

# Fuzzy Controller Design for a Automotive Air Suspension

## 자동차 에어 서스펜션에 대한 퍼지 제어기 설계

H. Liu and J. C. Lee

류하오 · 이재천

접수일: 2012년 3월 6일, 수정일: 2012년 4월 6일, 게재확정일: 2012년 5월 1일

**Key Words** : air spring(에어 스프링), fuzzy logic control(퍼지 로직 제어), ride comfort(승차감), semi-active hybrid control air suspension(반능동식 하이브리드제어 에어 서스펜션), skyhook damping control(스카이훅 댐핑 제어), stiffness control(강도 제어)

**Abstract**: 본 연구의 목적은 에어 서스펜션 시스템의 제어 특성을 분석하는 것이다. 우선 에어 서스펜션 시스템의 수학적 모델을 구하였다. 그리고 퍼지 제어 알고리즘을 적용하여 반능동식 하이브리드 제어 에어 서스펜션을 구하였다. 차체 가속도에 따라 퍼지 제어기는 오리피스 개도를 변경하여 특정 영역에서 에어 스프링의 강도를 조정한다. 동시에 서스펜션 운동 상태에 따라 서스펜션 댐핑이 제어된다. 시뮬레이션 결과는 반능동식 하이브리드 제어 에어 서스펜션이 노면 접지능력의 상실이나 서스펜션 작동 공간의 증가 없이 최고의 승차감을 제공할 수 있음을 보여준다.

### Nomenclature

$A$  Orifice area  
 $A_e$  Air spring effective area  
 $c_{smax}$  Maximal damping coefficient  
 $c_{smin}$  Minimal damping coefficient  
 $c_s$  Damping coefficient  
 $C_p$  Specific heat at constant pressure  
 $C_v$  Specific heat at constant volume  
 $E$  Air overall energy  
 $h_{in}$  Enthalpy flowing into  
 $h_{out}$  Enthalpy flowing out  
 $k_{us}$  Tire stiffness  
 $m$  Air flow mass  
 $m_s$  Sprung mass  
 $m_{us}$  Unsprung mass  
 $M$  Air mass  
 $P$  Absolute pressure  
 $P_a$  Atmospheric pressure  
 $q_m$  Air mass flow rate  
 $Q$  Exchange heat

$R$  Gas constant  
 $T$  Air temperature  
 $U$  Air internal energy  
 $V$  Air volume  
 $W$  Work done by outside  
 $x_r$  Road surface displacement  
 $x_s$  Sprung mass displacement  
 $x_{us}$  Unsprung mass displacement

### Subscripts

1 Air spring  
 2 Auxiliary reservoir  
 0 Initial value

## 1. Introduction

Though passive suspensions have still been used widely for some passenger vehicles and trucks in recent years, one of the fastest and dramatic changes for commercial passenger vehicles, especially those of the luxury classes, has been the improvement in ride comfort quality. The designers determine optimal parameter values of springs and dampers in passive suspensions according to the expected suspension

이재천(교신저자): 계명대학교 기계자동차공학부  
 E-mail: ljcds@kmu.ac.kr, Tel: 053-580-6720  
 류하오 : 계명대학교 전자화자동차부품지역혁신센터

performance. In some driving conditions, passive suspensions can not provide good ride comfort for passengers. On the other hand, the air spring has been replacing the conventional spring to improve ride comfort and driving safety recently, owing to its advantages of adjustable carrying capacity, reduced weight, variable spring stiffness with almost constant natural frequency, reduced structurally transmitted noise, variability of ride heights, and so on <sup>1)</sup>.

Based on the above background, this paper theoretically applies fuzzy logic control algorithm to the air suspension system to improve ride comfort. And also a semi-active hybrid control air suspension combining stiffness and skyhook damping control is proposed in the paper. Its characteristics are further analyzed in simulation, which shows advantages of using the semi-active hybrid control algorithm. The paper outline is as below. The mathematical model of a quarter vehicle air suspension is derived in the section 2. And then in the section 3, the fuzzy controller based on skyhook damping control principle and air spring stiffness control are designed. On the condition of random road surface input, the simulation for the proposed semi-active hybrid control air suspension model, as well as the passive suspension, the ideal skyhook suspension and the air suspension without control, is performed in the section 4. The comparison of the four suspension models is analyzed by using three performance evaluation indexes. Conclusions are drawn in the last section.

