http://dx.doi.org/10.7236/JIWIT.2012.12.3.123
JIWIT 2012-3-16

Radon RBF Network에 의해 그린 보증 함수의 근사화

Approximation of Green Warranty Function by Radon Radial Basis Function Network

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요 약 오래 전부터 연료의 가격은 상승하고 있다. 제조업체는 보증을 통해 실용적인 대안을 찾고자 전기와 강력한 바이오 연료를 이용하여 차량의 성장가능을 연구하고 있다. 이제, 이러한 녹색 환경(emission) 관련된 보증은 보증기간이 확장되며, 이러한 보증을 '수퍼 보증" 이라 불린다. 본 논문의 주요 결과는 라돈 변환의 역행렬을 보증공간의 수치를 줄이기 위해 사용되며, 응용 프로그램 및 RBF 네트워크를 사용하여 대략적인 이변량의 보증 기능에 새로운 방법을 제시한다. 이 방법은 다음과 같은 단계로 구성되어 있다. 첫째, 라돈 변환을 이용하여, 이변량 보증 함수의 1 차원 함수를 줄일 수 있다. 둘째, 1 차원 함수의 각 신경 서브 네트워크와 신경 네트워크 기법을 사용하여 근사할 수 있다. 셋째, 이러한 신경 sub-networks 형태로 최종 근사 신경망 함께 결합 된다. 넷째, 라 돈 변환의 역함수 값을 사용 하여 최종 근사 신경 네트워크에 우리가 주어진 함수 근사화를 얻을 수 있다. 또한, 우리는 자동차 회사의 일부 그린 보증 데이터를 가지고 위의 방법을 적용한다.

Abstract As the price of traditional fuels soar, the alternatives are becoming more viable. And manufacturers are promoting the growing viability of electric and biofuel-powered vehicles through longer warranties. Now, these longer green environment (emission)warranties, sometimes called extended warranties or "super warranties," have been adapted. The main result of this paper is to present a new method to approximate a bivariate warranty function by using Radial Basis Function Network with application of Radon Transform and its inverse which is used to reduce the dimension of the warranty space. This method consist of the following stages: First, by using the Radon Transform, the bivariate warranty function can be reduced to one dimensional function. Second, each of the one dimensional functions is approximated by using neural network technique into neural sub-networks. Third, these neural sub-networks are combined together to form the final approximation neural network. Four, by using the inverse of radon transform to this final approximation neural network we get the approximation to the given function. Also, we apply the above method to some green warranty data of automotive vehicle company.

Key Words : Two-attributes Warranty, Green Warranty, Radial Basis Function network, Radon Transformation

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접수일자 2012.5.10, 수정완료 2012.6.2,
게재확정일자 2012.6.8.

Received: 10 May, 2012, Revised: 2 June, 2012,
Accepted: 8 June, 2012
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1. INTRODUCTION

In the real world, vague phenomenon is quite common in the production models. In order to process the vagueness, a production model that can be more closely related to the real vagueness and can take account of the vague factors that contribute to production costs, is required. Especially, in warranty policy, there is a lot of uncertainty and also there are many gaps in the data that can only be filled by qualitative assessments by warranty experts. The model can be extended or altered to fit in with the fuzzy situation. In this context, Lolas, et al. [8] discussed an approach used in the construction of the fuzzy logic knowledge base for a new reliability improvement expert system, whose main objectives are to be able to improve the reliability of new vehicle systems. In regard to two-attribute warranty policy, Rai and Singh [12] discussed a method to estimate hazard rate from incomplete and unclear warranty data. Recently, Lee and Moon [5] presented a new sets-as-points geometric view of fuzzy warranty sets under two-dimensional warranty policy.

Both of statistical modeling and fuzzy knowledge based approach facilitate development of robust products that are insensitive to changes in the environment and internal component variation. Carefully planned studies remove hindrances to high quality and productivity at every stage of production. This saves time and money. It is well recognized that quality must be engineered into products as early as possible in the design process. Presently, the use of warranty expert systems combined with artificial intelligence methods is the modern approach to incorporate uncertainty into the various warranty estimations necessary for the product improvement process. In particular, the main development of the initial greenery warranty system has been done by using fuzzy logic but for the lower level connections in the greenery warranty system there is only complete numerical data (such as actual warranty counts of age and usage), which makes the use of neural networks a good possibility.

