A Fair Radio Resource Allocation Algorithm for Uplink of FBMC Based CR Systems

Hosseinali Jamal, Seyed Ali Ghorashi and Seyed Mohammad-Sajad Sadough
Cognitive Telecommunications Research Group,
Department of Electrical Engineering,
Faculty of Electrical and Computer Engineering,
Shahid Beheshti University G.C., Evin 1983963113, Tehran, IRAN.
[E-mail: h.jamal@mail.sbu.ac.ir, a_ghorashi@sbu.ac.ir, s_sadough@sbu.ac.ir]

Received February 13, 2012; revised April 22, 2012; accepted May 16, 2012; published June 25, 2012

Abstract

Spectrum scarcity seems to be the most challenging issue to be solved in new wireless telecommunication services. It is shown that spectrum unavailability is mainly due to spectrum inefficient utilization and inappropriate physical layer execution rather than spectrum shortage. Daily increasing demand for new wireless services with higher data rate and QoS level makes the upgrade of the physical layer modulation techniques inevitable. Orthogonal Frequency Division Multiple Access (OFDMA) which utilizes multicarrier modulation to provide higher data rates with the capability of flexible resource allocation, although has widely been used in current wireless systems and standards, seems not to be the best candidate for cognitive radio systems. Filter Bank based Multi-Carrier (FBMC) is an evolutionary scheme with some advantages over the widely-used OFDM multicarrier technique. In this paper, we focus on the total throughput improvement of a cognitive radio network using FBMC modulation. Along with this modulation scheme, we propose a novel uplink radio resource allocation algorithm in which fairness issue is also considered. Moreover, the average throughput of the proposed FBMC based cognitive radio is compared to a conventional OFDM system in order to illustrate the efficiency of using FBMC in future cognitive radio systems. Simulation results show that in comparison with the state of the art two algorithms (namely, Shaat and Wang) our proposed algorithm achieves higher throughputs and a better fairness for cognitive radio applications.

Keywords: Cognitive radio, OFDM, FBMC, fairness, resource allocation, throughput

---

This work is supported by Iranian Education and Research Institute for Information and Communication Technology (ERICT) under grant number 500/6487.

http://dx.doi.org/10.3837/tiis.2012.06.001

1. Introduction

Cognitive radio (CR) is an emerging technology that helps to utilize the unused parts of the spectrum in a more efficient manner. More precisely, it aims to increase the spectrum efficiency by allowing a number of secondary users to use the spectrum holes of licensed frequency channels without causing a harmful interference to the licensed users [1][2]. Multicarrier communication techniques have been considered as a promising candidate for cognitive radio systems as the granules of resources in such systems are much more than single carrier ones [3]. Orthogonal frequency-division multiplexing (OFDM) as a well-known multicarrier system has been suggested as a candidate for CR systems. However, OFDM-based CR systems can introduce considerable amount of interference to the primary users (PU's) due to large sidelobes of its filter frequency response. Moreover, it is well known that the insertion of the cyclic prefix (CP) at the end of each OFDM symbol in order to combat channel delays caused by multipath fading, decreases the system capacity.

Recently, Filter Bank based Multicarrier (FBMC) systems are proposed for CR systems [4], [5]. It is not necessary to insert CP extension into the FBMC block and in comparison with cyclic prefix based OFDM system it shows higher robustness against frequency offsets. The most important advantage of using FBMC-based systems in CR networks is the low spectral leakage of their modulation prototype filters [6]. FBMC systems are based on offset quadrature amplitude modulation (OQAM) and in comparison with OFDM-based systems they achieve smaller inter-symbol interferences (ISI) as well as Intercarrier interferences (ICI), by utilizing well designed pulse shapes that satisfy the perfect reconstruction conditions [6]. The problem of uplink resource allocation has already been studied in the literature for OFDMA-based CR systems [7][8][9][10][11], without, however, considering fairness issues. The main idea of these works is a joint subcarrier and power allocation for cognitive users in order to minimize the overall computational operations. In [12], a multistep algorithm is introduced in order to allocate subcarrier, power, and rate for secondary users with fairness considerations in the uplink of an OFDMA system. In this algorithm, a fairness consideration is proposed to maximize the sum rate under individual rate and transmit power constraints. This algorithm considers a rate constraint for each user in order to level up each secondary user's data rate. In [13], a two-step algorithm for resource allocation of an OFDMA based system is introduced. In the first step, subcarriers are allocated to each user, based on the channel quality and the amount of interference induced to the primary channels. Then in the second step, the available power is allocated to each assigned subcarrier. In this algorithm the fairness issue is considered in the subcarrier allocation procedure in which each secondary user has a data rate above a minimum specified threshold. Then power is allocated to each subcarrier by considering the interference imposed to each primary user. In [14], subcarrier and power-allocation schemes considering fairness issues in a cooperative orthogonal frequency-division multiple-access (OFDMA) uplink system are investigated.

In this paper, we propose an efficient resource allocation algorithm with fairness consideration in FBMC-based and multicarrier-based CR systems. The objective is to maximize the total CR systems throughput while limiting the interference introduced to the primary system and to maximize the minimum data rate of each user in order to achieve an efficient fairness. The efficiency of the proposed algorithm is compared to OFDM based CR systems.

The rest of this paper is organized as follows. The problem formulation and the system model of the FBMC uplink is described in Section II. The proposed subcarrier allocation algorithm with a fair rate allocation is discussed and proposed in Section III. In Section IV, simulation results are provided and the computational complexity of the proposed algorithm is compared to existing algorithms and finally Section V summarizes the paper.

2. System Model and Problem Formulation

In this paper, we consider and simulate an uplink CR system with N subcarriers based on a model shown in **Fig. 1.** In this model, we consider just one primary user having a bandwidth equal to B Hz. The bandwidth of each subcarrier is Δf Hz. Assume a base station which provides CR communication for a number of M secondary users and also assume that there is only one pair of primary users including a transmitter and a receiver. The base station continuously senses the uplink channel occupancy condition of all subcarriers for each secondary user. We also consider a multipath fading channel between transmitters and receivers.

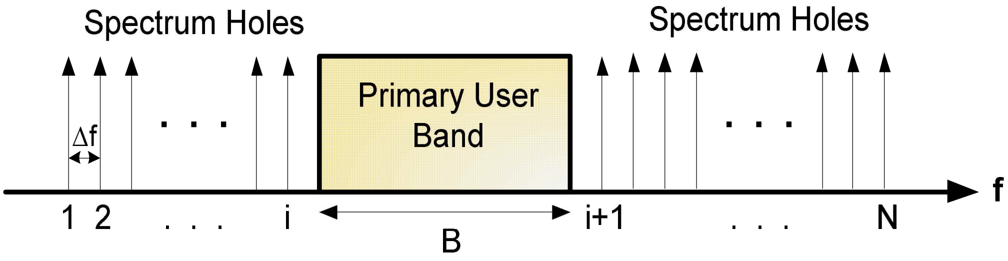

Fig. 1. Frequency distribution of the proposed model

Fig. 2 illustrates a view of the operation of CR base station which works in a given geographic area in which spectrum holes are available.

