

수명교체와 예비품 재고 정책의 통합 최적화

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Joint Optimization of Age Replacement and Spare Provisioning Policy

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Abstract

Joint optimization of preventive age replacement and inventory policy is considered in this paper. There are three decision variables in the problem: (i) preventive replacement age of the operating unit, (ii) order quantity per order and (iii) reorder point for spare replenishment. Preventive replacement age and order quantity are jointly determined so as to minimize the expected cost rate, and then the reorder point for meeting a desired service level is found. A numerical example is included to explain the joint optimization model.

1. Introduction

Maintenance policies for systems subject to stochastic failures have been treated for several decades [7], but most of them have implicitly assumed that spares are always available as needed. If, however as is often the case, spare replenishment is required to implement maintenance policies, we should consider spare provisioning policy together. In this paper joint optimization of preventive age replacement and spare provisioning policy is considered. The problem is different than what typically occurs in maintenance theory and practice, where maintenance policy is determined by engineers independently of the logisticians who opti-

mize inventory [2]. There are three decision variables in the problem: (i) PR age of the operating unit, (ii) order quantity per order and (iii) ROP for spare replenishment.

In the previous studies on the joint optimization of age replacement and inventory [1, 2, 6], order quantity per order is assumed only one. We examine joint optimization of age replacement and spare provisioning policy which does not restrict the number of order quantity. When to replace the operating unit and how much to order for spare replenishment are jointly determined so as to minimize the expected cost rate, and then the ROP for meeting a desired SL is found. A numerical example is included to explain the joint optimization model.

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Acronyms

PR preventive replacement

CR	corrective replacement
EOQ	economic order quantity
ROP	reorder point
SL	service level

Notation

$f(t), F(t), \bar{F}(t)$	pdf, Cdf, Sf of time to failure of a unit
$h(t)$	$f(t)/\bar{F}(t)$, hazard rate
T	PR age
Q	order quantity per order
R	ROP, reorder point
T^*, Q^*, R^*	optimal values of T, Q, R respectively
L	replenishment lead time
c_p	PR cost at age T
c_r	CR cost at failure(; $c_r \geq c_p$)
c_o	ordering cost per order
c_h	holding cost of a spare unit per unit time
μ_T, σ_T	mean and standard deviation of time to replacement of a unit
Y_R	time length of R successive replacements
$N(\mu, \sigma^2)$	s-normal distribution with mean μ and variance σ^2
z	safety factor for meeting a pre-specified SL
$[x]$	ceiling function, the smallest integer $\geq x$

2. MODEL AND ANALYSIS

1. Joint Optimization of Q and T

We first consider the joint optimization problem of order quantity and PR age, where Q units are purchased per order and each unit is replaced at failure or at age T whichever occurs first. If we regard the time between successive orders is a cycle, the expected cost per cycle is the sum of ordering, replacement and holding costs.

The expected replacement cost per cycle is

$$Q[c_r F(T) + c_p \bar{F}(T)] = c_p Q + (c_r - c_p) F(T) Q. \quad (1)$$

Since the average time to replacement of a unit is

$$\mu_T = \int_0^T t \cdot f(t) dt + T \cdot \bar{F}(T) = \int_0^T \bar{F}(t) dt, \quad (2)$$

the expected holding cost per cycle is

$$\begin{aligned} c_h [(Q-1) + (Q-2) + \dots + 1] \int_0^T \bar{F}(t) dt \\ = c_h Q(Q-1) \int_0^T \bar{F}(t) dt / 2. \end{aligned} \quad (3)$$

Thus the expected cost per cycle is

$$c_o + c_p Q + (c_r - c_p) F(T) Q + c_h Q(Q-1) \int_0^T \bar{F}(t) dt / 2. \quad (4)$$

Since the expected cycle length is $Q \cdot \mu_T$, the expected cost rate is

$$C(T, Q) = \frac{c_o + c_p Q + (c_r - c_p) F(T) Q + c_h Q(Q-1) \int_0^T \bar{F}(t) dt / 2}{Q \int_0^T \bar{F}(t) dt}. \quad (5)$$

Notice that if $Q=1$ and ordering cost is included in replacement costs, $C(T, Q)$ reduces to the classical age replacement of Barlow and Hunter [3]. A necessary condition that T minimizes (5) is obtained by setting the partial derivative of $C(T, Q)$ with respect to T equal to zero, that is,

$$h(T) \int_0^T \bar{F}(t) dt - \bar{F}(T) = (c_o + c_p Q) / (c_r - c_p) Q. \quad (6)$$

If $h(t)$ is strictly increasing the l.h.s. of (6) is also strictly increasing. Thus there exists a finite and unique T^* minimizing $C(T, Q)$ if $h(t)$ is strictly increasing to infinity.

Similarly, setting the partial derivative of $C(T, Q)$ with respect to Q equal to zero yields

$$Q = \sqrt{\frac{2c_o}{c_h \int_0^T \bar{F}(t) dt}} \quad (7)$$

Notice that (7) is the EOQ formula since

$\mu_T = \int_0^{T^-} \bar{F}(t) dt$. The optimal values of T and Q satisfying (6) and (7) can be obtained by the following iterative procedure.