Now, environmental concerns are affecting the warranty industry in dozens of ways. Government is also getting involved by mandating higher fuel efficiency, new fuel blends, lowered emissions levels, and longer warranties. It has become interested in this area, setting both maximum emissions standards and minimum exhaust equipment warranty durations for products registered in the country. These longer emissions warranties, sometimes called extended warranties or "super warranties," have also been adapted. The present work presents a novel method for approximating a two-attribute warranty function by using radial basis function network with one hidden layer and linear output in Green IT's point of view. In section2, we discuss a green warranty processing workflow regarding such super warranty problem. In section 3, we present the construction of new method for approximating a bivariate warranty function by using radial basis neural network. The method consists of four stages. First, by the use of discrete Radon transform, [11]], the problem of multidimensional approximation is replaced by, several simpler, one dimensional problems. Second, for each of the one dimensional problems the approximation sub-networks are found. Third, the sub-networks are combined together to form the final approximation network. Fourth, by using the inverse of Radon Transform to this final approximation neural network we get the approximation of the green warranty function.

II. GREEN WARRANTY PROCESSING WORKFLOW

When considering warranty issues, an important concept to keep in mind is the warranty chain. Like the supply chain for purchasing and manufacturing, the warranty chain extends the scope of warranty activities beyond the walls of a single company to encompass suppliers, manufacturers, OEMs, distributors, dealers, repair centers, policy carriers, and customers. Just as the integration of data and activities up and down the supply chain leads to cost savings, so does the integration of claims submission, claims processing, parts return, quality improvement, and supplier recovery along the entire warranty chain. Warranty claims processing refers to the entry, review, and acceptance or denial of warranty claims. In most organizations, this is a labor intensive, time consuming process that is prone to questions, delays, and back-and-forth communication and paperwork between manufacturers, distributors, dealers, and repair centers.

To identify actual warranty claims incurred by warranty claims processing, it helps to understand a warranty claims processing department as an equivalent to an accounts receivable or shipping department. Each warranty claim, like an invoice or shipment, takes on average a certain amount of time to complete. Therefore, the size of accounts receivable operations is determined by the number of invoices. In the same way, set warranty claims processing head-count to be sufficient to handle the number of claims processed each year. While no statistics exist on the amount of time spent on an individual warranty claim, given that employees are processing many claims at the same time, a warranty claim typically takes 30 to 90days to process from the time it is submitted to the time it is accepted or rejected, and paid. Labor costs are a major part of warranty claims processing.

The analysis for generating knowledge of the greenery warranty claims is usually carried out from the product cascading down to the component level, Fig. 1. Due to the space constraint full expansion in Fig.1 has only been done for the emission system and for the refinement module of the emission system. These are the components of the system that will be used for the case study example later in the paper. Other components such as the engine component have similar expansion architecture as the emission system from the system level until the component level. As shown in Fig.1 the system has four main levels, which are the component level, the sub-system/module level, the system level and the full product level.

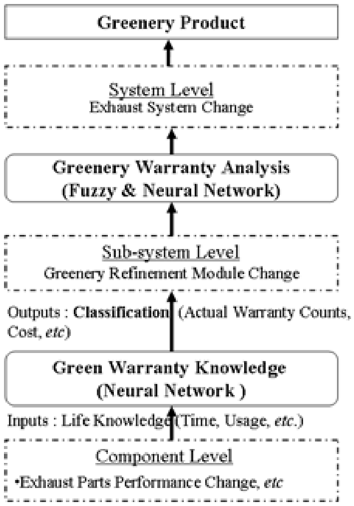

그림 1. 그린 보증 지식처리과정
Fig. 1. Greenery Warranty knowledge processing

Starting from the component level and moving through the architecture towards the complete product system it can be seen that each level connects with its superior level by a knowledge-base component. For the connection of the component level with the sub-system/module level a NN (neural network) learning system can be used. The reason is that at the component level there is very important to identify a lot of invalid warranty claims, in which actual warranty claims can only be filled by quantitative assessments by greenery part experts. For the rest of the connections above the sub-system module level there is a knowledge-based fuzzy logic system or a neural network module. The reason is that at the sub-system or system level there is a lot of uncertainty and also there are many gaps in the data that can only be filled by qualitative assessments by experts. This makes the use of fuzzy logic the best solution according to [4].

