Journal of the Korean Society of Mathematical Education Series D:
Research in Mathematical Education
Vol. 16, No. 1, March 2012, 31–50

The Effect of Solid Geometry Activities of Pre-service Elementary School Mathematics Teachers on Concepts Understanding and Mastery of Geometric Thinking Levels

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(Received December 16, 2011; Revised January 14, 2012; Accepted March 26, 2012)

The present study explored whether the implementation of focused activities (intervention programme) can enhance 22 pre-service mathematics teachers' proficiency in solid geometry thinking level as well as change for the better their feelings in this discipline. Over a period of 6 weeks the pre-service teachers participated in activities and diversified experiences with 3D shapes, using illustration aids and actual experience of building 3D shapes in relation to the various spatial thinking levels. The research objectives were to investigate whether the intervention programme, comprising task-oriented activities of solid geometry, enhance mathematics pre-service teachers' mastery of their geometric thinking levels as well as examine their feelings towards this discipline before and after the intervention programme. The findings illustrate that learners' levels of geometric thinking can be promoted, entailing control on higher thinking levels as well as a more positive attitude towards this field.

Keywords: Van Hiele theory, level of thinking in solid geometry, 3D shapes, pre-service mathematics teachers, reflection
MESC Classification: B52, B59, G42, G49
MSC2000 Classification: 97B50, 97G40

1. INTRODUCTION

Solid geometry is an important discipline which affects all of us in every-day life. We

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live in a three dimensional world; hence, developing spatial orientation is part of our responsibility as mathematics teachers. There are numerous theories that engage with the development of geometric thinking. One of them is Van Hiele theory (1987, 1999), which argues that the development and progress in geometric thinking are not age-dependant, but rather teaching-related. A series of studies of this topic (Clements & Battista, 1992, Koester, 2003) found that difficulties to identify and build geometric shapes are experienced already in elementary school, by both young pupils and teachers.

The present study focused on the first three spatial levels according to Van Hiele theory. In the paper, following a theoretical background, we present the effect of the activities on the concepts of understanding and the mastery of geometric thinking levels of the pre-service teachers.

2. THEORETICAL BACKGROUND

Solid geometry is a chapter in geometry which is studied since kindergarten throughout the years in elementary school, continues in high school and up to teacher education colleges in the field of mathematics. Basic geometric concepts are acquired during children's first years and in elementary school. Quite a few studies engaged in children's thinking about spatial insight in general and perceptions of terms associated with solid geometry shapes in particular (Hannibal, 1999; Shaw, 1990; Yackel & Wheatley, 1991). Spatial insight consists of two components, the first is spatial visualisation and the second is spatial orientation. Spatial visualisation is the ability to imagine the view of bi-dimensional shapes and bodies, a shift or changes in their sense (Del Grande, 1990).

The development of spatial visualisation is based on learners' experiencing and activities. The National Council of Teachers of Mathematics (NCTM, 2000) defined several standards which specify what pupils at various ages should know in geometry. These standards enable teachers to direct the learning material and include in it all the competences required in geometric thinking. The NCTM document indicates that the curriculum, designed for children from kindergarten and up to 12th grade, should enable pupils "to analyse characteristics and features of bi- and tri-dimensional geometric shapes and develop mathematical arguments about geometric relations (NCTM, 2000, p. 96).

Regarding the first years at school, children are expected to:

- Identify, implement, build, draw, compare and classify bi- and tri-dimensional shapes.
- Describe characteristics and parts of bi- and tri-dimensional shapes.
- Investigate and predict the results of composing and decomposing shapes.

In other words, the main emphases in geometry studies are: developing visual perception of the plane and space and developing thinking ways typical of plane and solid geometry (presenting hypotheses, generalisations and reasoning, drawing conclusions and so on). Some of the objectives of geometry teaching are: developing geometric skills as well as the ability to investigate shapes and bodies.

There are various theories about the development of geometric thinking. The present study used the Van Hiele Theory. Pierre Van Hiele and Dina van Hiele-Geldof, the Dutch mathematical educators who developed a theory during the late 1950s, attempted to explain the fact that numerous students were encountering difficulties associated with cognitive processes involved in geometric thinking in general and presentation of proofs in particular. In their theory, the Van Hieles related to plane geometry only. In recent years there have been studies which apply Van Hiele's plane geometry theory to additional branches of mathematics, i. e. solid geometry (Patkin, 2010), and arithmetic (Crowley, 1987; Guberman, 2008).