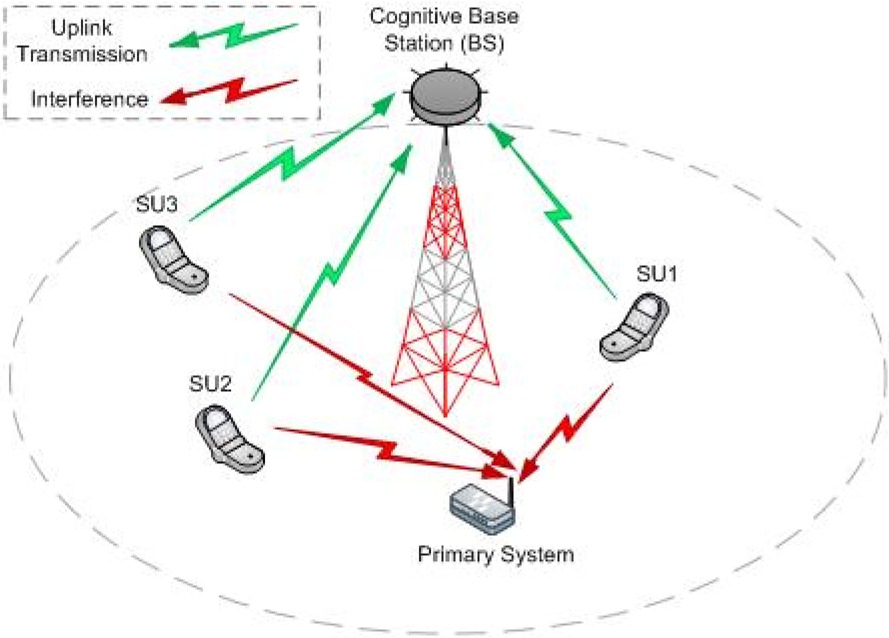

Fig. 2. The uplink of a typical CR network, interferences between secondary and primary users are illustrated

We cannot use primary bands until we find it as a frequency hole. Efficient spectrum sensing techniques can find out that if there is any spectrum hole or not [15][16][17]. Different spectrum sensing algorithms for cognitive radio applications are reviewed and compared in [15]. In [16] some high performance spectrum sensing methods based on different types of filter banks in cognitive radio systems are proposed. In [17] another spectrum sensing method based on a filter bank method is proposed. This method uses overlap FFT processing and energy detection method to estimate spectrum holes in a cognitive radio system.

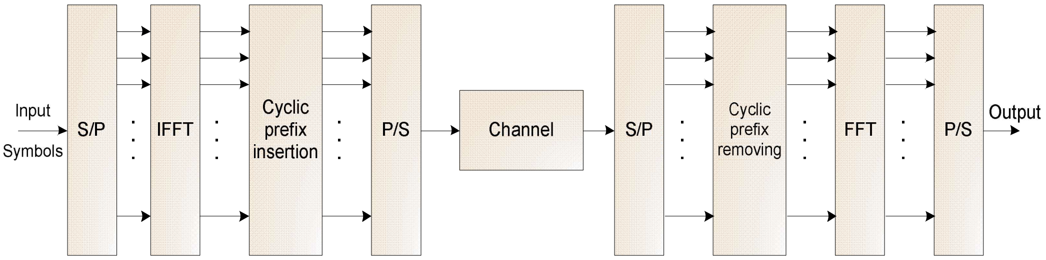

Fig. 3. Block diagram of OFDM-based systems

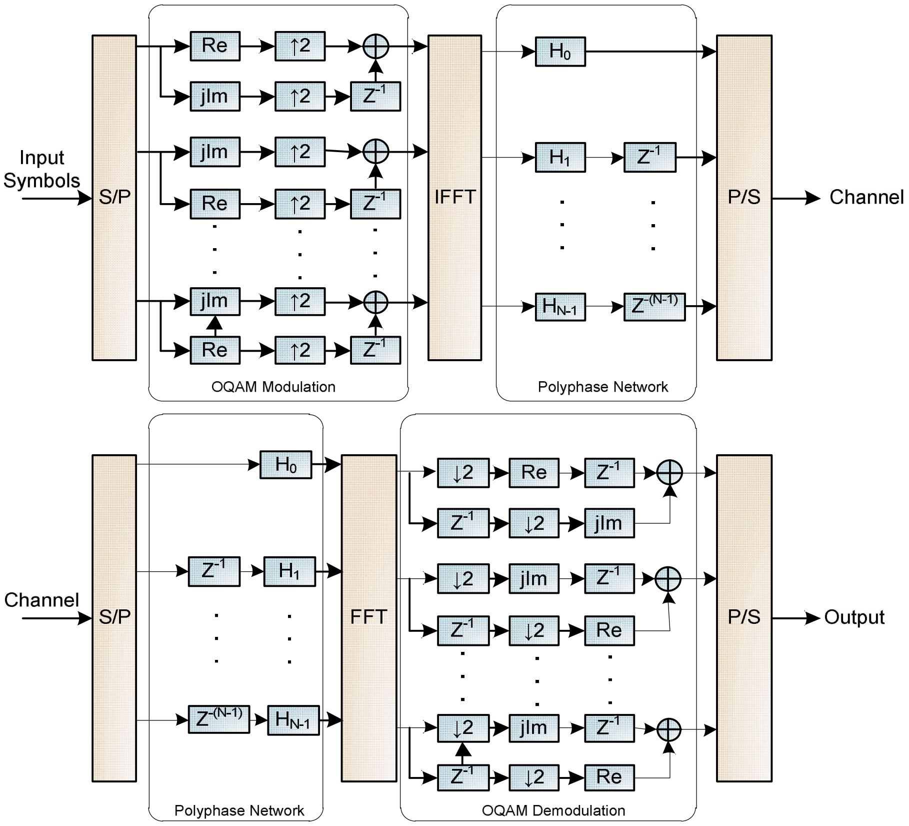

Fig. 4. block diagram of FBMC-based systems, transmitter and receiver

CR system can use free primary bands provided that the total interference introduced to the primary system does not exceed I_{th}, which denotes the maximum interference power that can be tolerated by the primary system. This comes from $I_{th} = T_{th}B$ where T_{th} is the interference temperature threshold for the primary user system and B is the primary user's total bandwidth. According to OFDM system block diagram that is shown in **Fig. 3**, in transmitter and receiver side we have two blocks for IFFT and FFT. There are also two blocks for cyclic prefix insertion and removing, but as it is shown in **Fig. 4**, In FBMC systems, instead of cyclic prefix which is needed in OFDM symbols, we have a Polyphase network block which eases the overall computational complexity in the FBMC system. There is also an OQAM block in FBMC system which converts complex input symbols into real ones (pre-processing OQAM).

The basic idea of this conversion is to transmit real symbols instead of complex ones through the radio channel. Orthogonality is achieved between adjacent subcarriers due to this time staggering [19]. The basic idea for using filter bank in multicarrier systems is due to its sharp power spectral density (PSD) figure. PSD side lobes of FBMC are negligible due to well-designed filters. It is well known that OFDM with rectangular filtering has large side lobes in comparison with FBMC and it produces more interference to adjacent subcarriers. In **Fig. 5** we can see the power spectral density of one subcarrier in OFDM and FBMC systems.

