International Journal of Fuzzy Logic and Intelligent Systems, vol. 12, no. 2, June 2012, pp. 162-166
http://dx.doi.org/10.5391/IJFIS.2012.12.2.162 pISSN 1598-2645 eISSN 2093-744X

Generalized Higher Order Energy Based Instantaneous Amplitude and Frequency Estimation and Their Applications to Power Disturbance Detection

Byeong-Gwan Iem

Dept. of Electronic Engineering, Gangneung-Wonju National Univ., Gangneung, 210-702, Korea

Abstract

The instantaneous amplitude (IA) based on the higher order differential energy operator is proposed. And its general form for arbitrary order is also proposed. The various definitions of the IA and the instantaneous frequency (IF) estimators are considered. The IA and IF estimators based on the energy operators need less computational cost than the conventional IF and IA estimators exploiting the Hilbert transform. The IF and IA estimators are compared in terms of the frequency and amplitude tracking accuracy of the AM-FM signals. For noiseless case, the IA and IF estimators based on the Teager-Kaiser energy operator show better tracking performance than the IF and IA estimators based on the higher energy operators. However, under noisy condition, the IF and IA estimator based on the higher order energy operators with the order 3 and 4 show better tracking than the Teager-Kaiser energy based estimators. The IF and IA estimators are applied to signals in the various power anomalies to show their usefulness as the disturbance detectors.

Keywords : Instantaneous frequency, instantaneous amplitude, higher order differential energy operator, power disturbances.

1. Introduction

The Instantaneous frequency and amplitude have been a study topic for decades partly because they are efficient and simple analysis tools to understand time-varying characteristics of non-stationary signals. The instantaneous frequency (IF) and instantaneous amplitude (IA) are a kind of time-frequency representations (TFR) in the sense that as functions of time, they show the time varying contents of a signal along the time axis when the signal has a single component at a time [1]. Typical application areas include the feature extraction like the fundamental frequency estimation in speech [2] and the power disturbance detection in power line [3]. For the power disturbance detection application, the IF and IA estimates should be constant if the signal is under normal condition. If any disturbance happens, then the IF and IA estimates should show abrupt changes, and the disturbance can be detected [3].

Conventionally, for an AM-FM signal $x(t) = a(t)e^{j\phi(t)}$, the instantaneous frequency can be defined as the derivative of the phase $\phi(t)$. The instantaneous amplitude is given as $|a(t)|$. For the discrete signal, the difference of phase can be used for the derivative. This conventional definition has been successfully used for the fundamental frequency estimation of speech signal [2] and the detection of power disturbance detection [3]. However, this definition requires the use of the Hilbert transform to get the phase information if the signal is real.

Kaiser introduced discrete energy separation algorithms (DESAs) to get the IF and IA using the Teager-Kaiser energy operator [4], and used the methods to analyze speech signals [5]. Maragos also proposed a method to obtain the IF and IA using the higher order differential energy operator [6]. These methods do not require the Hilbert transform, but need only additions, multiplication and the nonlinear operation like the inverse cosine or the square root. The IF and IA based on the higher order differential energy operator has been derived for other orders in [7], and the IF has been generalized for the arbitrary order in [8]. Furthermore, the IF based on the symmetric higher order differential energy operator has been derived in [9] and used for speech analysis [10]. However, the general form of the IA based on the higher order differential energy operator has not been studied yet.

In this paper, the IA based on the higher order differential energy operator will be proposed and it is generalized for the arbitrary order. And the various IF and IA estimators will be compared in terms of tracking accuracy for AM-FM signals through simulations. Depending on the order of the energy operators, the IA and IF show different estimation accuracy. The proper selection of IF and IA estimator can be achieved exploiting the performance comparison results. The usefulness of the IA and IF estimator will be shown by applying them to the power disturbance detection. Various methods have been applied to the power disturbance detection. Some of them are the FFT method [11], and the wavelet transform based methods [12,13]. They require considerable amounts of computational cost. With the IF and IA estimators proposed in this paper, the computational load can be reduced. The structure of the paper is as follows. In Section 2, the definitions of various IF and IA estimators will be reviewed. The general form of the IA and IF based on the higher order differential energy operator will be given in here. In Section 3, the estimation accuracy of the IF and IA estimators will be evaluated for AM-FM signals. DESA algorithms, the higher order energy operator based IF and IA