## 2. Modeling of Quarter Air Suspension

Considering a 2DOF quarter air suspension model shown in Fig. 1, the air spring and the variable damping damper in parallel connect the sprung mass (the vehicle body) with the unsprung mass, while the tire damping is neglected. Two accelerometers are used to measure accelerations of the sprung mass and the unsprung mass, respectively. The controller adjusts the orifice opening area and the damping of the air suspension system.

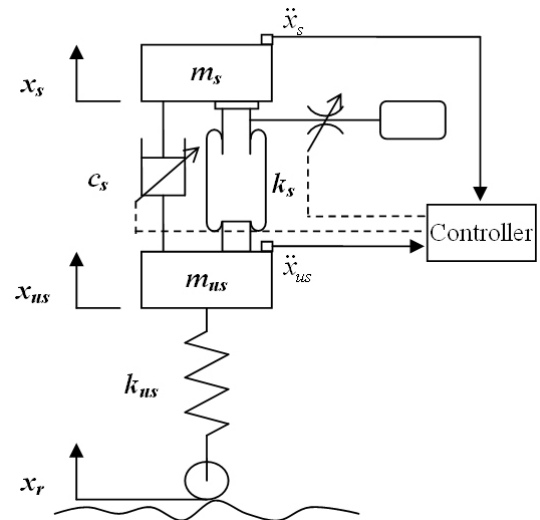


Fig. 1 Semi-active hybrid control air suspension model

### 2.1 Quarter Air Suspension

The differential equations describing the quarter air suspension are expressed as below.

$$m_s \ddot{x}_s + c_s (\dot{x}_s - \dot{x}_{us}) + (P_1 - P_a) A_e = 0 \tag{1}$$

$$m_{us} \ddot{x}_{us} - c_s (\dot{x}_s - \dot{x}_{us}) + k_{us} (x_{us} - x_r) - (P_1 - P_a) A_e = 0 \tag{2}$$

### 2.2 Air Spring with Auxiliary Reservoir

In Fig. 2, an orifice capable of adjustable opening area is installed between the air spring and the auxiliary reservoir. When the air spring is compressed or extended due to vibration, the air pressure inside the air spring varies, yielding the air pressure difference between the air spring and the auxiliary reservoir. The air flows through the orifice, and produces resistance, thus consuming some vibration energy due to road disturbance.

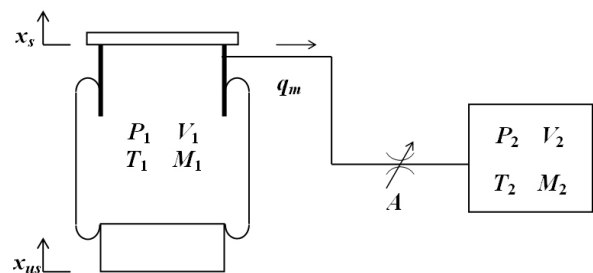


Fig. 2 Model of air spring with auxiliary reservoir

Firstly the model of air spring connected with auxiliary reservoir and the adjustable orifice is established. The energy conservation equation and the gas state equation can describe air change process in the air spring and the auxiliary reservoir. The gas flow rate equation is used to describe air mass exchange between the air spring and the auxiliary reservoir. In order to simplify the problem, assume the following items.

The air change process inside the air spring and the auxiliary reservoir is considered as adiabatic process in that the displacement excitation frequency is set over 0.2Hz<sup>2)</sup>.

The potential energy and kinetic energy of the air inside the air spring and the auxiliary reservoir are negligible.

The orifice is ideal. Therefore the theoretical flow rate equation is used to describe its pressure-flow property.