A radial basis function network (RBF) is a feedforward neural network with radial basis function as an activation function. Approximations of multidimensional function have been studied by many researchers such as Baxter (et al.) [1], Ciesielski (et al.) [2], Ellacott [4], Niyogi and Girosi [9] and Orr [10]. In particular, RBF has been used to the approximation of multidimensional function. Baxter (et al.), [1], it is known that the interpolation matrix (1) is invertible, where h is a radial function, they computed an upper bounds for $\|A^{-1}\|p$ where $p \geq 1$, and they show that when A is symmetric and positive definite then h decays sufficiently quickly.

$$A = [h(x_j - x_k)]_{j,k=1}^{n} \tag{1}$$

Ciesielski (et al.), [2], focused on a development of a constructive formula for the upper bound of L_∞ error approximation. Ellacott, [4], proved that a semilinear feedforward network with one hidden layer can uniformly approximate any continuous function in $C(K)$ where K is a compact set in R^s and s is a positive integer. Niyogi and Girosi [9], they derived a bound to generalization error for radial basis functions which apply to any approximation technique. Orr, [10], gave an introduction to radial basis function (RBF) neural networks with least squares bound. Johann Radon, [11], showed that if f is continuous and has a compact support, then the Radon Transform of f is uniquely determined by integrating along all lines in the domain $X \subset C(K)$.

III. RADON RBF DESIGN FOR GREEN WARRANTY FUNCTION

This section considers the approximation the construction of new method for approximating a bivariate warranty function by using Radon RBF. The suggested method consists of the use of discrete Radon transform, finding the approximation sub-networks for each of the one dimensional problems, combining the sub-networks together to form the final approximation network, and using the inverse of Radon transform to this final approximation neural network. Let X be a normed linear space. A function $f: X \to R$ is said to be radial if there exists a function $h: R^+ \to R$ such that $f(x) = h(\|x\|)$ for all $x \in X$, R^+ is a set of all positive real numbers. A radial basis function is any translate of f; that is, a function of the form

$$\mu(x) = f(x - \xi) = h(\|x - \xi\|) \tag{2}$$

Where ξ is any prescribed point of X, [5]. Such a function depends on the distance $\|x-\xi\|$, $\|\cdot\|$ usually is taken to be Euclidean norm, and this function is symmetric with respect to a center point ξ. Some examples of radial basis functions in one dimensional space are shown in Fig. 2.

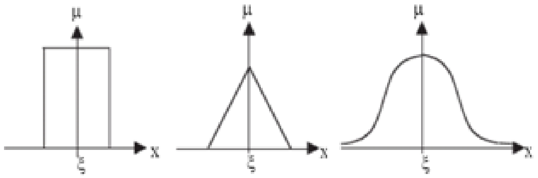

그림 2. 1차원 radial basis functions의 3가지 예
Fig. 2. Three examples of one dimensional radial basis functions

Radial basis functions are a special class of functions that has the following characteristic feature: Their response decrease (or increase) monotonically with distance from a central point. A common use of such functions is for interpolation (to approximate a given function). In this context, one usually has data prescribed at points $\xi_1, \xi_2, \cdots, \xi_n$ in X and attempts to interpolate these data by function of the form (3). Radial basis functions can be employed in the neural computation for approximating continuous functions.

