, it can be effectively calculated in practice.

As it is difficult to get an insight into the system's outage behavior from (11), we now focus on deriving a simple, closed-form lower bound for outage probability, from which the asymptotic outage probability at high SNR can be derived.

Theorem 2: The outage probability for the AF two-way relaying system with antenna correlation can be lower bounded by

$$
\begin{aligned}
P_{out}^{l}(\gamma_{th}) = 1- & \underbrace{\left[\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{k=0}^{j-1}\frac{\omega_{1,i,j}}{k!}\left(\frac{\bar{p}_s+\bar{p}_r}{\bar{p}_s\bar{p}_r\phi_{1,i}}\gamma_{th}\right)^{k}e^{-\frac{(\bar{p}_s+\bar{p}_r)\gamma_{th}}{\bar{p}_s\bar{p}_r\phi_{1,i}}}\right]}_{J_1} \\
& \times\underbrace{\left[\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1}\frac{\omega_{2,l,m}}{n!}\left(\frac{\bar{p}_s+\bar{p}_r}{\bar{p}_s\bar{p}_r\phi_{2,l}}\gamma_{th}\right)^{n}e^{-\frac{(\bar{p}_s+\bar{p}_r)\gamma_{th}}{\bar{p}_s\bar{p}_r\phi_{2,l}}}\right]}_{J_2}
\end{aligned}
\tag{14}
$$

Proof: please see Appendix B.

As illustrated in section 4, the above lower bound tightly correlates with the exact outage probability from medium to high SNR regime. To see the effect of antenna correlation at high SNR, we provide the asymptotic outage probability at high SNR values in the following proposition.

Proposition 1: The asymptotic outage probability at high SNR for the AF two-way relaying system is given by

$$P_{out}^{a}(\gamma_{th}) = c\left(\frac{\bar{p}_s+\bar{p}_r}{\bar{p}_s\bar{p}_r}\gamma_{th}\right)^{G_d} \tag{15}$$

$$G_d = \min\left(N_1, N_2\right) \tag{16}$$

$$c=\begin{cases}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{k=0}^{j-1}\frac{\omega_{1,i,j}\left(-1\right)^{N_1-k+1}}{k!\left(N_1-k\right)!\phi_{1,i}^{N_1}} & for\ N_1<N_2\\ \sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1}\frac{\omega_{2,l,m}\left(-1\right)^{N_2-n+1}}{n!\left(N_2-n\right)!\phi_{2,l}^{N_2}} & for\ N_2<N_1\\ \left[\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{k=0}^{j-1}\frac{\omega_{1,i,j}\left(-1\right)^{N-k+1}}{k!\left(N-k\right)!\phi_{1,i}^{N}}+\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1}\frac{\omega_{2,l,m}\left(-1\right)^{N-n+1}}{n!\left(N-n\right)!\phi_{2,l}^{N}}\right] & for\ N_1=N_2=N\end{cases} \tag{17}$$

Proof: please see Appendix C.

G_d in Proposition 1 denotes the system's diversity order. Proposition 1 suggests that the two-way relaying system with antenna correlation can achieve a diversity order of $\min\left(N_1,N_2\right)$, namely full spatial diversity order. In order to fully exploit the spatial diversity, we are required to mount multiple antennas at both sources. If only a single antenna is mounted at one source, the diversity order of one without spatial diversity gain is obtained.

3.2 Average Symbol Error Rate

Define P_{e1} and P_{e2} as the average SER at $T1$ and $T2$, respectively. The whole system's SER performance can be measured by the maximum of P_{e1} and P_{e2} [30], which is denoted as $P_e=\max\left(P_{e1},P_{e2}\right)$. Therefore, we focus on deriving the expression of P_e in this paper. As it is cumbersome to derive the exact expression of P_e, the asymptotic expression of P_e at high SNR will be derived to get an insight into the system's SER behavior.

Conditioned on the instantaneously received SNR γ, the conditional SER of a linear modulation format under AWGN can be written as $P_s(\gamma)=aQ\left(\sqrt{2b\gamma}\right)$ [28], where $Q(x)=\frac{1}{\sqrt{2\pi}}\int_x^{\infty}e^{-t^2/2}dt$, a and b are modulation type dependent constants. For example, $a=1,b=1$ corresponds to binary phase shift keying (BPSK) modulation, and $a=2,b=\sin^2\pi/M$ corresponds to Mary-phase shift keying (MPSK) modulation for $M\geq 4$. According to the result in [16], the average SER P_e can be calculated by

$$P_e=\frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^{\infty}\frac{e^{-b\gamma_{\min}}}{\sqrt{\gamma_{\min}}}F\left(\gamma_{\min}\right)d\gamma_{\min} \tag{18}$$

where $F\left(\gamma_{\min}\right)$ denotes the culmulative distribution function (CDF) of instantaneous SNR $\gamma_{\min}=\min\left(\gamma_1,\gamma_2\right)$.