Manuscript received Nov. 8, 2011; revised Jun. 9, 2012; accepted Jun. 14, 2012
*Corresponding Author: Byeong-Gwan Iem(ibg@gwnu.ac.kr)

estimators and the general forms of the IF and IA will be compared for both noiseless and noisy conditions. In Section 4, the IF and IA estimators will be used as power disturbance detectors to show their usefulness. Various power disturbance scenarios will be considered. They are the sag/swell, and the harmonic distortions. And in Section 5, the conclusion will be given.

2. Instantaneous Frequency and Amplitude Estimators

The instantaneous frequency and amplitude show a single frequency and a single amplitude value at a certain time instant. In other words, it is a time varying frequency and amplitude as functions of time. Here the review of their definitions is provided.

2.1. Conventional Instantaneous Frequency/Amplitude Estimators

For an AM-FM signal $x(t) = a(t)e^{j\phi(t)}$, the instantaneous frequency can be defined as

$$f(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt}$$

where $a(t)$ and $\phi(t)$ are the amplitude and the phase of the AM-FM signal. This relation can be easily shown in a pure sinusoid with a constant frequency. For real signal $x(t)$, the Hilbert transform is needed to get the analytic form $x_a(t) = x(t) + j\mathrm{HT}\{x(t)\} = a(t)e^{j\phi(t)}$. For discrete signal, the central difference equation is used as the IF estimator [1]

$$f[n] = \frac{1}{4\pi}(\phi[n+1] - \phi[n-1]). \quad (1)$$

2.2. Instantaneous Frequency/Amplitude Estimators based on the Teager-Kaiser Energy operator

Teager introduced an energy function defined as

$$\Psi\{x(t)\} = (x'(t))^2 - x(t)x''(t), \quad (2)$$

and used it to measure the amount of energy needed in creating an oscillating wave [13]. Its discrete version is defined as [4]

$$\Psi_d\{x[n]\} = x^2[n] - x[n+1]\,x[n-1]. \quad (3)$$

The subscript 'd' indicates the discrete version. Kaiser used the energy operator in (3) to derive an IF and IA estimator called the discrete time energy separation algorithm I (DESA I). The DESA I IF and IA estimators are defined respectively as [4]

$$f[n] \approx \frac{1}{2\pi}\cos^{-1}\left(1 - \frac{\Psi_d\{y[n]\} + \Psi_d\{y[n+1]\}}{4\Psi_d\{x[n]\}}\right) \quad (4)$$

$$|a[n]| \approx \sqrt{\frac{\Psi_d\{x[n]\}}{1 - \left(1 - \frac{\Psi_d\{y[n]\} + \Psi_d\{y[n+1]\}}{4\Psi_d\{x[n]\}}\right)^2}} \quad (5)$$

where $y[n] = x[n] - x[n-1]$. Thus, even for real valued signal, the Hilbert transform is not necessary to obtain the phase information. Their variation called DESA II is defined as

$$f[n] \approx \frac{1}{4\pi}\cos^{-1}\left(1 - \frac{\Psi_d\{x[n+1] - x[n-1]\}}{2\Psi_d\{x[n]\}}\right) \quad (6)$$

$$|a[n]| \approx \frac{2\Psi_d\{x[n]\}}{\sqrt{\Psi_d\{x[n+1] - x[n-1]\}}}. \quad (7)$$

These IF and IA estimators have been successfully used for the formant analysis of speech signals [5].