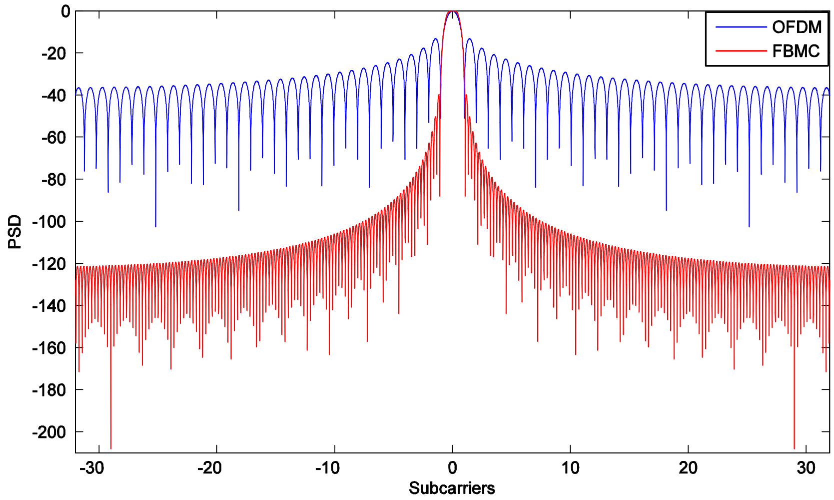

Fig. 5. Power spectral density of one subcarrier in OFDM and FBMC systems

We can see that main-lobe in the PSD of subcarrier in FBMC system is as wide as main-lobe in the PSD of the same subcarrier in the same OFDM system and its side lobes are much lower than side lobes of the same subcarrier in the OFDM system.

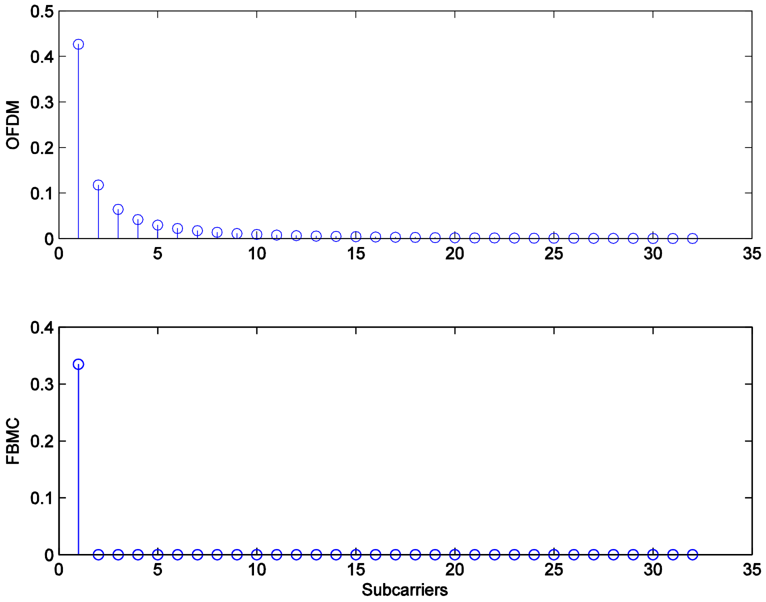

Fig. 6. Interference distribution of subcarriers in the SUs of OFDM and FBMC systems to the neighbor primary user subcarriers.

According to interference table provided in [20] which is later expounded in [21] for FBMC, each OFDM subcarrier induces interference to at most 8 adjacent primary users' subcarriers, while each FBMC subcarrier induces at most 1 primary user subcarrier.

In order to follow the theoretical analyses of OFDM and FBMC systems which are detailed later, the normalized interference distribution of the secondary user's subcarriers to their neighbor primary spectrum is depicted in **Fig. 6** (this will be detailed later in equation 6). It can be seen in this figure that in the case of OFDM system, each subcarrier of secondary user may induce interference to almost 8 adjacent primary users' subcarriers while in the case of FBMC this number is 1 subcarrier.

In **Fig. 7**, a scheme of FBMC side lobes and ICI that is induced to adjacent primary user subcarriers is shown. As obvious from these figures, in FBMC system only one adjacent primary user subcarrier is influenced with ICI is produced by secondary user subcarrier and this advantage leads us to benefit devices that operate even in low interference thresholds.

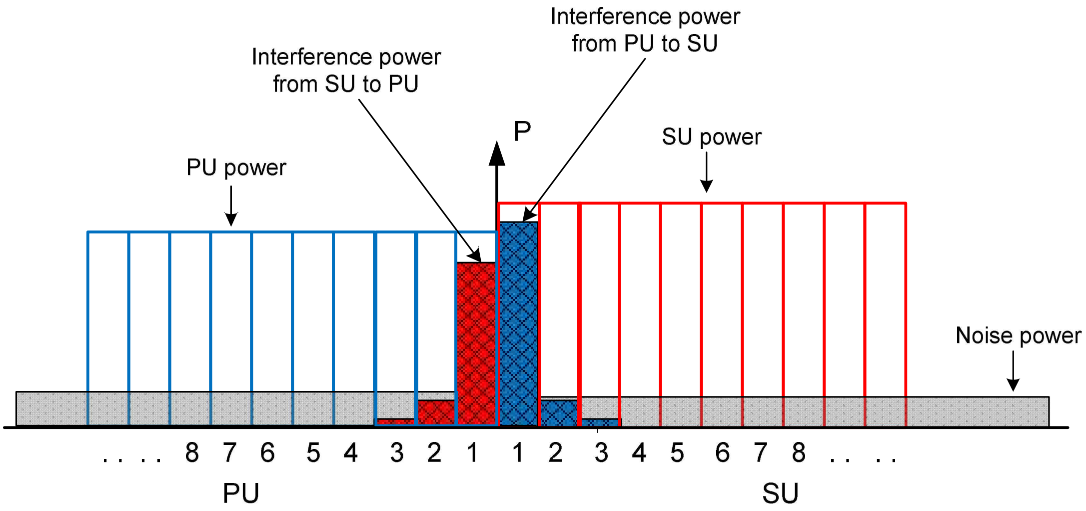

Fig. 7. Induced ICI from FBMC based secondary user out of band power spectrum to primary user subcarriers.

