International Journal of Fuzzy Logic and Intelligent Systems, vol. 12, no. 2, June 2012, pp. 149-153
http://dx.doi.org/10.5391/IJFIS.2012.12.2.149 pISSN 1598-2645 eISSN 2093-744X

Fuzzy Almost Strongly (r, s)-Semicontinuous Mappings

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Abstract

In this paper, we introduce the concept of fuzzy almost strongly (r, s)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The relationships among fuzzy strongly (r, s)-semicontinuous, fuzzy almost (r, s)-continuous, fuzzy almost (r, s)-semicontinuous, and fuzzy almost strongly (r, s)-semicontinuous mappings are discussed. The characterization for the fuzzy almost strongly (r, s)-semicontinuous mappings is obtained.

Key Words: fuzzy continuous, fuzzy topology, fuzzy almost strongly (r, s)-semicontinuous

1. Introduction

The concept of fuzzy set was introduced by Zadeh [1]. Chang [2] defined fuzzy topological spaces. These spaces and its generalizations are later studied by several authors, one of which, developed by Šostak [3], used the idea of degree of openness. This type of generalization of fuzzy topological spaces was later rephrased by Chattopadhyay and his colleagues [4], and by Ramadan [5].

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [6] and Çoker [7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [8] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth topological spaces and intuitionistic fuzzy topological spaces. Shi Zhong Bai [9] introduced the concept of fuzzy almost strongly semicontinuous mappings on Chang's fuzzy topological spaces.

In this paper, we introduce the concept of fuzzy almost strongly (r, s)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense. The relationships among fuzzy strongly (r, s)-semicontinuous, fuzzy almost (r, s)-continuous, fuzzy almost (r, s)-semicontinuous, and fuzzy almost strongly (r, s)-semicontinuous mappings are discussed. The characterization for the fuzzy almost strongly (r, s)-semicontinuous mappings is obtained.

Manuscript received Apr. 25, 2012; revised Jun. 7, 2012; accepted Jun. 10, 2012.
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2. Preliminaries

For the nonstandard definitions and notations we refer to [10, 11, 12, 13].

Let $I(X)$ be a family of all intuitionistic fuzzy sets in X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

Definition 2.1. ([8]) Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense*(SoIFT for short) $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2)$ on X is a mapping $\mathcal{T} : I(X) \to I \otimes I$ which satisfies the following properties:

(1) $\mathcal{T}_1(\underline{0}) = \mathcal{T}_1(\underline{1}) = 1$ and $\mathcal{T}_2(\underline{0}) = \mathcal{T}_2(\underline{1}) = 0$.

(2) $\mathcal{T}_1(A \cap B) \geq \mathcal{T}_1(A) \wedge \mathcal{T}_1(B)$ and $\mathcal{T}_2(A \cap B) \leq \mathcal{T}_2(A) \vee \mathcal{T}_2(B)$.

(3) $\mathcal{T}_1(\bigcup A_i) \geq \bigwedge \mathcal{T}_1(A_i)$ and $\mathcal{T}_2(\bigcup A_i) \leq \bigvee \mathcal{T}_2(A_i)$.

The $(X, \mathcal{T}) = (X, \mathcal{T}_1, \mathcal{T}_2)$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*(SoIFTS for short). Also, we call $\mathcal{T}_1(A)$ a *gradation of openness* of A and $\mathcal{T}_2(A)$ a *gradation of nonopenness* of A.

Definition 2.2. ([10, 11, 13]) Let A be an intuitionistic fuzzy set in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then A is said to be

(1) *fuzzy* (r, s)*-semiopen* if $\text{cl}(\text{int}(A, r, s), r, s) \supseteq A$,

(2) *fuzzy* (r, s)*-semiclosed* if $\text{int}(\text{cl}(A, r, s), r, s) \subseteq A$,

(3) *fuzzy* (r, s)*-regular open* if $\text{int}(\text{cl}(A, r, s), r, s) = A$,

(4) *fuzzy* (r, s)*-regular closed* if $\text{cl}(\text{int}(A, r, s), r, s) = A$,

(5) *fuzzy strongly* (r, s)*-semiopen* if $A \subseteq \text{int}(\text{cl}(\text{int}(A, r, s), r, s), r, s)$,

(6) *fuzzy strongly* (r, s)*-semiclosed* if
$A \supseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A, r, s), r, s), r, s)$.