Energy conservation equation for the air spring is as below<sup>3)</sup>

$$dQ_1 + d(h_{in} - h_{out}) + dW_1 = dE_1 \quad (3)$$

There is no heat exchange in adiabatic process.

$$dQ_1 = 0 \quad (4)$$

Enthalpy change in the air spring should be considered by two cases: air flow-out and air flow-in.

$$d(h_{in} - h_{out}) = -dh_{out} = -C_p T_1 dm = -\kappa C_v T_1 dm, (P_1 > P_2) \quad (5a)$$

$$d(h_{in} - h_{out}) = dh_{in} = C_p T_2 dm = \kappa C_v T_2 dm, (P_1 < P_2) \quad (5b)$$

Work done by outside:

$$dW_1 = -(P_1 - P_a) dV_1 \quad (6)$$

Air overall energy inside the air spring is equal to air internal energy, considering the second assumption.

$$dE_1 = dU_1 = d(C_v M_1 T_1) = C_v M_1 dT_1 + C_v T_1 dM_1 \quad (7)$$

Substituting Eq. (4) ~ (7) into Eq. (3), the energy conservation equation for the air spring can be expressed as.

$$-\kappa C_v T_1 \frac{dm}{dt} - (P_1 - P_a) \frac{dV_1}{dt} = C_v M_1 \frac{dT_1}{dt} + C_v T_1 \frac{dM_1}{dt}, (P_1 > P_2) \quad (8a)$$

$$\kappa C_v T_2 \frac{dm}{dt} - (P_1 - P_a) \frac{dV_1}{dt} = C_v M_1 \frac{dT_1}{dt} + C_v T_1 \frac{dM_1}{dt}, (P_1 < P_2) \quad (8b)$$

Differentiating the gas state equation for the air spring, the follow equation is obtained.

$$P_1 \frac{dV_1}{dt} + V_1 \frac{dP_1}{dt} = R(T_1 \frac{dM_1}{dt} + M_1 \frac{dT_1}{dt}) \quad (9)$$

Air volume inside the air spring is as below.

$$V_1 = V_{10} - \int A_e(x_{us} - x_s, P_1) dx \quad (10)$$

Similarly, the energy conservation equation for the auxiliary reservoir is

$$dQ_2 + d(h_{2in} - h_{2out}) + dW_2 = dE_2 \quad (11)$$

There is no heat exchange in adiabatic process, too.

$$dQ_2 = 0 \quad (12)$$

Enthalpy change in the auxiliary reservoir is also divided two cases.

$$d(h_{2in} - h_{2out}) = dh_{2in} = C_p T_1 dm = \kappa C_v T_1 dm, (P_1 > P_2) \quad (13a)$$

$$d(h_{2in} - h_{2out}) = -dh_{2out} = -C_p T_2 dm = -\kappa C_v T_2 dm, (P_1 < P_2) \quad (13b)$$

Work done by outside for the auxiliary reservoir is zero.

$$dW_2 = 0 \quad (14)$$

Gas overall energy inside the auxiliary reservoir is described as.

$$dE_2 = dU_2 = d(C_v M_2 T_2) = C_v M_2 dT_2 + C_v T_2 dM_2 \quad (15)$$

Substituting Eq. (12) ~ (15) into Eq. (11), the energy conservation equation for the auxiliary reservoir can be rewritten as.

$$\kappa C_v T_1 \frac{dm}{dt} = C_v M_2 \frac{dT_2}{dt} + C_v T_2 \frac{dM_2}{dt}, (P_1 > P_2) \quad (16a)$$

$$-\kappa C_v T_2 \frac{dm}{dt} = C_v M_2 \frac{dT_2}{dt} + C_v T_2 \frac{dM_2}{dt}, (P_1 < P_2) \quad (16b)$$

The gas state equation for the auxiliary reservoir is

$$V_2 \frac{dP_2}{dt} = R(T_2 \frac{dM_2}{dt} + M_2 \frac{dT_2}{dt}) \quad (17)$$

Ideal orifice flow rate equation can be considered as two cases of subsonic and supersonic, depending on the pressure drop of upstream and downstream <sup>4)</sup>.