It is seen from (18) that, to obtain the asymptotic average SER expression, we are required to obtain the approximate CDF of $\gamma_{\min}$ at high SNR. From the derivation of asymptotic outage probability (15), we get the approximate CDF of $\gamma_{\min}$ at high SNR as

$$F^{\infty}(\gamma_{\min}) = c\left(\frac{\bar{p}_s+\bar{p}_r}{\bar{p}_s\bar{p}_r}\gamma_{\min}\right)^{G_d} \tag{19}$$

Substituting (19) into (18) and solving the resultant integral by using the identity Eq. (3.381.4) in [31], the asymptotic average SER expression is given by

$$P_e = \frac{ac\sqrt{b}}{2\sqrt{\pi}}\left(\frac{\bar{p}_s+\bar{p}_r}{\bar{p}_s\bar{p}_r}\right)^{G_d}\frac{\Gamma(G_d+0.5)}{b^{G_d+0.5}} \tag{20}$$

where $\Gamma(\bullet)$ is the gamma function, and is defined as $\Gamma(z)=\int_0^{\infty}e^{-t}t^{z-1}dt$ ([31], Eq.(8.310.1)). According to the results in [32], the average SER at high SNR can be represented as $P_e=\left[G_c\bar{\gamma}\right]^{-G_d}$, where G_c is referred to as coding gain and $\bar{\gamma}$ denotes the average SNR. Without loss of generality, we assume $\bar{\gamma}=\bar{p}_s$ and then obtain the coding gain from (20) as

$$G_c = \left(\frac{\alpha}{1+\alpha}\right)\left(\frac{ac\sqrt{b}}{2\sqrt{\pi}}\frac{\Gamma(t+0.5)}{b^{G_d+0.5}}\right)^{-1/G_d} \tag{21}$$

where α is defined as $\alpha=\bar{p}_r/\bar{p}_s$.

3.3 Average Sum-Rate

Average sum-rate is an important performance measure reflecting the system's throughput, and is defined [14] as

$$R = \underbrace{E\left[\frac{1}{2}\log_2(1+\gamma_1)\right]}_{R_1}+\underbrace{E\left[\frac{1}{2}\log_2(1+\gamma_2)\right]}_{R_2} \tag{22}$$

Since it is difficult to derive the exact average sum-rate expression, we provide a closed-form lower bound in the following theorem.

Theorem 3: The average sum-rate for the AF two-way relaying system with antenna correlation is lower bounded by

$$R^l = R_1^l + R_2^l \tag{23}$$

$$\begin{aligned}
R_1^l =& \frac{1}{2}\log_2\left(\frac{\bar{p}_s}{\bar{p}_s+\bar{p}_r}\right)+\frac{1}{2\ln 2}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\frac{\omega_{2,l,m}}{(m-1)!}\hat{R}_{m-1}\left(\bar{p}_r\phi_{2,l}\right)\\
&+\frac{1}{2\ln 2}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{s=0}^{m-1}\left[\begin{array}{l}\frac{\omega_{1,i,j}\omega_{2,l,m}(-1)^{m-s-2}}{(m-s-1)!(j-1)!}\left(\frac{(\bar{p}_s+\bar{p}_r)\phi_{1,i}}{\bar{p}_r\phi_{2,l}}\right)^{m-s-1}\frac{\Gamma(m+j-s-1)}{(m+j-s-1)}\\ \times {}_2F_1\left(1,m+j-s-1;m+j-s;1-\frac{(\bar{p}_s+\bar{p}_r)\phi_{1,i}}{\bar{p}_r\phi_{2,l}}\right)\end{array}\right]\\
&-\frac{1}{2\ln 2}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{s=0}^{m-1}\sum_{k=1}^{m-s-1}\frac{\omega_{2,l,m}\omega_{1,i,j}(k-1)!}{(m-s-1)!(j-1)!}\left(-\frac{(\bar{p}_s+\bar{p}_r)\phi_{1,i}}{\bar{p}_r\phi_{2,l}}\right)^{m-s-k-1}\Gamma(m+j-s-k)
\end{aligned} \tag{24}$$

$$R_2^l = \frac{1}{2}\log_2\left(\frac{\bar{p}_s}{\bar{p}_s + \bar{p}_r}\right) + \frac{1}{2\ln 2}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\frac{\omega_{1,i,j}}{(j-1)!}\hat{R}_{j-1}\left(\bar{p}_r\phi_{1,i}\right)$$
$$+\frac{1}{2\ln 2}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{s=0}^{j-1}\left[\begin{array}{l}\frac{\omega_{1,i,j}\omega_{2,l,m}(-1)^{j-s-2}}{(j-s-1)!(m-1)!}\left(\frac{(\bar{p}_s+\bar{p}_r)\phi_{2,l}}{\bar{p}_r\phi_{1,i}}\right)^{j-s-1}\frac{\Gamma(m+j-s-1)}{(m+j-s-1)} \\ \times {}_2F_1\left(1, m+j-s-1; m+j-s; 1-\frac{(\bar{p}_s+\bar{p}_r)\phi_{2,l}}{\bar{p}_r\phi_{1,i}}\right)\end{array}\right] \tag{25}$$
$$-\frac{1}{2\ln 2}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{s=0}^{j-1}\sum_{k=1}^{j-s-1}\frac{\omega_{2,l,m}\omega_{1,i,j}(k-1)!}{(j-s-1)!(m-1)!}\left(-\frac{(\bar{p}_s+\bar{p}_r)\phi_{2,l}}{\bar{p}_r\phi_{1,i}}\right)^{j-s-k-1}\Gamma(m+j-s-k)$$

where the function $\hat{R}_\xi(a)$ is defined as ([31], Eq. (4.337.5))

$$\hat{R}_\xi(a) = \sum_{\mu=0}^{\xi}\frac{\xi!}{(\xi-\mu)!}\left[(-1)^{\xi-\mu-1}(1/a)^{\xi-\mu}e^{1/a}Ei(-1/a) + \sum_{k=1}^{\xi-\mu}(k-1)!(-1/a)^{\xi-\mu-k}\right]$$

and ${}_pF_q(a_1,\cdots a_p; b_1,\cdots b_q; x)$ is the generalized, hypergeometric function ([31], Eq. (9.14.1)).