$$x \mapsto \sum_{j=1}^{n} c_j h(\|x - \xi_j\|), \quad x \in X \tag{3}$$

The idea of radial basis function (RBF) network derive from the theory of function approximation, and this network consists of a large number of computing units arranged schematically in three layers as shown in Fig. 3. Each unit of the input layer can be connected to each unit of the hidden layer. This connection has associated with a weight, which is a real number. The weight attached to the link from input unit j to unit i on the hidden layer is denoted by w_{ij}, and is known as the radial basis function (RBF) center. In a typical operation, each unit on the input layer will contain a real number. Let the j-th input unit contain the real number xj. Then unit i on the hidden layer will receive from unit j on the input layer the quantity $(x_j - w_{ij})^2$. The total input that unit i receives from all the input units is then

$$c_i = \| x_j - w_{ij} \| = \sum_{j=1}^{s} (x_j - w_{ij})^2 .$$

Unit i on the hidden layer now processes this input with a radial basis function $\theta : R^+ \to R$ which is a given fixed univariate function (called the activation function) and the outputs are the real numbers $\theta_{ij}(\|x_j - w_{ij}\|)$ where (i=1, 2, ⋯, k and j=1, 2, ⋯,s). This output θ_{ij} is then transmitted, with a weight a_i, , to the output unit. The total output is then have the form

$$g(x) = \sum_{i=1}^{k} a_i \theta_i (\| x - w_i \|) \tag{4}$$

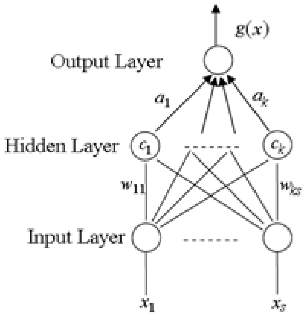

그림 3. 신경망의 층구조
Fig. 3. Layers in a neural network.

Any continuous function of s variables can be approximated by a function of the form g given in equation (4). For our RBF neural network the function θ_i and the centers wi are assumed to have been fixed. Thus by suitable adjusting the parameters ai this can be reproduce approximately, to any accuracy, any desired output with the use of artificial neural network(Fig. 3). From above analysis we can approximate any complicated type of functions by simple functions. For the simplification of the computational complexity take the case (one input, one hidden layer with n processing units and one output). Thus to approximate a continuous function $j{:}R \to R$ is by considering the approximation form of the neural network $B{:}R \to R$ as:

$$B(x) = \sum_{i=1}^{n} a_i \theta_i (\| x - w_i \|) \tag{5}$$

B is a linear combination of radial basis functions (RBF) and the corresponding network is known as a radial basis function network (RBF). Let us assume that the activation (transfer) function is the Gaussian function

$$\theta(x) = \exp\left(-x^2/(2\sigma^2)\right)$$

Here $\sigma > 0$ is a parameter whose value controls the smoothness properties of the interpolating function.

Let the function $j{:}R \to R$ be continuous on the compact set K in R. The approximation error of the function j by the network B given in (5) is defined as

$$E(x_j) = \sum_{j=1}^{s} (\varphi(x_j) - B(x_j))^2 , \tag{6}$$

The error E is minimum if and only if

$$\underline{a_M} = A^{-1} H^T \varphi \tag{7}$$

Here A is the variance matrix and H is the neural net matrix.

$$H = \begin{bmatrix} \theta_1(x_1) & \theta_2(x_1) & \cdots & \theta_n(x_1) \\ \theta_1(x_2) & \theta_2(x_2) & \cdots & \theta_n(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ \theta_1(x_s) & \theta_2(x_s) & \cdots & \theta_n(x_s) \end{bmatrix}$$

Numerically to compute a just need to find the inverse of the matrix $H^T H$. Now since $K(H^T H) = K^2(H)$, K is the condition number, which is defined to be $K(H) = \|H\| \|H^{-1}\|$,, and thus the computation of $(H^T H)^{-1}$

could be sensitive to the rounding error. Therefore, use the least square method. Assume H^TH is of full rank, to find (7). Our numerical results shows that as increasing the dimension of H^TH the condition number become bigger and thus we have an ill-condition system. However, this is not the case when use least square method or conjugate gradient method. If the matrix arise from using the above method (radial basis function neural network) is not of full rank that is rank$(H^TH)<n$ then we use the singular value decomposition technique.