We use this method in our proposed algorithm in order to simplify subcarrier allocation calculations. In order to calculate the amount of interference that is induced to primary user spectrum which will be calculated later, we should have the PSD of subcarriers in secondary users. If we consider *s(t)* as transmitted signal on j^{th} subcarrier in secondary user k and if we use rectangular prototype filter for subcarriers then power spectral density of subcarriers in OFDM based CR system can be written as:

$$\Phi_{ss}^{OFDM}(f) = P_{i,k} T_s \left(\frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \tag{1}$$

Where $P_{i,k}$ is i^{th} Subcarrier amplitude of secondary user k and $T_S = T_U + T_G$ is total OFDM symbol length where T_U is the effective OFDM length and T_G is the guard interval for cyclic prefix extension [22]. In FBMC system the power spectral density of the j^{th} subcarrier in *one* FBMC symbol can be written as:

$$\Phi_j(f) = P_j \, |H_j(f)|^2 \tag{2}$$

Where P_j is the amount of power that is related to subcarrier j and $H_j\ (f)$ is the frequency response of the j^{th} prototype filter with coefficients $h[n]$ for n = 0, ..., W-1. $W = KN$ is the length of prototype filter, N is the number of systems subcarriers, and K is an overlapping factor that specifies the length of each Polyphase component or the number of subcarrier symbols which overlap in the time domain. In our algorithm, we use the prototype filter proposed in the Phydias project [23]. This filter is symmetric around the $(KN/2)_{\mathrm{th}}$ coefficients and the first coefficient is zero. Then, according to [23] the magnitude of frequency response can be written as:

$$\left|H_j(f)\right| = h\left[\frac{W}{2}\right] + 2\times\sum_{n=1}^{\frac{W}{2}-1} h\left[\left(\frac{W}{2}\right)-n\right]\cos\left(2\pi n\left(f-\frac{j}{N}\right)\right) \tag{3}$$

Where h[n] is defines as follows:

$$\begin{aligned}
&h[0]=0\\
&h[i]=1+2\sum_{k=1}^{K-1}(-1)^k H_k\cos\left(2\pi\frac{ki}{KM}\right);\ 1\le i\le KM-1\\
&where:\\
&H_1=0.971960\\
&H_2=0.707\\
&H_3=0.235147
\end{aligned} \tag{4}$$

In the case of OFDM based CR system $\Phi_{ss}(f)$ (PSD of subcarriers) is defined (1) and for FBMC based CR system it can be written as follows:

$$\Phi_{ss}^{FBMC}(f) = P_{i,k}\left|H_0(f)\right|^2 \tag{5}$$

Where $P_{i,k}$ is the power of the i^{th} subcarrier of secondary user k and $H_0(f)$ is the frequency response of the main prototype filter in FBMC system.

In order to get the amount of interference induced from secondary users to PU band, I(S→P), we should calculate the out of band interference of each subcarrier for secondary users in PU band region (characterized by red bars in **Fig. 7**). We have M secondary users and N free subcarriers which have not been used, yet. According to frequency distribution model in **Fig. 1**, in OFDM and FBMC based CR systems, normalized proportion of the i^{th} subcarrier of the k^{th} secondary user interference to primary user band is:

$$\Delta_{SU_{i,k}\to PU}(n) = \frac{1}{P_{tot}}\int_{(n\Delta f-B/2)}^{(n\Delta f+B/2)}\left|\gamma_{i,k}\right|^2\Phi_{ss}(f)\,df \tag{6}$$

Hereafter, in order to simplify equations, we just mention (6) as $\Delta_{i,k}$. In (6), P_{tot} is the total power of subcarrier and n is the subcarrier distance index and by multiplying it by Δf, we get the spectrum distance between the subcarrier and the PU band. Parameter $\gamma_{i,k}$ in (6) is the fading channel gain between the i^{th} subcarrier in the k^{th} secondary user band and PU system (characterized by red flashes in **Fig. 2**). The function $\Phi_{ss}\ (f)$ is the PSD of OFDM or FBMC subcarriers which have been defined in equations 1 and 5, respectively.

It should also be mentioned that in addition to interference induced from secondary users to PU band, there exist some interferences from primary user band to secondary users.

Therefore the amount of induced interference to the i^{th} subcarrier of the k^{th} SU from primary user is:

$$I_{PU\rightarrow SU_{i,k}}\left(n\right)=\int_{(n-1/2)\Delta f}^{(n+1/2)\Delta f}\left|\lambda_{i,k}\right|^{2}\psi\left(f\right)df \tag{7}$$

Where $\Psi(f)$ is the PSD of primary user and $\lambda_{i,k}$ is the fading channel gain from PU to the i^{th} subcarrier of secondary user k. It is considered that PU and SU band are perfectly synchronized.

Using the well-known Shannon capacity formula, we can compute the achievable throughput for each subcarrier in the uplink operation mode:

$$C_{i,k}=\Delta f\log_{2}\left(1+\frac{P_{i,k}\left|A_{i,k}\right|^{2}}{\sigma_{i}^{2}}\right) \tag{8}$$

Where $A_{i,k}$ is the fading channel gain between the i^{th} subcarrier of secondary user k and base station (characterized by green flashes in **Fig. 2**) and $\sigma_i^2 = \sigma_{AWGN}^{2} + I_i$ is the variance of the additive white Gaussian noise (AWGN) plus the variances of all interferences induced by the PU. Practically, there exists interference from primary user into subcarriers of the FBMC or OFDM systems. As it is clear in **Fig. 1**, frequency band of the primary user is continuous, therefore the interference from the primary user to the secondary users arise from PSD of subcarriers in primary user that exist at the end of the primary user's frequency band. Without loose of generality, we assume noise variance to be the same for all cognitive users and we will ignore the interference induced by the PUs to the SUs in order to simplify the simulations. But in reality these interferences exist and maybe affect the performance of neighbor SU subcarriers. Therefore, in the following analysis I_i doesn't affect the variance of signals and there would be just signal to noise ratios (SNR) instead of signal to noise plus interference ratios (SNIR).

Our objective is to maximize the total throughput of the CR system. In the uplink mode, each user has a specific amount of power and it is not allowed to cross this power level. The optimization problem, then, can be formulated as follows:

$$\begin{aligned}
&P1:\ \underset{\alpha_{i,k},P_{i,k}}{Max}:\sum_{k=1}^{M}\sum_{i=1}^{N}\alpha_{i,k}C_{i,k}\\
&Subject\ to:\\
&1: P_{i,k}\geq 0 \qquad \forall i,k\\
&2: \alpha_{i,k}\in\{0,1\} \qquad \forall i,k\\
&3: \sum_{k=1}^{M}\alpha_{i,k}\leq 1 \qquad \forall i\\
&4: \sum_{i=1}^{N}P_{i,k}\leq \overline{P_k} \qquad \forall k\\
&5: \sum_{k=1}^{M}\sum_{i=1}^{N}\alpha_{i,k}P_{i,k}\Delta_{i,k}\leq I_{th}\\
&6: Max\left(C_k^{\min}\right) \qquad where: C_k\geq C_k^{\min}
\end{aligned} \tag{9}$$

Where $\alpha_{i,k}$ is the subcarrier allocation indicator, which is 1 when user k uses the i^{th} subcarrier, and zero otherwise. Constraint 3 indicates that each subcarrier is allocated to just one user. As mentioned earlier, each cognitive user has a specific amount of total power and constraint 4 is related to this subject. The total amount of interference that the primary user can tolerate is equal to I_{th} and this threshold level is prescribed by the PU total bandwidth and its interference temperature. Constraint 6 aims at maximizing the minimum throughput of all cognitive users and parameter C_k represents the total amount of the throughput that the cognitive user k produces, that is formulated as follows:

$$C_k = \sum_{i=1}^{N} \alpha_{i,k} C_{i,k} \tag{10}$$

This optimization problem is NP-hard and in order to simplify its solution, we need to first allocate subcarriers to cognitive users according to the aforementioned constrains. Then, the power allocation procedure can operate. In the following section, we introduce our proposed subcarrier allocation algorithm and then the power allocation procedure is discussed.

