A review on several methods for fast generation of digital Fresnel holograms

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Abstract *– Computer generated holography (CGH) is technology for generating holograms of synthetic, three dimensional (3D) objects which may not exist in the physical world. The process, however, requires heavy amount of computation as the resolution of a hologram is significantly higher than that of a typical optical image. This paper reviews four modern techniques for fast generation of digital Fresnel holograms which are important in the development of holographic video systems. The methods that will be described include the virtual window, sub-line, wavefront recording plane (WRP), and the interpolative WRP schemes. These works share the common objective to generate digital Fresnel hologram at a speed that is close to the video frame rate, and with complexity which is realizable with affordable computing and reconfigurable hardware devices. The author will present the principles and realization of these works, as well as some potential area of research in digital holography.*

Keywords: Computer generated holography, Virtual window, Sub-line, Wavefront recording plane, Interpolative wavefront recording plane.

1   Introduction

In the past two decades, the study on computer generated holography (CGH) has been identified as an important area of research [1]. The technology enables a three dimensional (3D) object scene, which may be synthetic or acquired from a physical environment, to be recorded in a digital image (known as a digital hologram). The latter can be taken to reconstruct the original 3D scene by illuminating with a coherent beam. The most common hologram is the Fresnel hologram, as it can be generated directly with the basic Fresnel diffraction equation. This process, on its own, is not a complicated one. An effective approach, refer as 'ray tracing' in some literatures, accumulates the diffraction fringe patterns emerges from each object point on the hologram plane. Suppose a scene comprises of a collection of self-illuminating object points $O = \left[o_0\left(x_0,y_0,z_0\right), o_1\left(x_1,y_1,z_1\right), \ldots\ldots, o_{N-1}\left(x_{N-1},y_{N-1},z_{N-1}\right) \right]$, the diffraction pattern $D(x,y)$ on the hologram plane is given by

$$D(x,y) = \sum_{j=0}^{N_P-1} \frac{a_j}{r_j} \exp\left(ikr_j\right) = \sum_{j=0}^{N_P-1}\left[\frac{a_j}{r_j}\cos\left(kr_j\right) + i\frac{a_j}{r_j}\sin\left(kr_j\right)\right], \quad (1)$$

where a_j and r_j represents the intensity of the 'jth' point in O and its distance to the position (x,y) on the diffraction plane, $k = \dfrac{2\pi}{\lambda}$ is the wavenumber and λ is the wavelength of the light. An off-axis hologram is generated by adding an oblique planar wave to the diffraction patterns, and taking the real part of the product as

$$H(x,y) = RE\left[D(x,y)R(y)\right] \quad (2)$$

It can be inferred from Eq. (1) that the amount of computation involved in the hologram generation process is enormous, especially when the hologram size (i.e. the range of x and y) is large and the number of object points is numerous. In the past, lots of research attempts have been conducted to overcome the above problems, which include, but not limited to the works developed in [2]-[8]. As it is not possible to describe all of them in here, we have selected the following works which are focusing on the development of video holographic systems.

a. The virtual window [6].
b. The sub-line approach [8].
c. The wavefront recording plane (WRP) method [9].
d. The interpolative WRP method [10].

The above methods will be presented in sub-sections 2.1 to 2.4. The optical reconstructed images of digital holograms generated with the IWRP method, which is so far the fastest method amongst its peers, is illustrated in section 3. These will be followed by a conclusion summarizing the essence of the paper.

2 Fast methods for generating digital Fresnel holograms

In this paper, 4 modern methods for fast generation of digital Fresnel hologram will be presented. The details of each scheme will be highlighted in the following sub-sections.