Using the discrete Radon Transform will decompose a multidimensional function into scalar functions for each p and ξ. These functions $f_l : R \rightarrow R$ (l=1, 2, ……, L, where L is the number of projections) are approximated using the function $B(x)$ given in the equation (5). Now each function $f_l : R \rightarrow R$ is approximated by a sub-network $B^l : R \rightarrow R$, given in (5), and these sub-networks B^l are combines into the final approximation network which have the form

$$N(x) = \sum_{l=1}^{L} B^l \left(\| x - w^l \| \right), \tag{8}$$

which is an approximation to the discrete Radon Transform of the function $f(x)$. It is necessary to invert the Radon Transform, that is, to solve for f in terms $\hat{f}$. The Radon Transform inversion formula has the form

$$f(x) = \begin{cases} \dfrac{1}{2(2\pi i)^{s-1}} \left(\dfrac{\partial}{\partial p} \right)^{\frac{s-1}{2}} \displaystyle\int_{|\xi|=1} \hat{f}(p,\xi) d\xi, & if\ s\ is\ odd. \\ \dfrac{1}{2(2\pi i)^{s}} \displaystyle\int_{|\xi|=1} d\xi \int_{-\infty}^{\infty} \dfrac{\left(\dfrac{\partial}{\partial p} \right)^{s-1} \hat{f}(p,\xi)}{p - \xi \cdot x} dp, & if\ s\ is\ even. \end{cases}$$

where i denotes the imaginary number, and ξ is a unit vector in R^s that defines the orientation of a hyper plane with equation $p = \cdot x = {}_1x_1 + {}_2x_2 + \cdots + {}_sx_s$ (p is the orientation of the hyper plane). Thus often using the inverse Radon Transform to the network $N(x)$ in equation (8) to get back to the dimension of the space that begin with and this will lead us to the approximation of the function $f(x)$, where xR^s.

[Step 1] Input (a: The vector xR^s, b: The network target yR, c: The error goal, d: The angles λ of the projections)

[Step 2] Compute the discrete Radon Transform to the input vector x and the angles λ to get the function f_l.

[Step 3] Apply a radial basis neural network B^l for each of the Radon Transform f_l using equation (5).

[Step 4] Combine these subnetworks B^l, which is found in Step 3, into final neural network $N(x)$ by using (8).

[Step 5] Compute the discrete inverse Radon Transform to the neural network $N(x)$, which has been computed in Step 4, and thus get the approximate to the given function.

IV. RESULTS

Despite the fact that warranties are so commonly used, the actual amount of warranty in many situations remains an unsolved problem. This may seem surprising since the fulfillment of warranty claims may cost companies large amounts of money. Underestimating true warranty amounts will result in losses for a company: on the other hand, overestimating them will result in uncompetitive products. The data relevant to the modeling of warranty amounts in automotive vehicle industry are usually highly confidential, since they are commercially sensitive. Much warranty analysis takes place in internal research divisions in large companies.

Most often automotive warranty is specified in terms of {*Tmax, Mmax*} with *Tmax* being a specified maximum time period and *Mmax* a specified maximum mileage. Table 1 provides a two-way warranty counts for the warranty claims of O_2 sensors analyzed in the component level. These are based on actual warranty claims reported about particular type cars during recent years in automotive company, South Korea, and exclude invalid claims. Oxygen sensors are a product that have been around for more than 20 years, yet most motorists do not even know they have one or more of these devices on their vehicle – let alone what it does. The only time most people even become aware of an oxygen sensor's existence is if they get a check engine light and there is a code that indicates an O_2 sensor problem their vehicle fails an emissions test because of a sluggish or dead O_2 sensors. But in most cases, they will not have a clue as to know to diagnose or test this mysterious little device that is often blamed for all kinds of drive ability and emissions ills.