Proof: please see Appendix D.

4. Simulation Results

In this section, Monte Carlo simulations are conducted to verify our analytical results. As our analytical expressions are valid for arbitrary correlation models, the correlation matrices $\mathbf{\Phi}_1$ and $\mathbf{\Phi}_2$ are constructed using the practical channel model in [33]. The correlation matrix $\mathbf{\Phi}_i, i=1,2$ is determined by the relative antenna spacing with respect to the carrier wavelength, the angles of arrival (AOA) and angles of departure (AOD). The $(p,q)th$ element of $\mathbf{\Phi}_i, i=1,2$ is given by [33]

$$(\mathbf{\Phi}_i)_{p,q} = e^{-j2\pi(q-p)d_i\cos(\bar{\theta}_i)}e^{-0.5\left[2\pi(q-p)d_i\sin(\bar{\theta}_i)\sigma_i\right]^2}$$

in which d_i denotes the relative antenna spacing between adjacent antennas at T_i, $\bar{\theta}_i$ and σ_i^2 denote the mean angle and angle spread of AOA/AOD at T_i, respectively. Note that the mean angles and angle spreads of AOA and AOD at T_i have the same value, because the channels are reciprocal. For simplicity, we assume that all the nodes in the system have the same transmitting power, namely $p_s = p_r = p$. The SNRs in all figures are defined as p/N_0 and the threshold γ_{th} in all simulations is set to 3.

Fig. 2 illustrates how fast $\psi_1(x)$ and $\psi_2(x)$ in theorem 1 converge to zero.

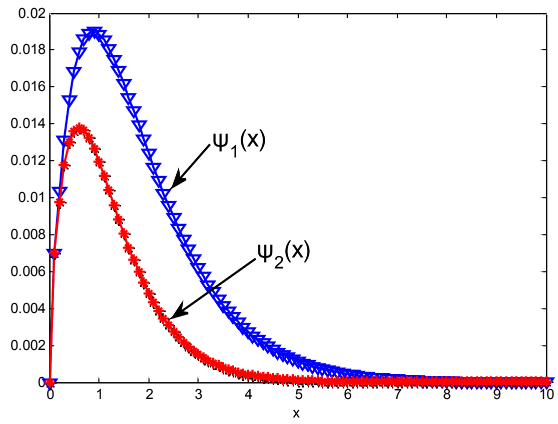

Fig. 2 The convergence of $\psi_1(x)$ and $\psi_2(x)$

The parameters corresponding to **Fig. 2** are as follows: SNR equals to 20dB, $\phi_{1,i}$ and $\phi_{2,l}$ are randomly generated in MATLAB, $j=2, n=1$ for $\psi_1(x)$ and $m=2, k=1$ for $\psi_2(x)$. It is obvious that $\psi_1(x)$ and $\psi_2(x)$ approach zero quite quickly as x increases, which indicates that the exact outage probability in Theorem 1 can be numerically calculated by truncating the integration range in practice.

Fig. 3 plots the outage probability curves for various antenna configurations. The exact outage probability curves are obtained by truncating the integration range in theorem 1 to a fixed value, beyond which there is no change of the integration results. As can clearly be seen from this figure, the exact curves agree excellently with the simulation results. We also plot the lower bound (14) in this figure, which is shown to be tight from medium to high SNR regime. As expected, the outage probability significantly decreases as the number of antenna increases, which indicates the benefits of employing multiple antennas in a two-way relaying system.

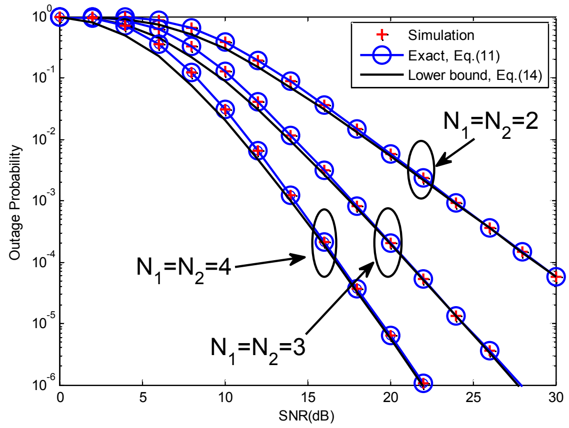

Fig. 3. Outage probability for various antenna configurations with

$$\sigma_1^2 = \sigma_2^2 = \pi/10,\ d_1 = d_2 = 1/2 \text{ and } \bar{\theta}_1 = \bar{\theta}_2 = \pi/2$$

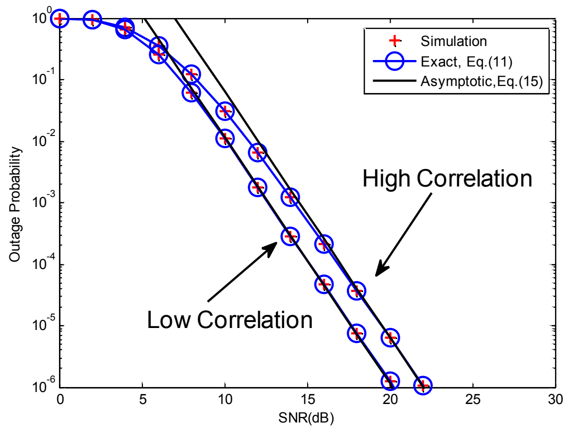

Fig. 4. Outage probability for low correlation ($\sigma_1^2 = \sigma_2^2 = \pi/5$) and high correlation ($\sigma_1^2 = \sigma_2^2 = \pi/10$) with $d_1 = d_2 = 1/2$, $\bar{\theta}_1 = \bar{\theta}_2 = \pi/2$ and $N_1 = N_2 = 4$