Procedure to find (T^*, Q^*)

- Step 0. Put $i=1$ and $Q_i=1$.
- Step 1. Put $Q=Q_i$ and find T_i , the value of T satisfying (6).
- Step 2. Put $T=T_i$ and find Q_{i+1} , the integer closest to the value of Q satisfying (7).
- Step 3. If $Q_{i+1}=Q_i$, then $Q^*=Q_i$ and $T^*=T_i$.
Otherwise go to Step 1 after replacing i with $i+1$.

2. Determination of R

We consider the familiar (Q, R) policy [4], in which a fixed quantity Q is ordered when the inventory position drops to R . By the central limit theorem, the distribution of time length taken by R successive replacements is

$$Y_R \approx N(R\mu_T, R\sigma_T^2). \tag{8}$$

where

$$\sigma_T^2 = \int_0^T (t - \mu_T)^2 f(t) dt + (T - \mu)^2 \bar{F}(T) \tag{9}$$

If an order is placed when inventory level including the operating unit is R , the probability that stockout occurs before the arrival of replenishment order is

$$\Pr\{Y_R \geq L\} = \Pr\left\{ \frac{Y_R - R\mu_T}{\sqrt{R}\sigma_T} \geq \frac{L - R\mu_T}{\sqrt{R}\sigma_T} \equiv z \right\}. \tag{10}$$

A widely used term for determining ROP is SL, which means the desired probability of no stockout in a cycle. Since stockout cost often includes such intangibles as loss of goodwill it might be more appropriate to specify service constraints, instead of or in addition to the use of cost parameters [2].

The values of safety factor for some typical SL [8] are shown in [TABLE 1].

<TABLE 1> SL AND THE CORRESPONDING SAFETEY FACTOR

SL	safety factor (z)
90 %	1.28
95 %	1.65
99 %	2.33

If a desired SL is exogenously given by operation policy, the ROP to meet the SL is determined by the following equation.

$$z = (L - R\mu_T) / \sqrt{R}\sigma_T \tag{11}$$

or

$$\mu_T R + z\sigma_T \sqrt{R} - L = 0 \tag{12}$$

The value of satisfying (12) is

$$R = \left(\frac{\sqrt{z^2 \sigma_T^2 + 4\mu_T L} - z\sigma_T}{2\mu_T} \right)^2. \tag{13}$$

Hence the optimal value of the ROP to meet a pre-specified SL is

$$R^* = \left\lceil \left(\frac{\sqrt{z^2 \sigma_T^2 + 4\mu_T L} - z\sigma_T}{2\mu_T} \right)^2 \right\rceil. \tag{14}$$

3. NUMERICAL EXAMPLE

Consider a unit having a Weibull Sf, $\bar{F}(t) = \exp(-0.01t^4)$ with cost parameters $c_o = \$600$, $c_r = \$10,000$, $c_p = \$5,000$, $c_h = \$10$.

The procedure to find (T^*, Q^*) iterates as follows.

- Step 1. $Q_1 = 1 \rightarrow T_1 = 2.62$
- Step 2. $T_1 = 2.62 \rightarrow Q_2 = 7$
- Step 3. Go to Step 1 because $Q_2 \neq Q_1$.
- Step 1. $Q_2 = 7 \rightarrow T_2 = 2.59$
- Step 2. $T_2 = 2.59 \rightarrow Q_3 = 7$

Step 3. Since $Q_3 = Q_2$, the iteration stops here.

$$Q^* = Q_2 = 7 \text{ and } T^* = T_2 = 2.59.$$

Assuming that the replenishment lead time $L = 8$ and the desired SL is 95%, let us find the optimal value of R. $\mu_T = 2.38$ is obtained by substituting $T^* = 2.59$ into (2), and $\sigma_T^2 = 0.144$ is obtained by substituting $T^* = 2.59$ and $\mu_T = 2.38$ into (9). Substituting those values of μ_T and σ_T^2 with $z = 1.65$ for satisfying 95% SL into (13) gives $R = 2.91$. Hence the optimal values of the three decision variables are $T^* = 2.59$, $Q^* = 7$, $R^* = 3$. That is, each operating unit is replaced at failure or at age 2.59 whichever occurs first, and 7 units are ordered for spare replenishment when the number of remaining stock drops to 3 units.

4. NUMERICAL EXAMPLE

A joint optimization of preventive age replacement and spare provisioning policy is considered in this paper. In general, there are three interrelated policies to be considered in the joint optimization of maintenance and spares inventory models[5]:

- (i) Ordering policy - When to order for spare units for replacements
- (ii) Stocking policy - How many spare units to order per ordering
- (iii) Maintenance policy - When to replace each unit

In order to obtain a more general ordering-stocking-maintenance policy, the extension to the case of unequal replacement intervals will be needed.

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