2.1 Constraint viewing zone with virtual window [6]

The concept of virtual window is suggested by Yoshikawa et al, who stated that it is only necessary to compute the diffraction patterns that are visible to the viewer. As an example, the object point in figure 1 is observable within the virtual window, which is only contributed by the diffraction patterns in the area bounded by the dotted square in the hologram.
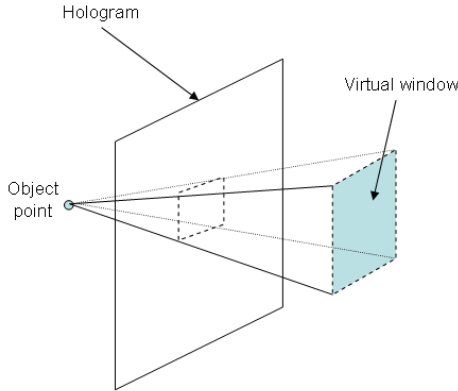

Figure 1 The concept of virtual window

As simple as it may sound, this ideal eliminates significant amount of redundant calculations as the window of visibility of an object point (and hence the optical wavefront emerged from it) could be much smaller than the hologram. As can be inferred from figure 1, if the viewpoint (governed by the virtual window) is restricted to a sufficiently small region, the amount of calculation will be decreased to an extent that can be conducted with a commodity personal computer (PC). Apart from its direct application, this concept is also applied either directly, or indirectly in other CGH techniques, such as the methods 'c' to 'd' which will be described in the later part of this paper.

2.2 Hologram generation from Sub-lines [8]

The sub-line method assumes that the object points are distributed within a narrow distance (depth) from the hologram. Under this assumption, the object scene is uniformly partitioned into a stack of horizontal scan plane as shown in figure 2.
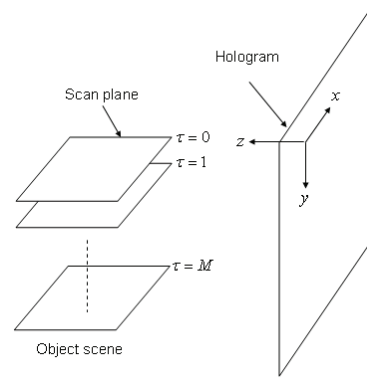

Figure 2 The sub-line approach

A 1D sub-line $O(x, \tau)$ is computed from each scan plane at vertical position τ as

$$O(x, \tau) = \sum_{j=0}^{N(\tau)-1} \frac{a_{j;\tau}}{r_0} \exp\left(ik \frac{(x - x_{j;\tau})^2}{2z_{j;\tau}} \right), \qquad (3)$$

where $N(\tau)$ is the number of object points on the scan plane at $y = \tau$. $a_{j;\tau}$ and $z_{j;\tau}$ are the amplitude of the 'jth' object point on the scan plane, and its perpendicular distance (along the direction z) to the hologram, respectively.

Next, the diffraction pattern is generated by convolving each column of the sub-lines with a 1D reference beam $B(y)$ as

$$D(x, y) = O(x, y) * B(y) \qquad (4)$$

The computation of the diffraction pattern only involved a pair of 1D process, which is significantly smaller than Eq. (1). The sub-line method is realized with field programmable gate array (FPGA) and GPU. A medium size hologram of 2048x2048 pixels (assuming same number of object points) can be generated at around 10 frames per second. The method, however, does not preserve the vertical parallax information.

2.3 Hologram generation from the wavefront recording plane (WRP) [9]

The WRP method, in certain ways, is similar to the virtual window approach. A hypothetic vertical plane $u_w(x, y)$, known as the WRP, is placed very close to the object scene, and the diffraction pattern contributed by each object point is accumulated onto the WRP. The arrangement is the same as that shown in figure 1, with the hologram replaced by the WRP. Suppose the visibility is restricted to a virtual window, it is only necessary to compute the diffraction pattern within a small region of size $W \times W$ on the WRP as

$$u_w(x, y) = \sum_{j=0}^{N-1} G_j, \quad \text{where}$$

$$G_j = \begin{cases} \dfrac{A_j}{R_{wj}} \exp\left(i\dfrac{2\pi}{\lambda}R_{wj}\right) & if\ |x - x_j|\ and\ |y - y_j| < \tfrac{1}{2}W \\ 0 & otherwise \end{cases} \quad (5)$$

$R_{wj} = \sqrt{\left(x - x_j\right)^2 + \left(y - y_j\right)^2 + d_j^2}$ is the distance of the point from the WRP. $0 < x_j < X$ and $0 < y_j < Y$ are the horizontal and vertical positions of the jth object point.