Definition 2.3. ([13]) Let $(X, \mathcal{T}_1, \mathcal{T}_2)$ be a SoIFTS. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the *fuzzy strongly* (r, s)*-semiinterior* is defined by

$$\mathrm{ssint}(A, r, s) = \bigcup\{B \in I(X) \mid B \subseteq A,$$

$$B \text{ is fuzzy strongly } (r, s)\text{-semiopen}\}$$

and the *fuzzy strongly* (r, s)*-semiclosure* is defined by

$$\mathrm{sscl}(A, r, s) = \bigcap\{B \in I(X) \mid A \subseteq B,$$

$$B \text{ is fuzzy strongly } (r, s)\text{-semiclosed}\}.$$

Theorem 2.4. ([11]) (1) The fuzzy (r, s)-closure of a fuzzy (r, s)-open set is fuzzy (r, s)-regular closed for each $(r, s) \in I \otimes I$.
(2) The fuzzy (r, s)-interior of a fuzzy (r, s)-closed set is fuzzy (r, s)-regular open for each $(r, s) \in I \otimes I$.

Definition 2.5. ([11, 12, 14]) Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called

(1) a *fuzzy strongly* (r, s)*-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy strongly (r, s)-semiopen set in X for each fuzzy (r, s)-open set B in Y,

(2) a *fuzzy almost* (r, s)*-continuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-open set in X for each fuzzy (r, s)-regular open set B in Y,

(3) a *fuzzy almost* (r, s)*-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy (r, s)-semiopen set in X for each fuzzy (r, s)-regular open set B in Y.

Definition 2.6. ([11]) Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in a SoIFTS $(X, \mathcal{T}_1, \mathcal{T}_2)$ and $(r, s) \in I \otimes I$. Then an intuitionistic fuzzy set A in X is called a *fuzzy* (r, s)*-neighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy (r, s)-open set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$.

Definition 2.7. ([9]) Let $f : (X_1, \delta_1) \to (X_2, \delta_2)$ be a mapping from a fuzzy space X_1 to another fuzzy space X_2. Then f is called a *fuzzy almost strongly semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy strongly semiopen set of X_1 for each fuzzy regular open set B of X_2.

3. Fuzzy almost strongly (r, s)-semicontinuous mappings

Now, we define the notion of fuzzy almost strongly (r, s)-semicontinuous mappings on intuitionistic fuzzy topological spaces in Šostak's sense, and then we investigate some of their properties.

Definition 3.1. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is called a *fuzzy almost strongly* (r, s)*-semicontinuous* mapping if $f^{-1}(B)$ is a fuzzy strongly (r, s)-semiopen set in X for each fuzzy (r, s)-regular open set B in Y.

Definition 3.2. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is said to be *fuzzy almost strongly* (r, s)*-semicontinuous at an intuitionistic fuzzy point* $x_{(\alpha,\beta)}$ in X if for each fuzzy (r, s)-regular open set B in Y with $f(x_{(\alpha,\beta)}) \in B$, there is a fuzzy strongly (r, s)-semiopen set A in X such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq B$.

Theorem 3.3. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r, s) \in I \otimes I$. Then f is fuzzy almost strongly (r, s)-semicontinuous if and only if f is fuzzy almost strongly (r, s)-semicontinuous at each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X.