Fig. 4 plots the outage probability for low and high correlation scenarios. It is observed that the asymptotic outage probability (15) approaches the simulated curves at high SNR, which validates the correctness of (15). As SNR increases, the curves corresponding to high correlation decrease at the same speed as those corresponding to low correlation, which demonstrates that antenna correlation does not affect the decreasing speed of outage probability at high SNR. In addition, it is observed that with the correlation increasing, the outage probability increases.

Fig. 5 illustrates the decreasing speeds of outage probability for various antenna configurations. Comparing the asymptotic curve corresponding to $N_1 = 4, N_2 = 3$ with that corresponding to $N_1 = 3, N_2 = 3$, we find that both curves demonstrate the same decreasing speed. This figure also shows that the curves corresponding to $N_1 = 1, N_2 = 2$ have lower decreasing speeds than those corresponding to $N_1 = 2, N_2 = 2$. The above observations illustrate that the system's outage performance is limited by the sources with less antennas, which is also revealed from (16).

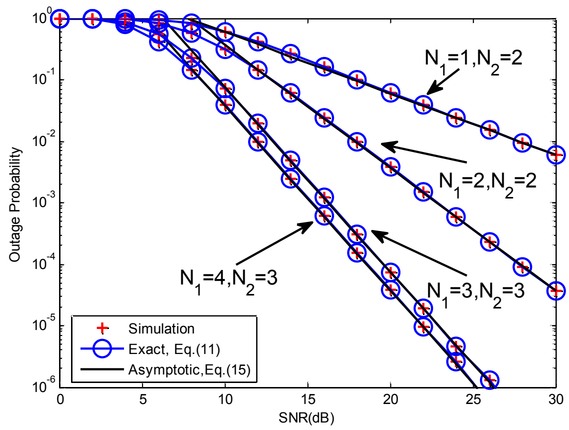

Fig. 5. Outage probability with $d_1 = d_2 = 1/2$, $\bar{\theta}_1 = \bar{\theta}_2 = \pi/2$ and $\sigma_1^2 = \sigma_2^2 = \pi/5$

Fig. 6 plots the average SER curves for quadrature phase shift keying (QPSK) modulation under various scenarios. In this simulation, where we assume $\sigma_1^2 = \sigma_2^2 = \sigma^2$, the asymptotic curves approximate the simulated curves quite well at high SNR, which verifies the accuracy of (20). Comparison of the curves corresponding to $N_1 = 3, N_2 = 3, \sigma^2 = \pi/5$ with those corresponding to $N_1 = 3, N_2 = 3, \sigma^2 = \pi/10$, shows that the increase in antenna correlation degrades the average SER performance. This demonstrates that antenna correlation degrades the coding gain. Besides, as the minimum number of antennas at $T1$ and $T2$ increases from 3 to 4, the SER decreasing speed increases. This phenomenon can be explained by the diversity order G_d in (20). We also find that the curves corresponding to $N_1 = 3, N_2 = 4, \sigma^2 = \pi/10$ have better SER performance than those corresponding to $N_1 = 3, N_2 = 3, \sigma^2 = \pi/10$, which illustrates that although mounting one extra antenna at $T2$ has no benefit for diversity order, it does provide a higher coding gain.

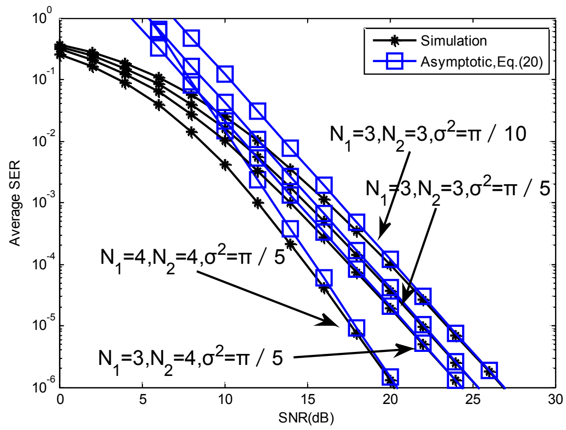

Fig. 6. Average SER for various scenarios with $d_1 = d_2 = 1/2$ and $\bar{\theta}_1 = \bar{\theta}_2 = \pi/2$

In **Fig. 7**, the average sum-rate is plotted for various antenna configurations. It shows that the curves corresponding to (23) bound the simulated curves quite tightly from medium to high SNR, which verifies the accuracy of (23). As the number of antennas at either $T1$ or $T2$ increases, the average sum-rate increases. This indicates that mounting multiple antennas can enhance the system's throughput.