2.3. Instantaneous Frequency/Amplitude Estimators based on the Higher Order Differential Energy operator

The higher order differential energy operator has been defined as [6]

$$\Gamma^{(k)}\{x[n]\} = x[n]x[n+k-2] - x[n-1]x[n+k-1]. \quad (8)$$

For $k=1$, the discrete higher order differential energy operator is equal to zero. For $k=2$, the higher order differential energy operator becomes $\Gamma^{(2)}\{x[n]\} = \Psi_d\{x[n]\}$ in (3). And for $k=3$, $\Gamma^{(3)}\{x[n]\} = x[n]x[n+1] - x[n-1]x[n+2]$.

In [6], the IF estimator has been defined using the higher order differential energy operator as

$$f[n] = \frac{1}{2\pi}\cos^{-1}\left(\frac{\Gamma^{(3)}\{x[n]\}}{2\cdot\Gamma^{(2)}\{x[n]\}}\right). \quad (9)$$

For general order k, the IF estimator is [7]

$$f^{(k)}[n] = \frac{1}{2\pi}\cdot\frac{1}{k-1}\cos^{-1}\left(\frac{\Gamma^{(2k-1)}\{x[n]\}}{2\cdot\Gamma^{(k)}\{x[n]\}}\right). \quad (10)$$

For k=2, the general form in (10) becomes (9). Using the higher order differential energy operator, the instantaneous amplitude can be obtained as

$$|a[n]| = \sqrt{\frac{\Gamma^{(2)}\{x[n]\}}{\left[1 - \left(\frac{\Gamma^{(3)}\{x[n]\}}{2\Gamma^{(2)}\{x[n]\}}\right)^2\right]}}. \quad (11)$$

And the general form of the IA is proposed as

$$|a^{(k)}[n]| = \left[\Gamma^{(k)}\{x[n]\}\right]^{1/2} \Big/ \left\{\left[1 - \left(\frac{\Gamma^{(2k-1)}\{x[n]\}}{2\Gamma^{(k)}\{x[n]\}}\right)^2\right]\cdot\left[1 - \left(\frac{\Gamma^{(k-1)}\{x[n]\} + \Gamma^{(k+1)}\{x[n]\}}{2\Gamma^{(k)}\{x[n]\}}\right)^2\right]\right\}^{1/4} \quad (12)$$

When k=2, the generalized IA estimator in (12) becomes (11). When k=3, the generalized IF/IA estimator (10) and (12) become

$$f^{(3)}[n] = \frac{1}{4\pi}\cos^{-1}\left(\frac{\Gamma^{(5)}\{x[n]\}}{2\cdot\Gamma^{(3)}\{x[n]\}}\right),$$

and

$$|a^{(3)}[n]| = \frac{\left[\Gamma^{(3)}\{x[n]\}\right]^{1/2}}{\left\{\left[1 - \left(\frac{\Gamma^{(5)}\{x[n]\}}{2\Gamma^{(3)}\{x[n]\}}\right)^2\right]\left[1 - \left(\frac{\Gamma^{(2)}\{x[n]\} + \Gamma^{(4)}\{x[n]\}}{2\Gamma^{(3)}\{x[n]\}}\right)^2\right]\right\}^{1/4}},$$

respectively.

3. Tracking Performance of Instantaneous Frequency and Amplitude Estimators

In this section, the tracking ability of various IF and IA estimators is compared through simulations. The test signal is an AM-FM signal with a sinusoidal instantaneous frequency and a slowly varying amplitude [5]. The signal is defined as

$$x[n]=\left[1+\kappa\cos\left(\frac{\pi}{100}n\right)\right]\cdot \cdot\cos\left[\frac{\pi}{5}n+20\lambda\sin\left(\frac{\pi}{100}n\right)\right]\ ;\ n=1,2,\ldots,400 \tag{13}$$

where $(\kappa,\lambda)\in\{(0.05i,0.05j)\,|\,i,j=1,2,\ldots,10\}$. Here, κ and λ control the amounts of AM and FM, respectively. They change values from 5% to 50% stepwise. For example, when $(\kappa,\lambda)=(0.05,0.05)$ *with i=j=1*, the amount of amplitude change is 0.05 (5%) compared to the constant amplitude 1, and the amount of frequency change is also 0.05(5%) compared to the constant frequency $f=20\,Hz$. The sampling frequency is 200 Hz. From the classical definition (1), the true IF and IA are obtained respectively as