3. Proposed Subcarrier Allocation Algorithm

In the downlink operation mode, subcarriers are allocated to users whose signal to noise ratio is higher than other users. However, in uplink, the concept is different due to each user's power constraint. In uplink subcarrier allocation, we should consider each user's power constraint and interference that each cognitive radio induces to primary user band. In our proposed subcarrier allocation algorithm, we use the concept that is mentioned earlier in which each FBMC subcarrier induces at most 1 primary user subcarrier. This algorithm consists of three steps. In the first step powers are initially set to all subcarriers according to the earlier mentioned fact that in FBMC multicarrier system each subcarrier induces interference to at most one adjacent subcarrier, then these adapted powers will be used in step two. In step two, each subcarrier will be allocated to the user which has the biggest channel gain between itself and the base station for that subcarrier. Finally, there is an iterative process in step three that all residue subcarriers is allocated to each user which has the least data rate among all users. In this algorithm, A_1 and A_2 are sets of allocated subcarriers to secondary users whose out of band spectrum would induce interference to primary user accordingly in FBMC and OFDM systems and Ω is a set which indicates all unused subcarriers.

```
Proposed Algorithm:     Subcarrier allocation
1) $N_1$= Set of free subcarriers to the left sight of PU's frequency band in frequency distribution model
   $N_2$= Set of free subcarriers to the right sight of PU's frequency band in frequency distribution model
   ---------------------------------------------------------------
   If $|N_1| \neq \emptyset$ then $N_1^{FBMC} = \{i\}$
   Else $N_1^{FBMC} = \emptyset$
   If $|N_2| \neq \emptyset$ then $N_2^{FBMC} = \{i+1\}$
   Else $N_2^{FBMC} = \emptyset$
   ---------------------------------------------------------------
   If $|\min\{8, |N_1|\}| < 9$ then $N_1^{OFDM} = \{1 \text{ to } i\}$
   Else $N_1^{OFDM} = \{i-7 \text{ to } i\}$
   If $|\min\{8, |N_2|\}| < 9$ then $N_2^{OFDM} = \{i+1 \text{ to } N\}$
   Else $N_2^{OFDM} = \{i+1 \text{ to } i+8\}$
   ---------------------------------------------------------------
```

```
    Set $A_1 = N_1^{FBMC} \cup N_2^{FBMC}$
    Set $A_2 = N_1^{OFDM} \cup N_2^{OFDM}$
    Set $\Omega = \{1, 2, \ldots, N\}$
    -------------------------------------------------------------------
    Now for each user k, we compute the following pre-allocated powers for all unused
    subcarriers of FBMC and OFDM systems:
    $\begin{cases} P_{i,k}^{FBMC} = \frac{I_{th}}{|A_1|\Delta_{i,k}} & \forall\{i \in A_1\} \\ P_{i,k}^{FBMC} = \frac{\overline{P_k} - \sum_{i \in A_1} P_{i,k}^{FBMC}}{N - |A_1|} & \forall\{i \in \Omega - A_1\} \end{cases}$
    $\begin{cases} P_{i,k}^{OFDM} = \frac{I_{th}}{|A_2|\Delta_{i,k}} & \forall\{i \in A_2\} \\ P_{i,k}^{OFDM} = \frac{\overline{P_k} - \sum_{i \in A_2} P_{i,k}^{FBMC}}{N - |A_2|} & \forall\{i \in \Omega - A_2\} \end{cases}$
    -------------------------------------------------------------------
2) Compute throughputs for each FBMC and OFDM systems:
    Set $B = \{1,2,\ldots,M\}$
    Set $C_k = 0 \quad \forall \{k \in B\}$
        For k =1 to M {
        a) Find $i$ which satisfying $|A_{i,k}| \geq |A_{j,k}|$ for all $j \in \Omega$
        b) $C_k = C_{i,k}$ , $\Omega = \Omega - \{i\}$
           }
    -------------------------------------------------------------------
3) Considering fairness:
        While $\Omega \neq \emptyset$ {
        a) Find $k^*$ satisfying $C_{k^*} \leq C_k \quad \forall \{k \in B\}$
        b) For this $k^*$, find $j$ so that $|A_{j,k^*}| \geq |A_{s,k^*}| \quad \forall \{s \in \Omega\}$
        c) $C_{k^*} = C_{k^*} + C_{s,k^*} \quad , \Omega = \Omega - \{s\}$
           }
```

After the above subcarrier allocation procedure, we can virtually consider convex optimization for one user multicarrier system and therefore optimization problem *P1* will be updated as follows:

$$
\begin{aligned}
&P2: \ \underset{p_{i,k}}{Max} : \sum_{i=1}^{N} C_{i,k} \\
&subject\ to: \\
&1: P_{i,k} \geq 0 \qquad \forall k \\
&2: \sum_{i \in N_k} P_{i,k} \leq \overline{P_k} \quad \forall k \\
&3: \sum_{i=1}^{N} P_{i,k} \Delta_{i,k} \leq I_{th}
\end{aligned}
\tag{11}
$$

Where k in all of the above parameters represents the user that subcarrier i is already allocated to it, and N_k represents the set of subcarriers that are allocated to user k. Problem *P2* is a convex optimization problem [24]. The lagrangian of problem *P2* can be written as follows:

$$L = -\Delta f \sum_{i=1}^{N} \log_2\left(1 + \frac{P_{i,k}^{*}\left|A_{i,k}\right|^2}{\sigma_i^2}\right) + \alpha\left(\sum_{i=1}^{N} P_{i,k}^{*}\Delta_{i,k} - I_{th}\right) + \sum_{k=1}^{M}\beta_k\left(\sum_{i\in N_k} P_{i,k}^{*} - \overline{P_k}\right) - \sum_{i=1}^{N} P_{i,k}^{*}\mu_i \tag{12}$$

Where α, μ_i and β_k are non-negative Lagrange multipliers. The Karush-Kuhn-Tucker (KKT) conditions for this lagrangian problem are as follows:

$$\begin{aligned} & P_{i,k}^{*} \geq 0; \quad \alpha \geq 0; \quad \beta_k \geq 0; \quad \mu_i \geq 0; \quad \mu_i P_{i,k}^{*} = 0 \\ & \alpha\left(\sum_{i=1}^{N} P_{i,k}^{*}\Delta_{i,k} - I_{th}\right) = 0 \\ & \beta_m\left(\sum_{i\in N_k} P_{i,k}^{*} - \overline{P_k}\right) = 0, \quad \forall k \end{aligned} \tag{13}$$

Now in order to find the solution of the problem we should find powers which lead the derivation of lagrangian problem to zero,

$$\frac{\partial L}{\partial P_{i,k}^{*}} = \frac{-1}{\dfrac{\sigma_i^2}{\left|A_{i,k}\right|^2} + P_{i,k}^{*}} + \alpha\Delta_{i,k} + \sum_{k=1}^{M}\beta_k - \mu_i = 0 \tag{14}$$

We should consider also the total power and interference constraints in (11) for the above equation. After rearranging (14) and considering the KKT conditions the optimal solution of this problem can be written as follows:

$$P_{i,k}^{*} = \left[\frac{1}{\alpha\Delta_{i,k} + \sum_{k=1}^{M}\beta_k} - \frac{\sigma_i^2}{\left|A_{i,k}\right|^2}\right]^{+} \tag{15}$$

Where $[x]^{+}$ is max (0, x). The Lagrange multipliers can be found numerically with several methods such as ellipsoid or interior point method [24].