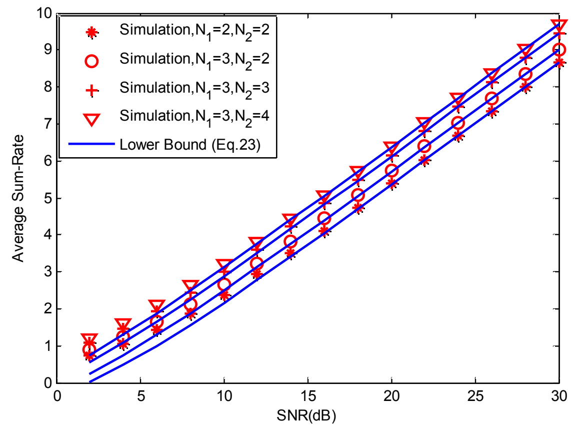

Fig. 7. Average sum-rate for various antenna configurations with

$$d_1 = d_2 = 1/2,\ \overline{\theta}_1 = \overline{\theta}_2 = \pi/2 \text{ and } \sigma_1^2 = \sigma_2^2 = \pi/10$$

Fig. 8 compares the average sum-rate for various antenna correlation scenarios. For simplicity, we assume $\sigma_1^2 = \sigma_2^2 = \sigma^2$. The curves at high SNR show that as the antenna correlation increases, i.e., σ^2 increases from $\pi/10$ to $\pi/30$, the average sum-rate decreases. This phenomenon indicates that antenna correlation not only degrades the system's transmission reliability (outage probability and average SER, seen from **Fig. 4** and **Fig. 6**), but also degrades the throughput (average sum-rate). As a comparison benchmark, the simulated average sum-rate curve with $N_1 = N_2 = 3, \sigma^2 = \pi/10$ is also plotted. The curves corresponding to $N_1 = N_2 = 4, \sigma^2 = \pi/30$ almost reach the benchmark curve, which demonstrates that the advantage of mounting multiple antennas diminishes when high antenna correlation exists.

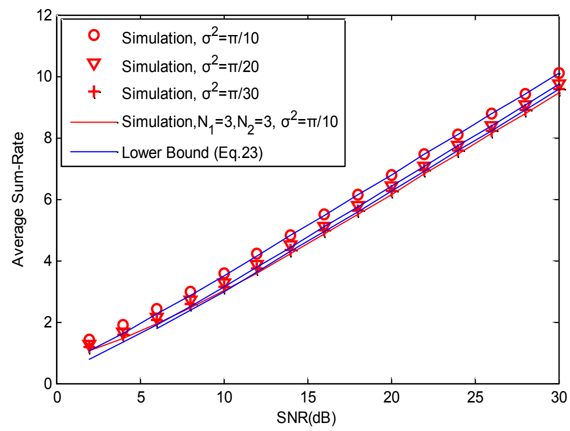

Fig. 8. Average sum-rate for different antenna correlation scenarios with

$$d_1 = d_2 = 1/2,\ \overline{\theta}_1 = \overline{\theta}_2 = \pi/2 \text{ and } N_1 = N_2 = 4$$

5. Conclusions

In this paper, we have analyzed the performance of an AF two-way relaying system, by taking into account the antenna correlation. Specifically, we considered a system consisting of two sources with multiple antennas and a relay with a single antenna. The exact outage probability expression was derived first. To get better insight, we provided a simple closed-form, lower bound for the outage probability, which was shown to be quite tight from medium to high SNR regime. To obtain further insight into the system's behavior at high SNR, the asymptotic outage probability and average SER expressions were derived in closed form. To quantify the effect of antenna correlation on the system's throughput, we also derived a tight closed-form, lower bound for the average sum-rate. Simulation results were presented to verify our derived expressions. Both analytical and simulation results have shown that antenna correlation has a detrimental effect on the outage probability, average SER and average sum-rate. Through the analysis of outage probability and average SER at high SNR, it was also revealed that full diversity order can be achieved even with antenna correlation. Since antenna correlation occurs in practice due to insufficient antenna spacing, our analytical expressions enabled us to achieve greater insight into the system's transmission reliability and throughput with antenna correlation, and provided the guidelines for system design in practice. Furthermore, our analytical results could be utilized as a theoretical basis in system optimization, for example, power allocation with the goal of minimizing outage probability.

Appendix

Appendix A: Proof of Theorem 1

First, we derive the expression of $P_1(\gamma_{th})$ .Substituting (5) into $P_1(\gamma_{th}) = \Pr\left[\gamma_1 < \gamma_{th}, \gamma_1 < \gamma_2\right]$, and after some elementary manipulations, we obtain

$$P_1(\gamma_{th}) = \int_0^{\delta} f_{z_1}(z_1)dz_1 \int_0^{z_1} f_{z_2}(z_2)dz_2 + \int_{\delta}^{\infty} f_{z_1}(z_1)dz_1 \int_0^{\frac{(\bar{p}_r+\bar{p}_s)\gamma_{th}z_1+\gamma_{th}}{\bar{p}_s\bar{p}_r z_1-\bar{p}_s\gamma_{th}}} f_{z_2}(z_2)dz_2 \quad (26)$$

where δ is derived by solving the inequality $z_1 \le \frac{(\bar{p}_r+\bar{p}_s)\gamma_{th}z_1+\gamma_{th}}{\bar{p}_s\bar{p}_r z_1-\bar{p}_s\gamma_{th}}$, and is given by $\delta = \left[(2\bar{p}_s+\bar{p}_r)\gamma + \sqrt{(2\bar{p}_s+\bar{p}_r)^2\gamma^2 + 4\bar{p}_s\bar{p}_r\gamma}\right] \Big/ 2\bar{p}_s\bar{p}_r$. Substituting (9) and (10) into (26) and solving the resultant integral with the help of Eq. (2.321.2) in [31], $P_1(\gamma_{th})$ is expressed as