$$f[n]=\frac{1}{10}+\frac{1}{10}\lambda\cos\left(\frac{\pi}{100}n\right),$$

$$a[n]=1+\kappa\cos\left(\frac{\pi}{100}n\right).$$

First, we show the effect of κ and λ values. Figure 1 shows the test signal x[n], the true IF, and true IA when $(\kappa,\lambda)=(0.05,0.05)$, $(\kappa,\lambda)=(0.5,0.05)$, and $(\kappa,\lambda)=(0.5,0.5)$. Figure 1 (a), (b), and (c) show the test signals for those three cases. (d), (e), and (f) are the true IA, and (g), (h) and (i) are the true IF, respectively. By changing κ from 0.05 to 0.5, but fixing λ =0.05 (the first and the second column in Fig. 1), the IF does not change (see (g) and (h)), but the IA does (see (d) and (e)). If the amount of FM is changed from $\lambda=0.05$ to $\lambda=0.5,$ but the AM part fixed to $\kappa=0.5$ (the second and the third column in Fig. 1), the change in the amount of frequency modulation can be easily observed between (h) and (i). The amplitude modulation has not been changed in (e) and (f).

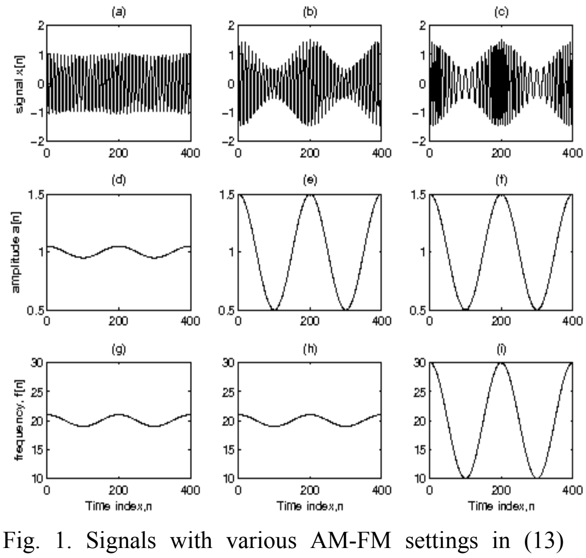

Fig. 1. Signals with various AM-FM settings in (13) (a) $(\kappa,\lambda)=(0.05,0.05)$, (b) $(\kappa,\lambda)=(0.5,0.05)$, (c) $(\kappa,\lambda)=(0.5,0.5)$, (d) IA of signal (a), (e) IA of signal (b), (f) IA of signal (c), (g) IF of signal (a), (h) IF of signal (b), and (i) IF of signal (c).

Next, we compare the estimation accuracy in terms of the mean absolute value of the frequency/amplitude estimation error [5]. The mean absolute value of the frequency estimation error is defined as

$$\boldsymbol{Error(\%)}=\frac{1}{40000}\sum_{i=1}^{10}\sum_{j=1}^{10}\sum_{n=1}^{400}\left|\left[f_{ij}[n]-\hat{f}_{ij}[n]\right]\Big/f_{ij}[n]\right|$$

where $f_{ij}[n]$ and $\hat{f}_{ij}[n]$ are the true IF and the IF estimates with the setting $(\kappa,\lambda)\in\{(0.05i,0.05j)\,|\,i,j=1,2,\ldots,10\}$. The mean absolute value of the amplitude estimation error is also defined likewise. For the noisy cases, zero mean white Gaussian noise has been used. The results are shown in Table 1 and 2. In the tables, “HIGHER” means the IF and IA based on the higher order differential energy operator (10) and (12), respectively. For the IF and IA based on the higher order differential energy operator, estimators with order k=2, 3 and 4 have been used. There is no explicit relationship between the order and the performance improvement. And the IF and IA with orders higher than 4 show no significant improvement in performance.