4. Complexity Analysis and Simulation Results

In an optimal implementation, subcarrier and power allocation are performed jointly. Therefore, with M selection for each subcarrier, we have totally M^N evaluation functions. Thus, after power allocation, the complexity of the algorithm would be of the order of $O(N^3M^N)$ when the optimal solution is implemented, where N^3 comes from the complexity of solving lagrangian problem for each evaluation function. This obviously means a considerable amount of calculations. Another jointly subcarrier and power allocation algorithm proposed by Wang *et al.* [18] has a complexity larger than $O(N^2M)$ and lower than $O(N^3M)$. In subcarrier allocation algorithm proposed by Shaat *et al.* [13], there are M evaluation functions for each subcarrier, therefore the complexity would be of the order of

$O(NM)$. In our proposed algorithm in this paper, we have M evaluation functions for $(N–M)$ subcarriers which are left after step 2 in the proposed algorithm. After subcarrier allocation, we can consider the whole system virtually as a single secondary user system and we will have power allocation complexity of $O(N^3)$ for these allocated subcarriers. **Table 1** shows the computational complexity for different algorithms.

Table 1. Complexity comparison of different algorithms

Algorithm	**Complexity**
Optimal (*Most complex*)	$O(N^3M^N)$
Wang [18]	*Between* $O(N^2M)$ and $O(N^3M)$
Shaat [13]	$O(NM)+O(N^3)$
Proposed in this paper	$O((N\text{-}M)M)+O(N^3)$

We have used the proposed model shown in **Fig. 1**, for a single cell cognitive system presented in **Fig. 2**. The overall results are averaged for 500 iterations. The value of σ_i^2 is assumed to be 10^{-6} and without loose of generality, we assumed that the PU will not induce any interference to the secondary system. In this model, we have M = 10 SUs and the bandwidth of primary system is B = 10 MHz. Then, we have totally 96 subcarriers where 32 of them have been obtained as occupied band. This means that these are already used by the primary system. Therefore, we have N = 64 subcarriers left for SUs. We compared this proposed algorithm with two other subcarrier allocation algorithms, Shaat *et al.* [13] and Wang *et al.* [18]. According to **Fig. 8**, it can be seen that FBMC has a higher achieved capacity even in low allowable interference thresholds, in comparison with OFDM systems.

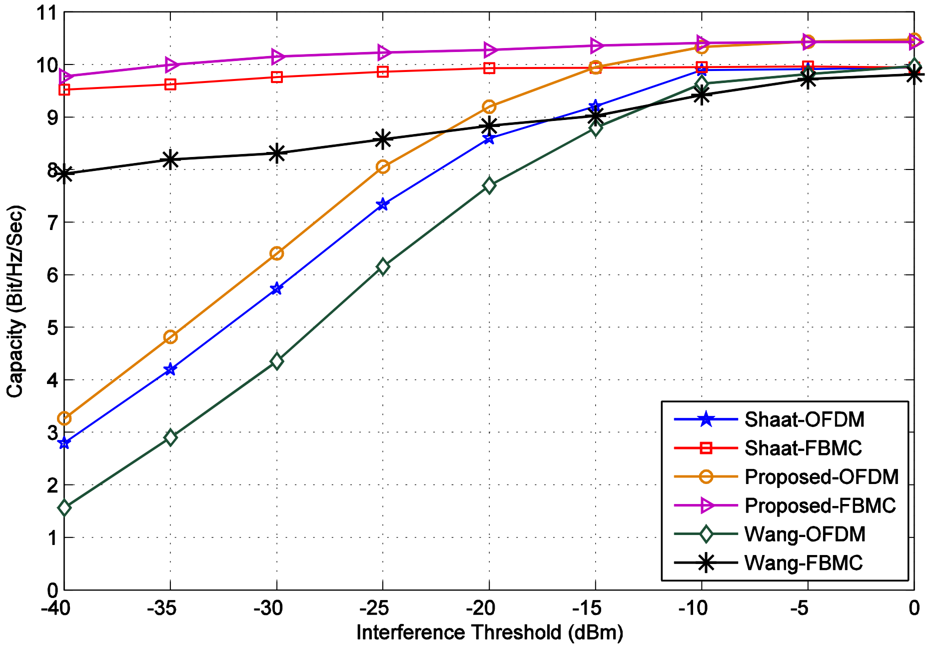

Fig. 8. Capacity versus interference threshold for different resource allocation methods.

Also, simulation results show that our proposed algorithm results in a better capacity achievement in comparison with other two competitive algorithms. Wang's algorithm is a well-known power allocation method. However it doesn't consider the fairness issue and the objective of its resource allocation algorithm is to maximize the total throughput. This algorithm is considered in the simulation part of this paper, just to be compared with other algorithms in terms of its fairness. Wang's algorithm consists of two steps, subcarrier and power allocation steps. In power allocation step, it proposes a sub-optimal algorithm in order to assign power to the allocated subcarriers. Therefore, capacity increase in proposed and Shaat's algorithms upon Wang's algorithm is due to the power allocation step. The power allocation in those algorithms is based on Lagrangian method, which is an optimal one, while power allocation in Wang's algorithm is a sub-optimal one. In our proposed algorithm, similar to Shaat algorithm, after subcarrier allocation to secondary users, we apply the same optimal power allocation algorithm in order to compare performances of these two subcarrier allocation algorithms. As the power allocation algorithms in the proposed algorithm and Shaat's algorithm are the same, it is clear that the capacity increase of the proposed algorithm in comparison with Shaat's algorithm, comes from the subcarrier allocation step. This subcarrier allocation was proposed based on interference thresholds on primary band and the idea of considering the maximum number of interfering subcarriers on primary user band. For the rest of the results, the optimal solution of the problem 1 (*P1*) will not be simulated due to the high computational complexity, especially when the number of subcarriers and secondary users are increased.

In **Fig. 9**, as it could be seen, power allocation in FBMC has higher achieved capacity than OFDM in higher transmitted powers. Also, simulation results show that our proposed power allocation algorithm results in a better capacity achievement in comparison with other two competitive algorithms.