$$\begin{aligned}
P_1(\gamma_{th}) = {} & 1 - \sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1} \omega_{1,i,j}\omega_{2,l,m} \binom{n+j-1}{n} \frac{\phi_{1,i}^n \phi_{2,l}^j}{\left(\phi_{1,i}+\phi_{2,l}\right)^{n+j}} \\
& + \sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1}\sum_{k=0}^{n+j-1} \omega_{1,i,j}\omega_{2,l,m} \frac{k!}{(j-1)!n!} \binom{n+j-1}{k} \left(\frac{\phi_{1,i}\phi_{2,l}}{\phi_{1,i}+\phi_{2,l}}\right)^{k+1} \frac{1}{\phi_{1,i}^j\phi_{2,l}^n} \delta^{n+j-1-k} e^{-\frac{\phi_{1,i}\phi_{2,l}}{\phi_{1,i}+\phi_{2,l}}\delta} \quad (27) \\
& - \sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1} \omega_{1,i,j}\omega_{2,l,m} \frac{1}{(j-1)!n!} \frac{1}{\phi_{1,i}^j\phi_{2,l}^n} \int_{\delta}^{\infty} x^{j-1} \left[\frac{\left(\bar{p}_s+\bar{p}_r\right)\gamma_{th}x+\gamma_{th}}{\bar{p}_s\bar{p}_r x-\bar{p}_s\gamma_{th}}\right]^n e^{-\frac{x}{\phi_{1,i}}-\frac{(\bar{p}_s+\bar{p}_r)\gamma_{th}x+\gamma_{th}}{\phi_{2,l}(\bar{p}_s\bar{p}_r x-\bar{p}_s\gamma_{th})}} dx
\end{aligned}$$

Following the same steps taken in (26-27), we get the expression of $P_2(\gamma_{th})$, as given in (13)■

Appendix B: Proof of Theorem 2

The instantaneous SNRs γ_1 and γ_2 in (5) can be tightly upper bounded by [34]

$$\begin{aligned} \gamma_1 &= \frac{\bar{p}_s \bar{p}_r z_1}{(\bar{p}_s + \bar{p}_r) z_1 + \bar{p}_r z_2} \\ \gamma_2 &= \frac{\bar{p}_s \bar{p}_r z_1 z_2}{(\bar{p}_s + \bar{p}_r) z_1 + \bar{p}_r z_2} \end{aligned} \tag{28}$$

By using the inequality $xy/(x+y) \le \min(x,y)$ [35], the instantaneous SNRs of (28) can be upper bounded by

$$\begin{aligned} \gamma_1 &\le \min\left[\bar{p}_s z_1, \bar{p}_s \bar{p}_r z_2/(\bar{p}_s + \bar{p}_r)\right] \\ \gamma_2 &\le \min\left[\bar{p}_s z_2, \bar{p}_s \bar{p}_r z_1/(\bar{p}_s + \bar{p}_r)\right] \end{aligned} \tag{29}$$

Substituting (29) into the definition of outage probability $P_{out} = \Pr\left[\min(\gamma_1, \gamma_2) < \gamma_{th}\right]$, the outage probability can be lower bounded by

$$\begin{aligned} P_{out} &= 1 - \Pr\left[\min(\gamma_1, \gamma_2) \ge \gamma_{th}\right] \\ &\ge 1 - \Pr\left[\min\left[\bar{p}_s z_1, \bar{p}_s \bar{p}_r z_2/(\bar{p}_s + \bar{p}_r), \bar{p}_s z_2, \bar{p}_s \bar{p}_r z_1/(\bar{p}_s + \bar{p}_r)\right] \ge \gamma_{th}\right] \quad (a) \\ &\ge 1 - \Pr\left[\bar{p}_s \bar{p}_r z_2/(\bar{p}_s + \bar{p}_r) \ge \gamma_{th}, \bar{p}_s \bar{p}_r z_1/(\bar{p}_s + \bar{p}_r) \ge \gamma_{th}\right] \quad (b) \\ &\ge 1 - \Pr\left[\bar{p}_s \bar{p}_r z_1/(\bar{p}_s + \bar{p}_r) \ge \gamma_{th}\right] \Pr\left[\bar{p}_s \bar{p}_r z_2/(\bar{p}_s + \bar{p}_r) \ge \gamma_{th}\right] \quad (c) \end{aligned} \tag{30}$$

Step (b) in (30) holds because $\bar{p}_r/(\bar{p}_s + \bar{p}_r) \le 1$, and step (c) in (30) holds because z_1 and z_2 are independent. Substituting (9) and (10) into (30), solving by using Eq. (2.321.2) in [31] and making elementary manipulations, yields the lower bound of outage probability, as presented in (14).■

Appendix C: Proof of Proposition 1

The proof of Proposition 1 is based on the outage probability's lower bound, as given in (14). Let $x = \frac{\bar{p}_s + \bar{p}_r}{\bar{p}_s \bar{p}_r} \gamma_{th}$. As the transmitting power approaches infinity, x approaches zero. Using Taylor series expansion, J_1 in (14) can be expanded as

$$J_1(x) = \sum_{i=1}^{t_1} \sum_{j=1}^{u_{1,i}} \sum_{k=0}^{j-1} \sum_{r=0}^{N_1-1-k} \frac{\omega_{1,i,j} (-1)^r}{k! r!} \left(\frac{x}{\phi_{1,i}}\right)^{k+r} + \sum_{i=1}^{t_1} \sum_{j=1}^{u_{1,i}} \sum_{k=0}^{j-1} \sum_{r=N_1-k}^{\infty} \frac{\omega_{1,i,j} (-1)^r}{k! r!} \left(\frac{x}{\phi_{1,i}}\right)^{k+r} \tag{31}$$