Proof. Let f be fuzzy almost strongly (r, s)-semicontinuous, $x_{(\alpha,\beta)}$ an intuitionistic fuzzy point in X, and B a fuzzy (r, s)-regular open set in Y with $f(x_{(\alpha,\beta)}) \in B$. Since f is fuzzy almost strongly (r, s)-semicontinuous, $f^{-1}(B)$ is a fuzzy strongly (r, s)-semiopen set in X. Putting $A = f^{-1}(B)$. Then A is fuzzy strongly (r, s)-semiopen in X, $x_{(\alpha,\beta)} \in A$, and $f(A) = f(f^{-1}(B)) \subseteq B$. Since $x_{(\alpha,\beta)}$ is an arbitrary intuitionistic fuzzy point in X, we conclude that f is fuzzy almost strongly (r, s)-semicontinuous at each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X.

Conversely, let B be a fuzzy (r, s)-regular open set in Y and $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then $f(x_{(\alpha,\beta)}) \in B$. From the assumption, there is a fuzzy strongly (r, s)-semiopen set $A_{x_{(\alpha,\beta)}}$ in X such that $x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}}$ and $f(A_{x_{(\alpha,\beta)}}) \subseteq B$. Thus

$$\begin{aligned} f^{-1}(B) &= \bigcup\{x_{(\alpha,\beta)} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq \bigcup\{A_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\} \\ &\subseteq f^{-1}(B). \end{aligned}$$

Hence $f^{-1}(B) = \bigcup\{A_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\}$, which is a fuzzy strongly (r, s)-semiopen set in X. Therefore f is fuzzy almost strongly (r, s)-semicontinuous. □

Remark 3.4. It is clear that the following implications are true:

(1) fuzzy strongly (r,s)-semicontinuous $\Rightarrow$ fuzzy almost strongly (r,s)-semicontinuous.

(2) fuzzy almost (r,s)-continuous $\Rightarrow$ fuzzy almost strongly (r,s)-semicontinuous.

(3) fuzzy almost strongly (r,s)-semicontinuous $\Rightarrow$ fuzzy almost (r,s)-semicontinuous.

However, the following examples show that all of the converses need not be true.

Example 3.5. Let $X = \{x,y,z\}$ and let A_1, A_2, A_3, and A_4 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0.1, 0.8),\ \ A_1(y) = (0,1),\ \ A_1(z) = (0.2, 0.6);$$

$$A_2(x) = (0.5, 0.5),\ \ A_2(y) = (0.5, 0.5),\ \ A_2(z) = (0.5, 0.5);$$

$$A_3(x) = (0.2, 0.7),\ \ A_3(y) = (0.3, 0.6),\ \ A_3(z) = (0.4, 0.5);$$

and

$$A_4(x) = (0.6, 0.3),\ \ A_4(y) = (0.5, 0.5),\ \ A_4(z) = (0.7, 0.1).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, A_2, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_3, A_4, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$, $f(y) = y$, and $f(z) = z$. Note that

$$\text{int}(\text{cl}(A_3, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_3,$$

$$\text{int}(\text{cl}(A_4, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = \underline{1} \neq A_2 \ \text{ in } \ (X, \mathcal{U}).$$

Thus A_3 is fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open but A_4 is not a fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open set in $(X, \mathcal{U})$. Since

$$\begin{aligned} f^{-1}(A_3) = A_3 \ &\subseteq\ \text{int}(\text{cl}(\text{int}(A_3, \tfrac{1}{2}, \tfrac{1}{3}), \tfrac{1}{2}, \tfrac{1}{3}), \tfrac{1}{2}, \tfrac{1}{3}) \\ &=\ \text{int}(\text{cl}(A_1, \tfrac{1}{2}, \tfrac{1}{3}), \tfrac{1}{2}, \tfrac{1}{3}) \\ &=\ \text{int}(A_2^c, \tfrac{1}{2}, \tfrac{1}{3}) = A_2, \end{aligned}$$