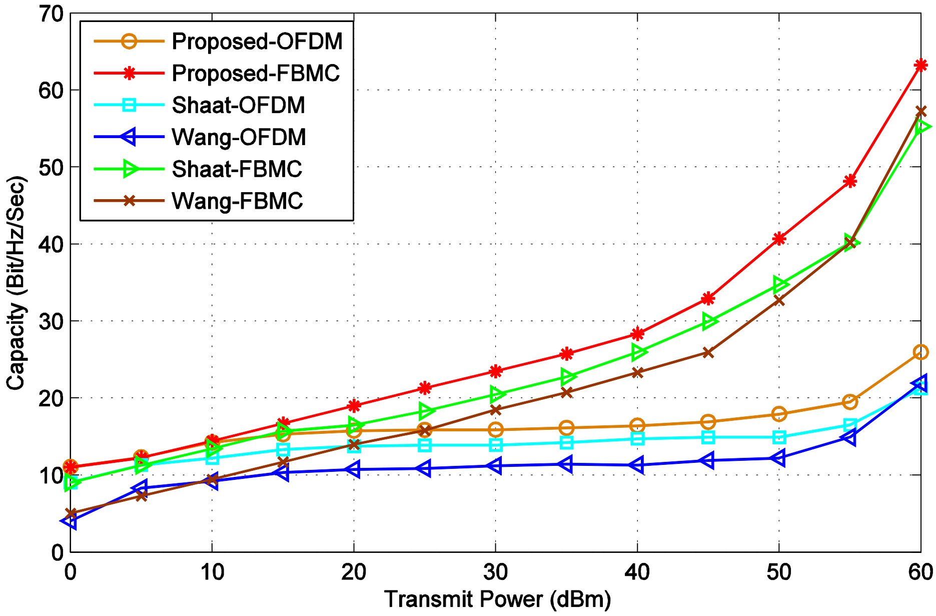

Fig. 9. Capacity versus power for different resource allocation methods.

In **Fig. 10**, the fairness issue for I_{th} = -40 dBm among secondary users of our proposed algorithm and two other algorithms are shown. We observe that the proposed algorithm has the most fairness in comparison with other algorithms, and in a statistic point of view, it has lower variance in comparison with other algorithms, among all secondary users. As it is mentioned earlier, Wang's algorithm doesn't consider fairness issue and it is clear in **Fig. 10** that this algorithm has the worst fairness, compared to the other algorithms. The fairness issue in the proposed algorithm is actually considered in the step 3 of the proposed subcarrier allocation algorithm (see step. 3). In step 3 the remaining subcarriers which have the best channel gains are iteratively allocated to weak secondary users that have the least capacity. The goal in this step is to minimize the capacity variance among different secondary users. Again, the subcarrier allocations for Shaat's and our proposed algorithms are different, but the power allocation is the same. Therefore, we can see in **Fig. 10** that the difference between the fairness of algorithms results from subcarrier allocation part. In order to compare the fairness criterion in several power allocation algorithms that are shown in **Fig. 10,** the standard deviation of the users' capacities are shown in **Table 2**.

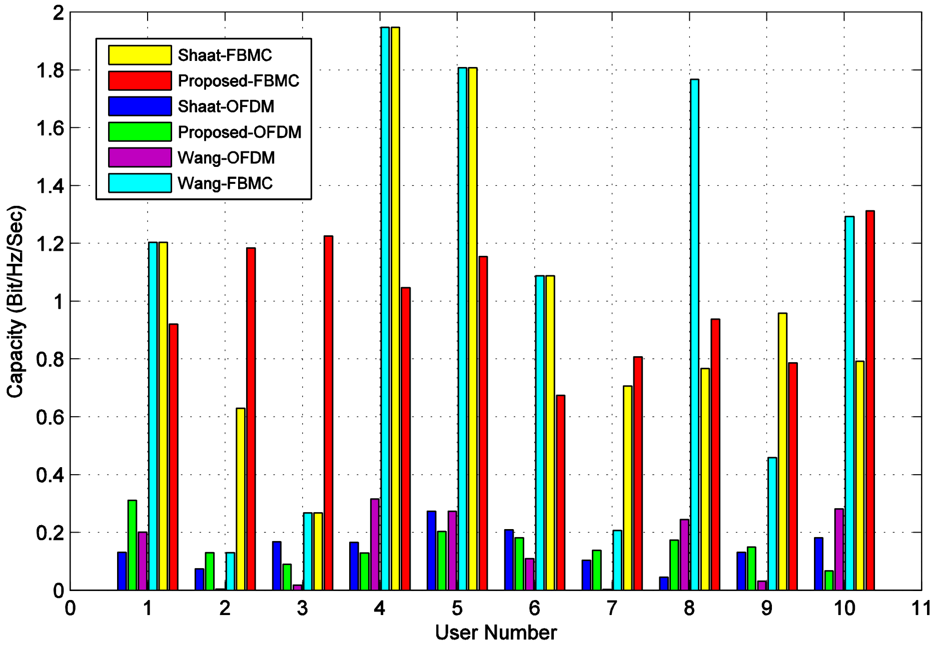

Fig. 10. Fairness among secondary users for different resource allocation methods.

Table 2. Standard deviation of the capacity for different secondary users, for different resource allocation algorithms

Methods	Standard deviation
Proposed-FBMC	0.2
Shaat-FBMC	0.5
Wang-FBMC	0.67
Proposed-OFDM	0.06
Shaat-OFDM	0.08
Wang-OFDM	0.12

We can see in this table that the proposed algorithm has the least standard deviation of the capacity for different secondary users, and Wang algorithm has the biggest standard deviation.

5. Conclusion

In this paper, we proposed an efficient subcarrier allocation algorithm for uplink in a single cell CR FBMC-based system and we compared it with two other algorithms (Shaat *et al.* [13]

and Wang *et al*. [18]). Our proposed algorithm is separated into three steps. In the first step, we initialize and define the adjacent subcarriers to the PUs then we predefined each subcarriers power according to the interference that adjacent subcarriers induced to the primary system. In step 2, we predefined each user capacity according to the channel quality. Finally in step 3, we assigned the remaining subcarriers to users that have the lowest capacity. It was shown that our proposed algorithm outperforms other algorithms in terms of total achievable throughput and the difference of achievable data rate between all secondary users is minimized with this proposed algorithm.

References

[1] J. Mitola, “Cognitive Radio: An integrated agent architecture for software defined radio,” *PhD dissertation*, *Royal Institute of Technology*, 2000. Article (CrossRef Link).

[2] B. Wang, and K.J.R. Liu, “Advances in cognitive radio networks: A survey,” *IEEE Journal of Selected Topics in Signal Processing*, vol.5, no.1, pp.5-23, Feb.2011. Article (CrossRef Link).

[3] T. Weiss, and F. K. Jondral, “Spectrum pooling: an innovative strategy for the enhancement of spectrum efficiency,” *IEEE Communications Magazine*, vol.42, no.3, pp.8-14, Mar.2004. Article (CrossRef Link).

[4] P. Amini, P. Kempter, R. R. Chen, and L. Lin, “Filter bank multitone: a physical layer candidate for cognitive radios”, in *Proc. of the SDR Forum technical Conference*, pp.14-18, Nov.2005. Article (CrossRef Link).

[5] B. Farhang-Boroujeny, “OFDM versus filter bank multicarrier,” *IEEE Signal Processing Magazine*, vol.28, no.3, pp.92-112, May.2011. Article (CrossRef Link).