Substituting the $\omega_{1,i,j}$ values into (31) and expanding the summation, the first term in (31) sums to one [18]. Thus, (31) can be expressed as

$$J_1(x) = 1 - \sum_{i=1}^{t_1} \sum_{j=1}^{u_{1,i}} \sum_{k=0}^{j-1} \frac{\omega_{1,i,j} (-1)^{N_1-k+1}}{k!(N_1-k)!} \left(\frac{x}{\phi_{1,i}}\right)^{N_1} + o(x^{N_1+1}) \tag{32}$$

Following the same approach, J_2 in (14) can be expanded as

$$J_2(x) = 1 - \sum_{l=1}^{t_2} \sum_{m=1}^{u_{2,l}} \sum_{n=0}^{m-1} \frac{\omega_{2,l,m} (-1)^{N_2-n+1}}{n!(N_2-n)!} \left(\frac{x}{\phi_{2,i}}\right)^{N_2} + o(x^{N_2+1}) \tag{33}$$

Substituting (32) and (33) into (14), we have

$$P_{out}^{l}(\gamma_{th}) = \sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{k=0}^{j-1}\frac{\omega_{1,i,j}\left(-1\right)^{N_1-k+1}}{k!\left(N_1-k\right)!}\left(\frac{x}{\phi_{1,i}}\right)^{N_1} + \sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1}\frac{\omega_{2,l,m}\left(-1\right)^{N_2-n+1}}{n!\left(N_2-n\right)!}\left(\frac{x}{\phi_{2,i}}\right)^{N_2} + o\left[x^{\min\left(N_1+1,N_2+1\right)}\right] \quad (34)$$

If $N_1 < N_2$, we have

$$P_{out}^{l}(x) = \sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{k=0}^{j-1}\frac{\omega_{1,i,j}\left(-1\right)^{N_1-k+1}}{k!\left(N_1-k\right)!}\left(\frac{x}{\phi_{1,i}}\right)^{N_1} + o(x^{N_1+1}) \quad (35)$$

If $N_1 > N_2$, we have

$$P_{out}^{l}(x) = \sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1}\frac{\omega_{2,l,m}\left(-1\right)^{N_2-n+1}}{n!\left(N_2-n\right)!}\left(\frac{x}{\phi_{2,i}}\right)^{N_2} + o\left(x^{N_2+1}\right) \quad (36)$$

If $N_1 = N_2 = N$, we have

$$P_{out}^{l}(x) = \left[\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{k=0}^{j-1}\frac{\omega_{1,i,j}\left(-1\right)^{N-k+1}}{k!\left(N-k\right)!\phi_{1,i}^{N}} + \sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{n=0}^{m-1}\frac{\omega_{2,l,m}\left(-1\right)^{N-n+1}}{n!\left(N-n\right)!\phi_{2,l}^{N}}\right]x^{N} + o(x^{N}) \quad (37)$$

Substituting the definition of x into (35), (36) and (37), yields the desired result as presented in (15).■

Appendix D: Proof of the Theorem 3

First, we derive the lower bound of R_1. Substituting (5) into (22), R_1 can be written as

$$\begin{aligned} R_1 &= E\left[\frac{1}{2}\log_2\left(1+\frac{\overline{p}_s\overline{p}_r z_1 z_2}{(\overline{p}_s+\overline{p}_r)z_1+\overline{p}_r z_2+1}\right)\right] \\ &= \frac{1}{2\ln 2}\int_0^\infty\int_0^\infty \ln\left(1+\frac{\overline{p}_s\overline{p}_r z_1 z_2}{(\overline{p}_s+\overline{p}_r)z_1+\overline{p}_r z_2+1}\right) f_{z_1}\left(z_1\right) f_{z_2}\left(z_2\right) dz_1 dz_2 \quad (38) \\ &= \frac{1}{2\ln 2}\int_0^\infty\int_0^\infty \left[\ln\left(\frac{\overline{p}_s}{\overline{p}_s+\overline{p}_r}\right)+\ln\left(\frac{\overline{p}_s+\overline{p}_r}{\overline{p}_s}+\frac{\left(\overline{p}_s+\overline{p}_r\right)z_1\overline{p}_r z_2}{(\overline{p}_s+\overline{p}_r)z_1+\overline{p}_r z_2+1}\right)\right] f_{z_1}\left(z_1\right) f_{z_2}\left(z_2\right) dz_1 dz_2 \end{aligned}$$