we conclude that f is fuzzy almost strongly $(\frac{1}{2}, \frac{1}{3})$-semicontinuous. However, f is neither fuzzy almost $(\frac{1}{2}, \frac{1}{3})$-continuous nor fuzzy strongly $(\frac{1}{2}, \frac{1}{3})$-semicontinuous. For $f^{-1}(A_3) = A_3$ is not fuzzy $(\frac{1}{2}, \frac{1}{3})$-open in $(X, \mathcal{T})$ and

$$\begin{aligned} f^{-1}(A_4) = A_4 \ &\not\subseteq\ \text{int}(\text{cl}(\text{int}(A_4, \tfrac{1}{2}, \tfrac{1}{3}), \tfrac{1}{2}, \tfrac{1}{3}), \tfrac{1}{2}, \tfrac{1}{3}) \\ &=\ \text{int}(\text{cl}(A_2, \tfrac{1}{2}, \tfrac{1}{3}), \tfrac{1}{2}, \tfrac{1}{3}) \\ &=\ \text{int}(A_2^c, \tfrac{1}{2}, \tfrac{1}{3}) = A_2. \end{aligned}$$

Example 3.6. Let $X = \{x,y,z\}$ and let A_1 and A_2 be intuitionistic fuzzy sets in X defined as

$$A_1(x) = (0, 0.5),\ \ A_1(y) = (0.3, 0.5),\ \ A_1(z) = (0.3, 0.5);$$

and

$$A_2(x) = (0, 0.7),\ \ A_2(y) = (0.2, 0.7),\ \ A_2(z) = (0.2, 0.8).$$

Define $\mathcal{T} : I(X) \to I \otimes I$ and $\mathcal{U} : I(X) \to I \otimes I$ by

$$\mathcal{T}(A) = (\mathcal{T}_1(A), \mathcal{T}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_2, \\ (0,1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}(A) = (\mathcal{U}_1(A), \mathcal{U}_2(A)) = \begin{cases} (1,0) & \text{if } A = \underline{0}, \underline{1}, \\ (\frac{1}{2}, \frac{1}{3}) & \text{if } A = A_1, \\ (0,1) & \text{otherwise.} \end{cases}$$

Then clearly $\mathcal{T}$ and $\mathcal{U}$ are SoIFTs on X. Consider a mapping $f : (X, \mathcal{T}) \to (X, \mathcal{U})$ defined by $f(x) = x$, $f(y) = y$, and $f(z) = z$. Note that

$$\text{int}(\text{cl}(A_1, \frac{1}{2}, \frac{1}{3}), \frac{1}{2}, \frac{1}{3}) = A_1 \ \text{ in } \ (X, \mathcal{U}),$$

and hence A_1 is fuzzy $(\frac{1}{2}, \frac{1}{3})$-regular open in $(X, \mathcal{U})$. Since $f^{-1}(A_1) = A_1$ is fuzzy $(\frac{1}{2}, \frac{1}{3})$-semiopen in $(X, \mathcal{T})$, f is fuzzy almost $(\frac{1}{2}, \frac{1}{3})$-semicontinuous. However, f is not a fuzzy almost strongly $(\frac{1}{2}, \frac{1}{3})$-semicontinuous mapping. For

$$\begin{aligned} f^{-1}(A_1) = A_1 \ &\not\subseteq\ \text{int}(\text{cl}(\text{int}(A_1, \tfrac{1}{2}, \tfrac{1}{3}), \tfrac{1}{2}, \tfrac{1}{3}), \tfrac{1}{2}, \tfrac{1}{3}) \\ &=\ \text{int}(\text{cl}(A_2, \tfrac{1}{2}, \tfrac{1}{3}), \tfrac{1}{2}, \tfrac{1}{3}) \\ &=\ \text{int}(A_2^c, \tfrac{1}{2}, \tfrac{1}{3}) = A_2. \end{aligned}$$

Theorem 3.7. Let $f : (X, \mathcal{T}_1, \mathcal{T}_2) \to (Y, \mathcal{U}_1, \mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then the following statements are equivalent:

(1) f is fuzzy almost strongly (r,s)-semicontinuous.