Table 1: Mean absolute error in % of instantaneous frequency estimators. Taken from [8]

Algorithm	noise free	SNR=30dB	SNR=20dB
DESA I in (4)	0.37	7.63	27.60
DESA II in (6)	0.44	9.90	35.11
HIGHER, k=2	2.92	39.14	96.83
HIGHER, k=3	0.78	4.86	17.33
HIGHER, k=4	1.78	3.21	9.32

Table 2: Mean absolute error in % of instantaneous amplitude estimators.

Algorithm	noise free	SNR=30dB	SNR=20dB
DESA I in (5)	0.51	9.74	29.35
DESA II in (7)	0.57	13.27	42.90
HIGHER, k=2	2.68	25.59	37.22
HIGHER, k=3	0.77	8.57	18.06
HIGHER, k=4	0.87	8.87	19.05

In noise free case, the DESA I IF estimator shows the best result. And the higher order differential energy operator based IF estimator with order k=3 produces acceptable results. Among higher order differential energy operator based methods, the second order shows the worst result. It is because the second order differential energy operator based IF estimator uses the least number of data samples with asymmetry. Under noisy situation, the higher order differential energy operator based IF estimator with order k=4 shows the best results. And, the higher order method with order k=3 produces the better performance than the DESA methods. The IA estimators show similar results. The DESA I IA shows the best estimation for the noise free case. And IA based on the higher order energy operator with order k=3 and 4 shows the acceptable results. For the noisy case, the IA with k=3 shows the best estimation results.

4. Application to Power Disturbances

Here, the IF/IA estimators are applied to distorted signals to show their usefulness as disturbance detectors. The IF and IA used are the ones based on the higher order differential energy operator with the order 4. Note that the unit for the horizontal axis is “seconds” for all figures.

4.1. Sag

The normal signal is modeled as $x[n] = A[n]\cos(2\pi f_0 n + \phi)$. Without loss of generality, the phase is assumed to be $\phi = 0$. The frequency is 60 Hz and the sampling frequency is 10 kHz. Under the normal state, the amplitude is assumed to be $A[n] = 1$. For a certain period of time, the magnitude is decreased 30% of the normal value. But, the frequency does not change. The event happens in the time interval from 0.25 to 0.28 second. Fig. 2 shows the IF and IA estimating results. The IF and IA estimators can detect the start and the end of the sag event correctly. Between them, the IF is still 60 Hz since the change is not the frequency but the amplitude over the sagged time duration. The IA shows the decrease of the amplitude.

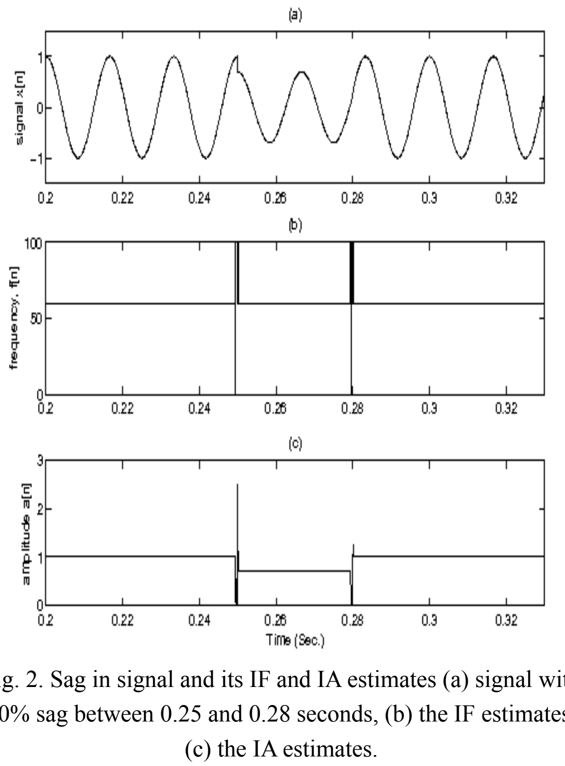

Fig. 2. Sag in signal and its IF and IA estimates (a) signal with 30% sag between 0.25 and 0.28 seconds, (b) the IF estimates, (c) the IA estimates.