Since $\ln\left(x\right)$ is a monotonically increasing function and $\left(\overline{p}_s+\overline{p}_r\right)/\overline{p}_r \geq 1$, then (38) can be lower bounded by

$$\begin{aligned} R_1 &\geq \frac{1}{2}\log_2\left(\frac{\overline{p}_s}{\overline{p}_s+\overline{p}_r}\right)+\frac{1}{2\ln 2}\int_0^\infty\int_0^\infty \ln\left(1+\frac{\left(\overline{p}_s+\overline{p}_r\right)z_1\overline{p}_r z_2}{(\overline{p}_s+\overline{p}_r)z_1+\overline{p}_r z_2+1}\right) f_{z_1}\left(z_1\right) f_{z_2}\left(z_2\right) dz_1 dz_2 \\ &= \frac{1}{2}\log_2\left(\frac{\overline{p}_s}{\overline{p}_s+\overline{p}_r}\right)+\underbrace{\frac{1}{2\ln 2}\int_0^\infty\int_0^\infty \ln\left(1+\overline{p}_r z_2\right) f_{z_1}\left(z_1\right) f_{z_2}\left(z_2\right) dz_1 dz_2}_{K_1} \quad (39) \\ &\quad \underbrace{-\frac{1}{2\ln 2}\int_0^\infty\int_0^\infty \ln\left(1+\frac{\overline{p}_r z_2}{(\overline{p}_s+\overline{p}_r)z_1+1}\right) f_{z_1}\left(z_1\right) f_{z_2}\left(z_2\right) dz_1 dz_2}_{K_2} \end{aligned}$$

Substituting (9) and (10) into K_1 in (39) and integrating with the help of Eq. (4.337.5) in [31], K_1 can be expressed as

$$K_1 = \frac{1}{2\ln 2}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}} \frac{\omega_{2,l,m}}{(m-1)!}\hat{R}_{m-1}\left(\bar{p}_r\phi_{2,l}\right) \tag{40}$$

The definition of $\hat{R}_{\xi}(a)$ has been written in Theorem 3. As $\frac{\bar{p}_r z_2}{(\bar{p}_s+\bar{p}_r)z_1+1} > \frac{\bar{p}_r z_2}{(\bar{p}_s+\bar{p}_r)z_1}$ and $\ln(x)$ is a monotonically increasing function, K_2 in (39) can be lower bounded by

$$\begin{aligned} K_2 &> -\frac{1}{2\ln 2}\int_0^\infty\int_0^\infty \ln\left(1+\frac{\bar{p}_r z_2}{(\bar{p}_s+\bar{p}_r)z_1}\right) f_{z_1}(z_1) f_{z_2}(z_2)\,dz_1 dz_2 \\ &= \frac{-1}{2\ln 2}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\frac{\omega_{2,l,m}}{(m-1)!}\int_0^\infty \hat{R}_{m-1}\left(\frac{\bar{p}_r\phi_{2,l}}{(\bar{p}_s+\bar{p}_r)z_1}\right) f_{z_1}(z_1)\,dz_1 \\ &= \underbrace{\frac{-1}{2\ln 2}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\frac{\omega_{2,l,m}}{(m-1)!}\sum_{s=0}^{m-1}\frac{(m-1)!}{(m-s-1)!}\left[\int_0^\infty\left((-1)^{m-s-2}\left(\frac{(\bar{p}_s+\bar{p}_r)}{\bar{p}_r\phi_{2,l}}z_1\right)^{m-s-1} e^{\frac{(\bar{p}_s+\bar{p}_r)}{\bar{p}_r\phi_{2,l}}z_1} \times Ei\left(-\frac{(\bar{p}_s+\bar{p}_r)}{\bar{p}_r\phi_{2,l}}z_1\right) f_{z_1}(z_1)\,dz_1\right)\right]}_{K_3} \\ &\quad \underbrace{-\frac{1}{2\ln 2}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\frac{\omega_{2,l,m}}{(m-1)!}\sum_{s=0}^{m-1}\frac{(m-1)!}{(m-s-1)!}\sum_{k=1}^{m-s-1}\int_0^\infty (k-1)!\left(-\frac{(\bar{p}_s+\bar{p}_r)}{\bar{p}_r\phi_{2,l}}z_1\right)^{m-s-k-1} f_{z_1}(z_1)\,dz_1}_{K_4} \end{aligned} \tag{41}$$

Solving the integration in K_3 with the help of Eq. (6.228.2) in [31], yields

$$K_3 = \frac{1}{2\ln 2}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{s=0}^{m-1}\left[\frac{\omega_{1,i,j}\omega_{2,l,m}(-1)^{m-s-2}}{(m-s-1)!(j-1)!}\left(\frac{(\bar{p}_s+\bar{p}_r)\phi_{1,i}}{\bar{p}_r\phi_{2,l}}\right)^{m-s-1}\frac{\Gamma(m+j-s-1)}{(m+j-s-1)} \times {}_2F_1\left(1, m+j-s-1; m+j-s; 1-\frac{(\bar{p}_s+\bar{p}_r)\phi_{1,i}}{\bar{p}_r\phi_{2,l}}\right)\right] \tag{42}$$

Solving the integration in K_4 with the help of Eq. (3.381.4) in [31], yields

$$K_4 = \frac{-1}{2\ln 2}\sum_{l=1}^{t_2}\sum_{m=1}^{u_{2,l}}\sum_{i=1}^{t_1}\sum_{j=1}^{u_{1,i}}\sum_{s=0}^{m-1}\sum_{k=1}^{m-s-1}\frac{\omega_{2,l,m}\omega_{1,i,j}(k-1)!}{(m-s-1)!(j-1)!}\left(-\frac{(\bar{p}_s+\bar{p}_r)\phi_{1,i}}{\bar{p}_r\phi_{2,l}}\right)^{m-s-k-1}\Gamma(m+j-s-k) \tag{43}$$

Adding $\frac{1}{2}\log_2\left(\frac{\bar{p}_s}{\bar{p}_s+\bar{p}_r}\right)$, K_1, K_3 and K_4, gives the lower bound of R_1 as R_1^l as presented in (24). Following the same approach as (38-43), yields the lower bound of R_2 as R_2^l as presented in (25).■

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