Subsequently, $u_w(x,y)$ is expanded to a full hologram $u(x,y)$ which is located at a relatively larger distance from the object scene.

$$u(x,y) = K F^{-1}\big[F[u_w(x,y)] \cdot F[h(x,y)]\big], \quad (6)$$

where $F[\cdot]$ and $F^{-1}[\cdot]$ denote the forward and inverse Fourier transform, respectively. $K = -\dfrac{i}{\lambda z_w} exp\left(i\dfrac{2\pi z}{\lambda}\right)$ is a constant and $h(x,y) = exp\left(i\frac{\pi}{\lambda z_w}\left(x^2 + y^2\right)\right)$ is an impulse function which is fixed for a given separation z_w between the WRP and the hologram. As reported in the article, these two processes can be conducted swiftly with the multiple core CPU and the GPU, and capable of generating a 2048x2048 hologram, representing 10^4 object points at 10 frames per second.

2.4 Near computation free hologram generation from the interpolative wavefront recording plane (IWRP) [10]

The IWRP method assumes that the resolution of the scene image is smaller than that of the hologram, which is generally true in practice. As a result, it is unnecessary to convert every object point of the scene to its wavefront on the WRP. On this basis, the object scene is down-sampled evenly by M times (where $M > 0$) along the horizontal and the vertical directions. Each object point projects a small, non-overlapping diffraction pattern representing a square area covered by each sample point onto the WRP as shown in figure 3. Different from the WRP method reported in [9], there is no need to accumulate the diffraction pattern and the process is computational free in practice. Subsequently, Eq. (6) is applied to expand the WRP into a full hologram. The method, implemented with GPU, is capable of generating hologram representing 10^6 object points at over 40 frames per second.
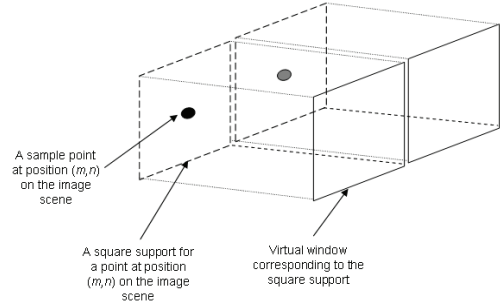

Figure 3 (exerpted from [10]). A pair of sample points, each associate with a square support and a virtual window on the scene image and the WRP, respectively.

3 Optical reconstruction of holograms generated with the IWRP method

According to [10], construction of the IWRP, and its subsequent expansion into a hologram, are conducted with the CPU(i7 920 @ 2.80GHz) and the GPU (Nvidia Geforce GTX260+), respectively. The method is capable of generating, a 2048x2048 hologram comprising of over 4×10^6 points, in less than 25ms (equivalent to 40 frames per second). To demonstrate the method, a digital hologram representing a synthetic globe with the image of the earth is generated with the optical setting listed in Table I. The radius of the globe is around 0.005m, and the front tip of the globe is located at 0.01m from the IWRP. The latter is at a distance of 0.3m from the hologram.

Table 1. Optical setting

Hologram Pixel size	7um
Wavelength	650nm
Angle of reference beam	1.2 degree

The hologram is optically reconstructed with a liquid crystal on silicon (LCoS) device which is modified from the Sony Bravia projector, and the reconstructed image is shown in figure 4.

Figure 4 (excerpt from [10]) Optical reconstruction of the hologram of a globe image generated with the IWRP method

4   Conclusions

In this paper, four modern methods for fast generation of digital Fresnel hologram are presented. These techniques emphases on reducing the complexity of the hologram generation process so that they can be realized with affordable computing and hardware devices. However, the description in here is by no means exhaustive. With the advancements on computing technology, vibrant development on methods and systems for fast hologram generation is expected in the coming years. At the same time, there are also research areas that address different problems in digital holography. One of the areas is to expand the hologram display by integrating multiple LCoS or spatial modulator (SLM) devices, a concept similar to the construction of video wall. Another potential area of development is on the compression of the data size of digital hologram, which is significantly higher than an optical image, especially if multiple displays are involved. Apart from video holography, printing of digital hologram is also a challenging process. At present, digital hologram is printed with the fringe printer, which is an expensive and time consuming process. Besides a stable, light shielded environment is mandatory. While all these streams of development can be conducted as separate research areas, there are also plenty of rooms for synergizing their findings, leading towards a more mature and practical holographic infrastructure.

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