(2) $f^{-1}(B)$ is fuzzy strongly (r,s)-semiclosed in X for each fuzzy (r,s)-regular closed set B in Y.

(3) For each fuzzy (r,s)-closed set B in Y,

$$\text{sscl}(f^{-1}(\text{cl}(\text{int}(B,r,s),r,s)),r,s) \subseteq f^{-1}(B).$$

(4) For each fuzzy (r,s)-open set B in Y,

$$f^{-1}(B) \subseteq \text{ssint}(f^{-1}(\text{int}(\text{cl}(B,r,s),r,s)),r,s).$$

(5) For each fuzzy (r,s)-semiopen set B in Y,

$$\text{sscl}(f^{-1}(B),r,s) \subseteq f^{-1}(\text{cl}(B,r,s)).$$

(6) For each fuzzy (r,s)-semiclosed set B in Y,

$$f^{-1}(\text{int}(B,r,s)) \subseteq \text{ssint}(f^{-1}(B),r,s).$$

Proof. It is clear that (1) ⇔ (2), (3) ⇔ (4), (5) ⇔ (6).

(2) ⇒ (3) Let B be a fuzzy (r,s)-closed set in Y. By Theorem 2.4, $\text{cl}(\text{int}(B,r,s),r,s)$ is fuzzy (r,s)-regular closed in Y. By (2), $f^{-1}(\text{cl}(\text{int}(B,r,s),r,s))$ is a fuzzy strongly (r,s)-semiclosed set in X. Hence
$\text{sscl}(f^{-1}(\text{cl}(\text{int}(B,r,s),r,s)),r,s)$

$$\begin{aligned} &= f^{-1}(\text{cl}(\text{int}(B,r,s),r,s)) \\ &\subseteq f^{-1}(\text{cl}(B,r,s)) = f^{-1}(B). \end{aligned}$$

(3) ⇒ (5) Let B be a fuzzy (r,s)-semiopen set in Y. Then $\text{cl}(B,r,s)$ is fuzzy (r,s)-closed in Y. By (3), we have
$\text{sscl}(f^{-1}(B),r,s)$

$$\begin{aligned} &\subseteq \text{sscl}(f^{-1}(\text{cl}(\text{int}(B,r,s),r,s)),r,s) \\ &\subseteq \text{sscl}(f^{-1}(\text{cl}(\text{int}(\text{cl}(B,r,s),r,s),r,s)),r,s) \\ &\subseteq f^{-1}(\text{cl}(B,r,s)). \end{aligned}$$

(5) ⇒ (2) Let B be a fuzzy (r,s)-regular closed set in Y. Then B is fuzzy (r,s)-closed and fuzzy (r,s)-semiopen in Y. By (5), we obtain

$$\begin{aligned} f^{-1}(B) \subseteq \text{sscl}(f^{-1}(B),r,s) &\subseteq f^{-1}(\text{cl}(B,r,s)) \\ &= f^{-1}(B). \end{aligned}$$

Thus we have $f^{-1}(B) = \text{sscl}(f^{-1}(B),r,s)$, which is a fuzzy strongly (r,s)-semiclosed set in X. □

Definition 3.8. Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in a SoIFTS $(X,\mathcal{T}_1,\mathcal{T}_2)$ and $(r,s) \in I \otimes I$. Then an intuitionistic fuzzy set A in X is called a *fuzzy strongly (r,s)-semineighborhood* of $x_{(\alpha,\beta)}$ if there is a fuzzy strongly (r,s)-semiopen set B in X such that $x_{(\alpha,\beta)} \in B \subseteq A$.

Theorem 3.9. Let $f:(X,\mathcal{T}_1,\mathcal{T}_2) \to (Y,\mathcal{U}_1,\mathcal{U}_2)$ be a mapping from a SoIFTS X to a SoIFTS Y and $(r,s) \in I \otimes I$. Then f is fuzzy almost strongly (r,s)-semicontinuous if and only if for each intuitionistic fuzzy point $x_{(\alpha,\beta)}$ in X and each fuzzy (r,s)-neighborhood B of $f(x_{(\alpha,\beta)})$, there is a fuzzy strongly (r,s)-semineighborhood A of $x_{(\alpha,\beta)}$ such that $x_{(\alpha,\beta)} \in A$ and $f(A) \subseteq \text{int}(\text{cl}(B,r,s),r,s)$.