When  $P_1 > P_2$

$$\begin{cases} q_m = AP_1 \frac{1}{\sqrt{T_1}} \sqrt{\frac{2\kappa}{R(\kappa-1)}} \sqrt{\left(\frac{P_2}{P_1}\right)^{\frac{2}{\kappa}} - \left(\frac{P_2}{P_1}\right)^{\frac{\kappa+1}{\kappa}}} & (1 > \frac{P_2}{P_1} > 0.528) \\ q_m = AP_1 \frac{1}{\sqrt{T_1}} \sqrt{\frac{\kappa}{R} \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{2(\kappa-1)}}} & (\frac{P_2}{P_1} \leq 0.528) \end{cases} \quad (18a)$$

When  $P_1 < P_2$

$$\begin{cases} q_m = AP_2 \frac{1}{\sqrt{T_2}} \sqrt{\frac{2\kappa}{R(\kappa-1)}} \sqrt{\left(\frac{P_1}{P_2}\right)^{\frac{2}{\kappa}} - \left(\frac{P_1}{P_2}\right)^{\frac{\kappa+1}{\kappa}}} & (1 > \frac{P_1}{P_2} > 0.528) \\ q_m = AP_2 \frac{1}{\sqrt{T_2}} \sqrt{\frac{\kappa}{R} \left(\frac{2}{\kappa+1}\right)^{\frac{\kappa+1}{2(\kappa-1)}}} & (\frac{P_1}{P_2} \leq 0.528) \end{cases} \quad (18b)$$

Considering total air inside the air spring and the auxiliary reservoir as a system, the mass conservation as following equation is built.

When  $P_1 > P_2$

$$\begin{cases} M_1 = M_{10} - \int q_m dt \\ M_2 = M_{20} + \int q_m dt \end{cases} \quad (19a)$$

When  $P_1 < P_2$

$$\begin{cases} M_1 = M_{10} + \int q_m dt \\ M_2 = M_{20} - \int q_m dt \end{cases} \quad (19b)$$

The air spring with the auxiliary reservoir shown in Fig. 2 can be mathematically expressed by the Eq. (8) ~ (10) and (16) ~ (19).

### 3. Design of Fuzzy Logic Controller

In order to improve ride comfort for passengers, the aim of stiffness control is to eliminate the sprung mass acceleration because lower stiffness can provide better ride quality. Furthermore, the aim of damping control is, according to skyhook damping control principle, to attenuate vibration energy due to road unevenness. Therefore, the semi-active hybrid control air suspension is developed based on fuzzy logic controller.

#### 3.1 Stiffness Controller Design

The stiffness control principle is described as following. When the orifice is completely closed, the air spring presents higher stiffness. On the other hand, when the orifice is fully opened, the air spring volume is enlarged owing to the auxiliary reservoir, yielding low stiffness <sup>1)</sup>. Therefore, the air spring stiffness can be adjusted in certain range by changing the orifice opening area.

Considering convenience, the two dimension fuzzy logic controller, namely two inputs, is used. One input is the sprung mass acceleration error signal (E), and another input is the change rate of the acceleration error signal (EC). The control output is the orifice opening area (U). Their universes of discourse are identical as below.