표 1.
Table 1. Two-way automotive warranty amounts

x_2: (1,000km)	x_1: Time (36 months)			
	0~2	2~4	4~6	6~8
0~5	1	0	0	0
5~20	7	1	0	0
20~60	13	46	26	1
60~100	1	31	37	17
100~120	1	7	12	9
120~	1	23	33	45

The proposed algorithm has been, numerically, implemented with the use of MATLAB version (7.0), the radial basis function neural network use the Gaussian function in the hidden layer and pure line function in the output layer. The numerical procedure is as follows:

1) Initially put the input angles $\lambda \in \{0°, 30°, 60°, 90°, 120°, 150°\}$ i.e L=6, error goal 10−5. Fig. 4 shows approximation of warranty function $f(x_1, x_2)$ using above λ's.
2) Compute the discrete Radon Transform to the function $f(x_1, x_2)$ using λ's and input vector x and thus get the one dimensional functions f_l.
3) Each of the one dimensional functions f_l is approximated by using a single hidden layer RBF.
4) The networks approximating functions f_l are combined together, using equation (8), to form the final neural network approximation $N(x)$.
5) Compute the discrete inverse Radon Transform to $N(x)$ and thus get the approximation to the function $f(x_1, x_2)$.

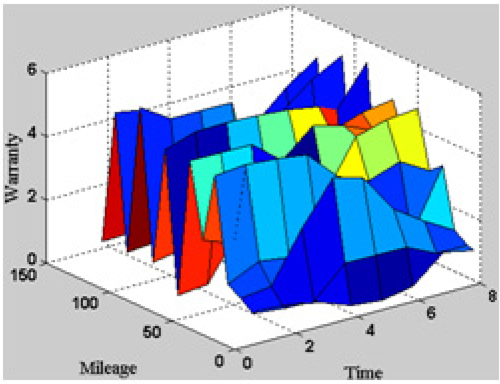

그림 4. L=6일 때 근사화
Fig. 4. Approximation with L = 6

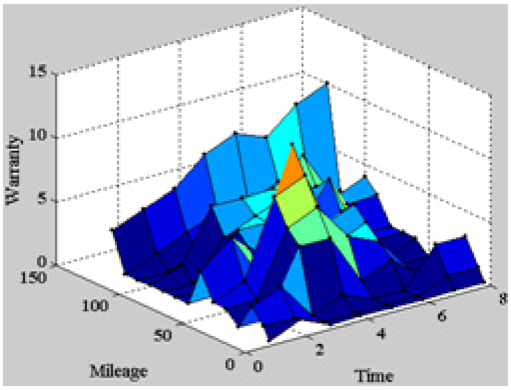

그림 5.L=10일 때 근사화
Fig. 5. Approximation with L=10.

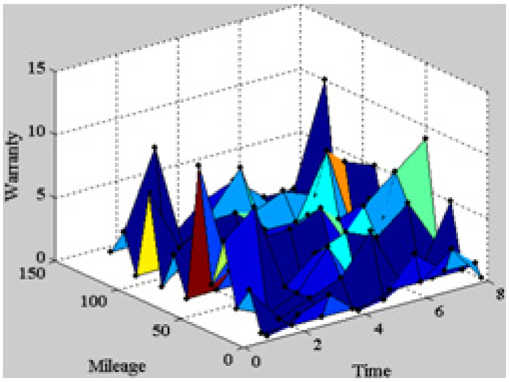

그림 6. L=15일 때 근사화
Fig. 6. Approximation with L=15.

Results for the approximating of the function $f(x_1, x_2)$ with L=10 and L=15 are shown in Fig. 5 and Fig. 6, respectively.

V. CONCLUSIONS

Green warranty problem can be easily identified as an unknown nonlinear multivariate function of the age and usage counts under two-attribute warranty policy. This paper develops a method, with the aid of neural network technique, to approximate the function of two-attribute warranty variables. Our numerical results are more accurate than those given in [6], and computationally our method is easier to implement with less flops than the method in [2]. So, the proposed approach can successfully be applied for environmental management of warranty claims system such as automotive vehicle.

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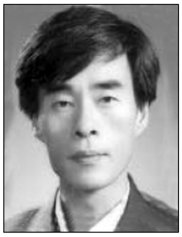

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