Proof. Let $x_{(\alpha,\beta)}$ be an intuitionistic fuzzy point in X and B a fuzzy (r,s)-neighborhood of $f(x_{(\alpha,\beta)})$. Then there is a fuzzy (r,s)-open set C in Y such that $f(x_{(\alpha,\beta)}) \in C \subseteq B$. Thus $x_{(\alpha,\beta)} \in f^{-1}(C) \subseteq f^{-1}(B)$. Since f is fuzzy almost strongly (r,s)-semicontinuous, by Theorem 3.7, we have

$$\begin{aligned} f^{-1}(C) &\subseteq \text{ssint}(f^{-1}(\text{int}(\text{cl}(C,r,s),r,s)),r,s) \\ &\subseteq \text{ssint}(f^{-1}(\text{int}(\text{cl}(B,r,s),r,s)),r,s). \end{aligned}$$

Put $A = f^{-1}(\text{int}(\text{cl}(B,r,s),r,s))$. By Theorem 2.4, $\text{int}(\text{cl}(B,r,s),r,s)$ is fuzzy (r,s)-regular open in Y, and hence $A = f^{-1}(\text{int}(\text{cl}(B,r,s),r,s))$ is a fuzzy strongly (r,s)-semiopen set in X. Thus
$x_{(\alpha,\beta)} \in f^{-1}(C)$

$$\begin{aligned} &\subseteq \text{ssint}(f^{-1}(\text{int}(\text{cl}(B,r,s),r,s)),r,s) \\ &= \text{ssint}(A,r,s) = A. \end{aligned}$$

Hence we conclude that A is a fuzzy strongly (r,s)-semineighborhood of $x_{(\alpha,\beta)}$ and

$$\begin{aligned} f(A) &\subseteq f(f^{-1}(\text{int}(\text{cl}(B,r,s),r,s))) \\ &\subseteq \text{int}(\text{cl}(B,r,s),r,s). \end{aligned}$$

Conversely, let B be a fuzzy (r,s)-regular open set in Y and $x_{(\alpha,\beta)} \in f^{-1}(B)$. Then B is fuzzy (r,s)-open in Y, and hence B is a fuzzy (r,s)-neighborhood of $f(x_{(\alpha,\beta)})$. From the assumption, there is a fuzzy strongly (r,s)-semineighborhood $A_{x_{(\alpha,\beta)}}$ of $x_{(\alpha,\beta)}$ such that $x_{(\alpha,\beta)} \in A_{x_{(\alpha,\beta)}}$ and

$$f(A_{x_{(\alpha,\beta)}}) \subseteq \text{int}(\text{cl}(B,r,s),r,s) = B.$$

Because $A_{x_{(\alpha,\beta)}}$ is a fuzzy strongly (r,s)-semineighborhood of $x_{(\alpha,\beta)}$, there is a fuzzy strongly (r,s)-semiopen set $C_{x_{(\alpha,\beta)}}$ such that

$$\begin{aligned} x_{(\alpha,\beta)} \in C_{x_{(\alpha,\beta)}} &\subseteq A_{x_{(\alpha,\beta)}} \\ &\subseteq f^{-1}(f(A_{x_{(\alpha,\beta)}})) \subseteq f^{-1}(B). \end{aligned}$$

Hence $f^{-1}(B) = \{C_{x_{(\alpha,\beta)}} \mid x_{(\alpha,\beta)} \in f^{-1}(B)\}$, which is a fuzzy strongly (r,s)-semiopen set in X. Therefore f is fuzzy almost strongly (r,s)-semicontinuous. □

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