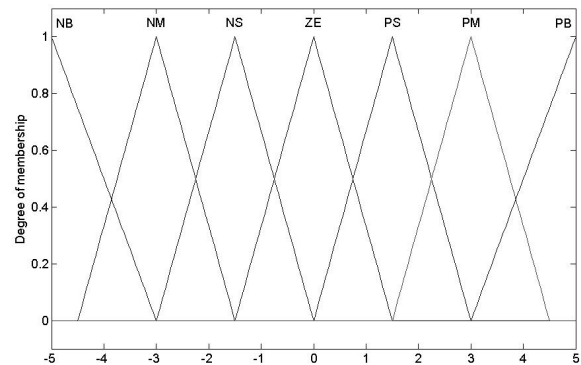


Fig. 3 Input membership function of stiffness control

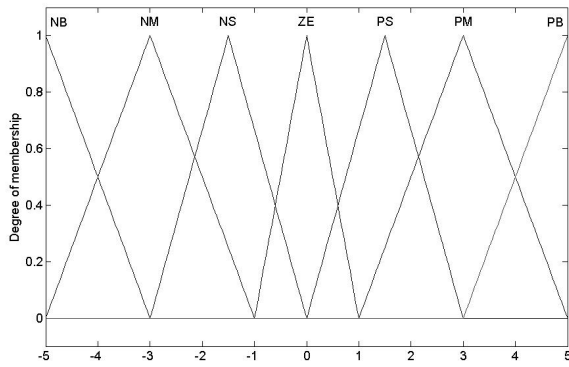


Fig. 4 Output membership function of stiffness control

{-5, -4, -3, -1, 0, 1, 2, 3, 4, 5}

Moreover, the inputs and output linguistic values of their fuzzy sets are also same as

{NB NM NS ZE PS PM PB}

Fig. 3 and Fig. 4 illustrate the membership functions of input and output, respectively.

Fuzzy rule base is designed on the basis of the following rules. If the acceleration error and the acceleration error change rate are positive or both of them are negative, which means that the sprung acceleration value will more deviate from zero, the air spring should set as low stiffness, namely the orifice should be opened more widely. If the acceleration error is positive and the acceleration error change rate is negative, or reversely, if the acceleration error is negative and the acceleration error change rate is positive, which means that the acceleration value has the trend to return to zero, the air spring stiffness is set as middle. If the acceleration error and the acceleration error change are close to zero, the air spring stiffness is set to stiffer to keep road holding ability. As results, the rule base for stiffness control can be obtained, as shown in Table 1.

Table 1 Fuzzy rule matrix of stiffness control

U		E						
		NB	NM	NS	ZE	PS	PM	PB
EC	NB	PB	PM	PM	PS	PS	ZE	ZE
	NM	PM	PS	ZE	ZE	ZE	NS	ZE
	NS	PM	ZE	NS	NM	NM	NS	PS
	ZE	PS	ZE	NM	NB	NM	ZE	PS
	PS	PS	NS	NM	NM	NS	ZE	PM
	PM	ZE	NS	ZE	ZE	ZE	PS	PM
	PB	ZE	ZE	PS	PS	PM	PM	PB

To obtain the crisp output control signal, the inference mechanism uses a Max-Min fuzzy inference, and defuzzification interface uses Center of Area (COA) method. The scale factor is used to convert the defuzzified output to the actual orifice opening area.

### 3.2 Damping Controller Design

Next the fuzzy controller using skyhook damping control principle is designed. Since the skyhook damping control requires the virtual reference (sky), it is impossible to be realized in real vehicle<sup>5)</sup>. However, the skyhook damping force can be achieved in some conditions. When the sprung mass velocity has the same direction as its relative velocity with the unsprung mass, the conventional damper can generate identical skyhook damping. In this case, the damping should set as maximal value. If the sprung mass velocity has the opposite direction as its relative velocity with the unsprung mass, the conventional damper can not produce the same damping as the skyhook damper. Thus it is better to set the damping as minimal as possible. As results, by using ideal skyhook damping control principle, the actual control algorithms is as following.

$$c_s = \begin{cases} c_{s,max} & \dot{x}_s(\dot{x}_s - \dot{x}_{us}) > 0 \\ c_{s,min} & \dot{x}_s(\dot{x}_s - \dot{x}_{us}) \leq 0 \end{cases} \quad (20)$$

The damping fuzzy controller has two inputs, the sprung mass velocity ( $\alpha$ ) and the relative velocity of the sprung mass and the unsprung mass ( $\beta$ ), and one output, the damping coefficient (U). The universe of discourses of the sprung mass velocity and its relative velocity with the unsprung mass are defined as

{-5, -4, -3, -1, 0, 1, 2, 3, 4, 5}.

And their fuzzy set is represented as

{NB NS ZE PS PB}.

Both membership functions of inputs are same, given in Fig. 5.