4.2. Swell

The signal has the increased amplitude for a short period of time. The amount of change is 30 % between 0.2 and 0.28 sec. Fig. 3 shows the IF/IA estimation values for swelled signal.

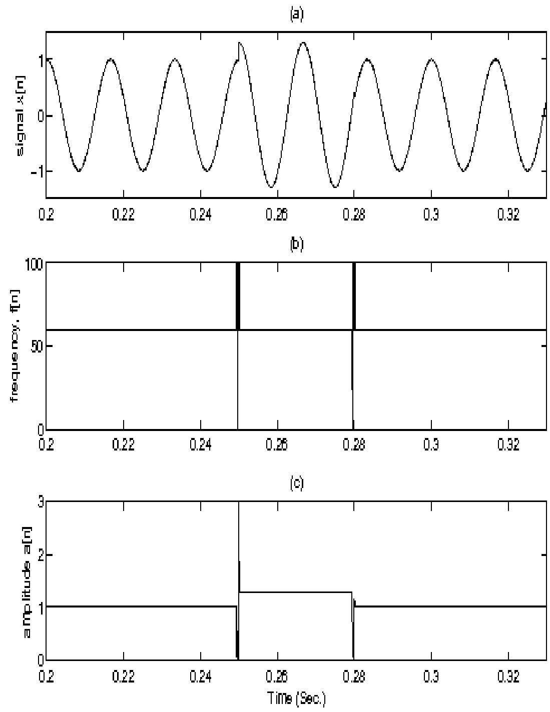

Fig. 3. Swell in signal and its IF and IA estimates (a) signal with 30% swell between 0.25 and 0.28 seconds, (b) the IF estimates, (c) the IA estimates.

Except the starting and ending moment of the swelling, the IF estimates are all 60 Hz since the frequency of the signal has not changed. The IA estimator shows the increase of the amplitude correctly.

4.3. Harmonic Distortion

The signal is distorted by harmonic terms of 60 Hz. The amount of harmonics can be measured by the total harmonic distortion (THD) defined as [12]

$$THD = \frac{\sqrt{\sum_{k=2}^{\infty} V_k}}{V_1}$$

where V_1 is the amplitude of the signal of frequency 60 Hz and V_k ’s with *k>1* are the amplitudes of the harmonics with frequencies $k \cdot 60$ Hz. If there is no harmonics, then THD becomes 0. We test the IF estimators with signals with THD=30%. We assumed that the number of harmonics is 10 terms from 2nd harmonic to 11-th harmonic. The amplitudes of harmonics, V_k , are obtained from the random number generator, and scaled to get the desired THD. Fig. 4 shows the IF/IA estimating results for the harmonic distorted signal. Both the IF and the IA show abrupt changes during the distortion interval. Thus, the detection can be achieved using the IF/IA estimators.

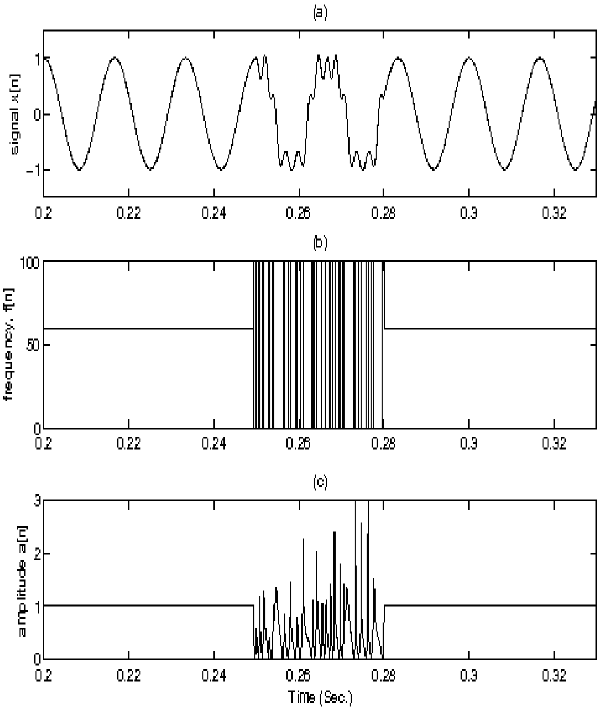

Fig. 4. Harmonically distorted signal and its IF and IA estimates (a) signal with THD=30%, (b) the IF estimates, (c) the IA estimates.