[6] M. Bellanger, “Filter banks and OFDM/OQAM for high throughput wireless LAN”, *3rd International Symposium on Communications, Control and Signal Processing*, pp.758-761, Mar.2008. Article (CrossRef Link).

[7] K. Kim, Y. Han, and S.L. Kim, “Joint subcarrier and power allocation in uplink OFDMA Systems,” *IEEE Commun. Lett,* vol.9, no.6, pp.526–528, Jun.2005. Article (CrossRef Link).

[8] Cho Yiu Ng, and Chi Wan Sung, “Low complexity subcarrier and power allocation for utility maximization in uplink OFDMA systems” *IEEE Transactions on Wireless Communications*, vol.7, no.5, pp.1667-1675, May.2008. Article (CrossRef Link).

[9] J. Huang, V. G. Subramanian, R. Agrawal, and R. Berry, “Joint scheduling and resource allocation in uplink OFDM systems for broadband wireless access networks” *IEEE Journal on Selected area in communications,* vol.27, no.2, pp.226-234, Feb.2009. Article (CrossRef Link).

[10] Y. Wang, L. Yang, and X. Liu, “Resource allocation algorithm for uplink OFDMA-based cognitive radio systems,” *IEEE 3rd International Conference on Communication Software and Networks*, pp.16-20, May.2011. Article (CrossRef Link).

[11] E. Yaacoub, and Z. Dawy, “A Survey on uplink resource allocation in OFDMA wireless networks,” *IEEE Communications Surveys & Tutorials*, no.99, pp.1-16, 2011. Article (CrossRef Link).

[12] L. Gao and S. Cui, “Efficient subcarrier, power, and rate allocation with fairness consideration for OFDMA uplink,” *IEEE Trans. Wireless Commun.*, vol.7, no.5, pp.1507–1511, May.2008. Article (CrossRef Link).

[13] M. Shaat, and F. Bader, “Fair and efficient resource allocation algorithm for uplink multicarrier based cognitive networks” *IEEE 21st International Symposium on Personal Indoor and Mobile Radio Communications*, pp.1212-1217, Sep.2010. Article (CrossRef Link).

[14] W. Shim, Y. Han, and S. Kim, “Fairness-aware resource allocation in a Cooperative OFDMA uplink system,” *IEEE Transactions on Vehicular Technology*, vol.59, no.2, pp.932-939, Feb. 2010. Article (CrossRef Link).

[15] T. Yucek, and H. Arslan, “A survey of spectrum sensing algorithms for cognitive radio applications,” *IEEE Communications Surveys & Tutorials*, vol.11, no.1, pp.116-130, First Quarter 2009. Article (CrossRef Link).

[16] F. Boroujeny, “Filter bank spectrum sensing for cognitive radios,”, *IEEE Transactions on Signal Processing*, vol.56, no.5, pp.1801-1811, May.2008. Article (CrossRef Link).

[17] T. Sato, and M. Umehira, “A new spectrum sensing scheme using overlap FFT filter-bank for dynamic spectrum access,”, *Sixth International ICST Conference on Cognitive Radio Oriented Wireless Networks and Communications*, pp.6-10, Jun.2011. Article (CrossRef Link).

[18] W. Wang, W. Wang, Q. Lu, and T. Peng, "An uplink resource allocation scheme for OFDMA-based cognitive radio networks,", *International J. Commun. Syst.*, vol.22, no.5, pp.603–623, 2009. Article (CrossRef Link).

[19] P. Siohan, C. Siclet, and N. Lacaille, "Analysis and design of OFDM/OQAM systems based on filter bank theory," *IEEE Transactions on Signal Processing*, vol.50, no.5, pp.1170-1183, May.2002. Article (CrossRef Link).

[20] Y. Medjahdi, M. Terre, D. Le Ruyet, D. Roviras, J.A. Nossek, and L. Baltar, "Inter-cell interference analysis for OFDM/FBMC systems,", *IEEE 10th Workshop on Signal Processing Advances in Wireless Communications*, pp.598-602, Jun.2009. Article (CrossRef Link).

[21] H.Zhang, D. Le Ruyet, D. Roviras, and H.Sun, "Uplink capacity comparison of OFDM / FBMC based cognitive radio networks," *IEEE International Conference on Communications*, pp.1-5, May.2010. Article (CrossRef Link).

[22] T. Weiss, and J. Hillenbrand, "Mutual interference in OFDM-based spectrum pooling systems,", *VTC-Spring*, vol.4, May.2004. Article (CrossRef Link).

[23] M. Tanda, T. Fusco, M. Renfors, J. Louveaus, and M. Bellanger, "Physical layer for dynamic access and cognitive radio,", *PHYDYAS Project*, 2008. Article (CrossRef Link).

[24] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge, Cambridge Univ. Press, U.K., 2004. Article (CrossRef Link).

Hosseinali Jamal received his B.Sc. degree in Electrical Eng. from Tafresh (Amirkabir University of technology), Iran, in 2009 and his M.Ss. degree in Telecommunications from Shahid Beheshti University, G. C., Tehran, Iran. Since May 2011, he has been with the Cognitive Radio Research Group of Shahid Beheshti University, Tehran, Iran. His current research interests include cognitive radio, resource allocation algorithms and convex optimization problems.

Seyed Ali Ghorashi received his B.Sc. and M.Sc. degrees in Electrical Eng. from University of Tehran, Iran, in 1992 and 1995, respectively. Then, he joined SANA Pro Inc., where he worked on modelling and simulation of OFDM based wireless LAN systems and interference cancellation methods in W-CDMA systems. Since 2000, he worked as a research associate at King's College London on "capacity enhancement methods in multi-layer W-CDMA systems" sponsored by Mobile VCE. In 2003 He received his PhD at King's College and since then he worked at Kings College as a research fellow. In 2006 he joined Samsung Electronics (UK) Ltd as a senior researcher and now he is a faculty member of Cognitive Telecommunication Research Group, Department of Electrical Engineering, Shahid Beheshti University G.C., at Tehran, Iran, working on wireless communications.

Seyed Mohammad-Sajad Sadough received his B.Sc. degree in Electrical Engineering (electronics) from Shahid Beheshti University, Tehran, I.R. Iran in 2002 and the M.Sc. and his Ph.D. degrees in Electrical Engineering (telecommunication) from Paris-Sud 11 University, Orsay, France, in 2004 and 2008, respectively.

From 2004 to 2007, he held a joint appointment with the National Engineering School in Advanced Techniques (ENSTA), Paris, France, and the Laboratory of Signals and Systems (LSS), at Supélec, Gif-sur-Yvette, France. He was a lecturer in the Department of Electronics and Computer Engineering (UEI), ENSTA, where his research activities were focused on improved reception schemes for ultra-wideband communication systems. From December 2007 to September 2008, he was a postdoctoral researcher with the LSS, Supélec-CNRS, where he was involved in the European research project TVMSL with Alcatel-Lucent France. Since October 2008, he has been a member of the Faculty of Electrical & Computer Engineering, Shahid Beheshti University, where he is currently an Assistant Professor in the Department of Telecommunication. Dr. Sadough's areas of research include signal processing, communication theory, and digital communication.