The universe of discourse of the damping coefficient is defined as

{-6, -5, -4, -3, -1, 0, 1, 2, 3, 4, 5, 6}.

And its fuzzy set of output membership function is as below, also shown in Fig. 6.

{NB NM NS ZE PS PM PB}.

Based on the actual semi-active damping control algorithm of Eq. (20), the fuzzy rule base is written in matrix form, given in Table 2. Max-Min fuzzy inference and COA defuzzification are also utilized to obtain the crisp output. The scale factor sets the output damping as the actual and reasonable range.

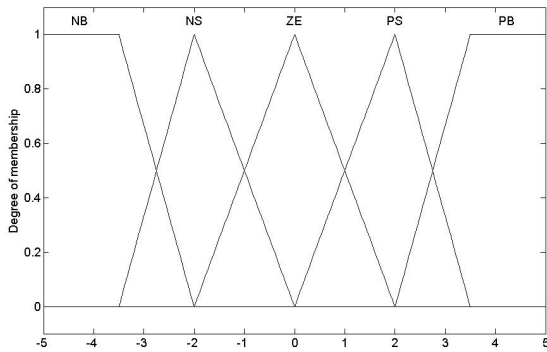


Fig. 5 Input membership function of damping control

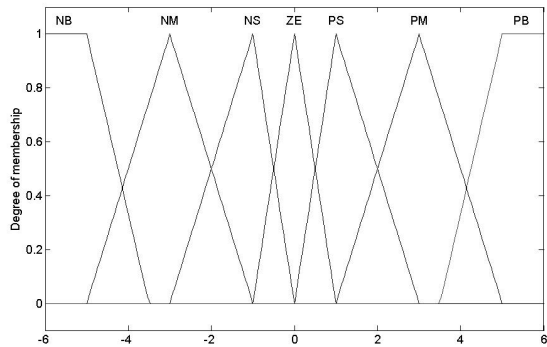


Fig. 6 Output membership function of damping control

Table 2 Fuzzy rule matrix of damping control

U		$\beta$				
		NB	NS	ZE	PS	PB
$\alpha$	NB	PB	PM	PS	NB	NB
	NS	PM	PS	NS	NM	NB
	ZE	PS	NS	ZE	NS	PS
	PS	NB	NM	NS	PS	PM
	PB	NB	NB	PS	PM	PB

#### 4. Simulation and Result Analysis

The proposed semi-active hybrid control air suspension in the section 2 is validated by simulation here. In order to clearly compare the control effect, the following different quarter vehicle suspension models

are considered.

Model 1: Passive suspension

This is conventional suspension model, which consists of a common spring and a passive damper, neglecting the tire damping.

Model 2: Ideal skyhook suspension

In this model a skyhook damper connects the vehicle body with the virtual inertial reference (sky).

Model 3: Air suspension (AS) without control

The conventional damper and the air spring connected with the auxiliary reservoir are used in the suspension. In the simulation, the orifice opening area is fixed as  $10\text{mm}^2$ .

Model 4: Semi-active hybrid air suspension (AS)

This model can control stiffness and damping simultaneously, as shown in Fig. 1.

The random B level road excitation, which is generated according to the road unevenness coefficient given in ISO 8608:1995<sup>6)</sup>, is implemented on the above four models in simulation, where the vehicle driving speed is  $20\text{m/s}$  ( $72\text{ km/h}$ ). The main parameters in the simulation are listed in Table 3. The vehicle body displacement responses of four models in the beginning of 3 second are illustrated in Fig. 7.