5. Conclusions

The various definitions of the instantaneous frequency and amplitude estimators have been reviewed. And the instantaneous amplitude estimator based on the higher order differential energy operator and its general form have been proposed. The proposed IA estimators with certain order show better amplitude tracking performance than existing IA estimators under noisy condition. The IF and IA based on the higher order differential energy operator have been applied to detect the various power disturbance patterns. They successfully detect the onset time of the disturbances, and show different IF/IA signatures depending on the type of disturbances. Exploiting these unique IF/IA combination patterns, the kind of disturbance can be identified, which remains to be further studied.

References

[1] F. Hlawatsch and G.F. Boudreaux-Bartels, "Linear and quadratic time-frequency representations," *IEEE Signal Processing Magazine*, 1990.

[2] L. Qiu, H. Yang, S. Koh, "Fundamental frequency determination based on instantaneous frequency estimation", *Signal Processing*, vol. 44, pp.233-241, 1995.

[3] L. Qiu, H. Yang, S. Koh, "Fundamental frequency determination based on instantaneous frequency estimation", *Signal Processing*, vol. 44, pp.233-241, 1995.

[4] P. Maragos, J. F. Kaiser, and T. F. Quatieri, "On amplitude and frequency demodulation using energy operators," *IEEE Trans. Signal Processing*, vol. 41, pp. 1532-1550, Apr. 1993.

[5] P. Maragos, J. F. Kaiser, and T. F. Quatieri, "Energy separation in signal modulations with application to speech analysis," *IEEE Trans. Signal Processing*, vol. 41, pp. 3024-3051, Oct. 1993.

[6] P. Maragos, and A. Potamianos, "Higher order differential energy operators," *IEEE Signal Processing Letters*, vol. 2, pp. 152-154, Aug. 1995.

[7] B. Iem, "Frequency/Amplitude Separation Algorithm using the Higher Order Differential Energy Operator and Its Application," *Trans. of KIEE*, vol. 56, pp. 1498-1502, Aug. 2007.

[8] B. Iem, "Generalization of an instantaneous frequency estimator based on the higher order differential energy operator," *Proc. of IEEE TENCON 2008*, Nov. 2008.

[9] B. Iem, "An instantaneous frequency estimator based on the symmetric higher order differential energy operator," *IEICE Trans. on Fundamentals*, vol. E93-A, pp. 227-232, Jan. 2010.

[10] B. Iem, "Estimation of Fundamental Frequency Using an Instantaneous Frequency Based on the Symmetric Higher Order Differential Energy Operator," *Trans. of KIEE*, vol. 60, pp. 2374-2379, Dec. 2011.

[11] K. Srinivasan, "Digital measurement of the voltage flickers," *IEEE Trans. Power Delivery*, vol. 6, pp. 1593-1598, 1991.

[12] S. Santoso, E.J. Powers and W.M. Grady, "Electric power quality disturbance detection using wavelet transform analysis," Proc. of *IEEE TF/TS Symposium*, pp. 166-169, Philadelphia, PA, Oct. 1994.

[13] O. Poisson, P. Rioual, and M. Meunier, "New signal processing tools applied to power quality analysis," *IEEE Trans. Power Delivery*, vol. 14, pp. 561-566, Apr. 1999.

Byeong-Gwan Iem
Professor of the Gangnung-Wonju National University
Research Area: Digital Signal Processing and its related applications
E-mail : ibg@gwnu.ac.kr