Table 3 Main parameters in simulation

$c_{smax}$	2200 Ns/m	$c_{smin}$	1200 Ns/m
$c_s$	1700 Ns/m	$c_{sky}$	1700 Ns/m
$k_s$	37000 N/m	$k_{us}$	150000 N/m
$m_s$	475 kg	$m_{us}$	60 kg
$A$	$10\text{ mm}^2$	$V_2$	1 liter

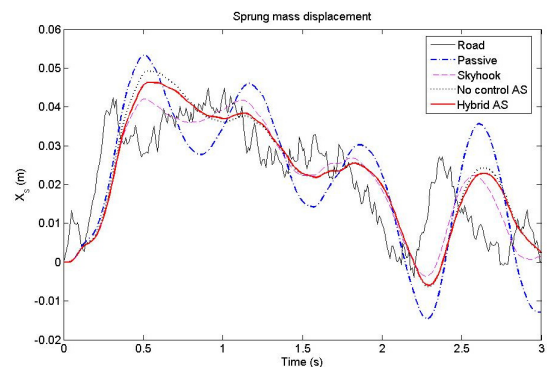


Fig. 7 Sprung mass displacement response to random road input

In order to synthetically assess suspension characteristics, three performance evaluation indexes, namely sprung mass acceleration (A), suspension dynamic deflection (D) and tire dynamic load (L), are introduced <sup>7)</sup>. The root mean square (RMS) values of the indexes are calculated to investigate four suspension model responses to the random road excitation, as listed in Table 4.

Table 4 RMS value of displacement response

Suspension model	RMS(A) (m/s <sup>2</sup> )	RMS(D) (m)	RMS(L) (N)
Passive	0.87454	0.0089307	548.11
Skyhook	0.65459	0.0061676	479.78
No control AS	0.67124	0.012586	493.1
Hybrid AS	0.65187	0.009383	495.38

It is found in Fig. 7 and Table 4 that the passive suspension shows the worst performance, while the skyhook damping suspension presents the best comprehensive performance, which, unfortunately, can not be realized for the real vehicle. The air suspension model without control achieves good ride comfort and tire dynamic load, whereas its suspension dynamic deflection is the biggest. The semi-active hybrid control air suspension possesses the lowest sprung mass acceleration, which means that it can provide the best ride comfort quality for passengers. Meanwhile, it keeps the suspension dynamic deflection and the tire dynamic load in a reasonable level. The simulation results show that the semi-active hybrid control air suspension realizes trade-off for the three performance evaluation indexes, and obtains the best ride comfort quality, without losing road holding ability and increasing suspension working space.

## 5. Conclusion

In order to improve ride comfort for passenger vehicles, air suspension characteristics are investigated in the paper. The proposed semi-active hybrid control air suspension utilizes the fuzzy logic controller to

adjust the suspension stiffness and the damping at the same time according to motion states of the sprung mass and the unsprung mass. The simulation shows that this type air suspension can provide the best ride comfort quality, simultaneously keeping road holding force and restraining too big suspension dynamic deflection.

## Acknowledgment

This research was supported by the Ministry of Knowledge Economy (MKE) and Korea Institute for Advancement of Technology (KIAT) through the Center for Automotive Mechatronics Parts (CAMP) at Keimyung University.

## References

- 1) Q. Giuseppe, S. Massimo, "Air Suspension Dimensionless Analysis and Design Procedure", *Vehicle System Dynamics*, Vol. 35, No. 6, pp. 443~475, 2001.
- 2) R.A. Williams, "Automotive active suspensions part 2: practical considerations", *Proc. IMechE, Part D: J. Automobile Engineering*, Vol. 211, pp. 427~444, 1997.
- 3) Y. A. Cengel, M.A. Boles, "Thermodynamics: an Engineering Approach (Fifth Edition)", McGraw-Hill, Singapore, 2006.
- 4) H. Liu, J.C. Lee, et al., "High Precision Pressure Control of a Pneumatic Chamber using a Hybrid Fuzzy PID Controller", *International Journal of Precision Engineering and Manufacturing*, Vol. 8, No. 3, pp. 8~13, 2007.
- 5) D. Karnopp, M.J. Crosby, et al., "Vibration control using semi-active force generators", *ASME Journal of Engineering for Industry*, Vol. 96, No. 2, pp. 619~626, 1974.
- 6) ISO 8608:1995, "Mechanical vibrations - Road surface profiles - Reporting of measured data", 1995.
- 7) R. Rajesh, "Vehicle Dynamics and Control", Springer, New York, 2006.