In 1959 the Van Hieles' stipulated five levels in a hierarchical order. Today, however, the common reference is only to four levels (Gutierrez, 1992; Van Hiele, 1987): recognition or visualisation level, analysis level, ordering level and rigour and deduction level:

1 **Recognition or visualisation level** - at this initial level, learners know to identify geometric shapes and distinguish between them. Each of the concepts or shapes is perceived as a whole, as it appears. Learners are capable of distinguishing between different 3D shapes and of naming them. At this level, they are unable to specify the features of these bodies.

2 **Analysis or description level** - at this level, learners know to analyse the features of 3D shapes. However, they lack the ability to attribute features of a specific body to the features of the group to which it belongs. For example: learners know that the sides of a box are rectangular but do not know to generalise that each cube is a box.

3 **Ordering or informal conclusion level** – at this level, learners identify a hierarchical order of inclusion between shape groups according to their features and definitions. They do not know, though, to prove claims relating to the properties of geometrical shapes. For example: at this level learners understand the relation between a prism and a cube (namely, every cube is a prism) but do not know to prove the property attributed to the four cube diagonals which are equal and cross each other.

4 **Rigour and deduction or formal conclusion level** – at this level, learners are familiar with the role of fundamental concepts, axioms, definitions, theorems and proofs as well as their interrelations. They can use hypotheses in order to prove theorems and understand the meaning of necessary and sufficient conditions. Moreover, they are capable of presenting reasons and arguments for the different stages of the proof.

According to Van Hiele theory, partial mastery of a certain level is a necessary but not sufficient condition for mastering a higher level. People cannot be versed in level X before they are familiar with level X-1. That is, they must be well acquainted with all the previous levels; otherwise they are referred to as "inconsistent".

It is worthwhile mentioning that, unlike other learning theories, mainly that of Piaget, the Van Hiele theory is grounded on the assumption that advancing from one level to the next depends more on teaching than on age or biological maturity (Van Hiele, 1999). According to Geddes, Fuys, Lovett & Tischler (1982), various teaching types can affect progress differently.

Studies indicate that pupils at any age as well as teachers encounter difficulties in solid geometry (Koester, 2003). Clements & Battista (1992) researched geometry and spatial thinking of kindergarten and elementary school children concerning the performance of the same teaching assignment. Their study showed that, although there was an almost 8-year gap between the young participants (kindergarten children) and the older participants (6th-graders), the scores for the same assignments increased only minimally. Patkin (2010) explored "personal knowledge" of elementary school mathematics teachers. The findings illustrated that, while exposing their self knowledge, the teachers indicated lack of control and comprehension of solid geometry. Nevertheless, after becoming acquainted with the Van Hiele theory, including experiencing, being in situations which encouraged critical reflection on thinking (meta-cognition), they enhanced their thinking levels, demonstrating openness and wish to learn, cope and improve. Crowley (1987) argues that the type of activities given to learners is important. In her study of solid geometry thinking, she showed that compliance between the learners' level of comprehension and the level of their assignments is essential if we want them to experience meaningful learning.

Balacheff (1987) discusses the great importance we have to attribute to learners' ability to verbalise, explain and reason. Moreover, he maintains that every mathematical subject, including geometry, should be taught through activities and situations whereby learners use verbalisation and reasoning for their explanations and comprehension; he also distinguishes between several levels of explanations and proofs. Teachers should be greatly aware of the different levels of reasoning which learners apply (Hershkowitz, Ben-Chaim, Hoyles, Lappan, Mitchelmore & Vinner, 1990).

According to the importance of mastery in the Van-Hiele levels and the comprehension of plane and solid geometry, we developed in our college a course in solid geometry for pre-service teachers. In the course they were exposed to applied activities integrating verbalisation, discourse, explanations and reasoning on different levels, from the intuitive to the logical comprehension levels.

The present study focused on the mastery of the first three spatial levels according to Van Hiele theory: recognition, analysis and ordering.

Research questions

1. Is there any improvement in pre-service mathematics teachers' mastery of thinking levels following a task-oriented focused activity in solid geometry?
2. Is there any improvement in pre-service mathematics teachers' feelings towards solid geometry following a task-oriented focused activity in the subject?

3. METHODOLOGY

Research population – consisted of 22 elementary school pre-service mathematics teachers in their first year at the teacher education college. All the participants hold a full matriculation certificate. Sixteen pre-service teachers took the matriculation exam at an advanced mathematics level, and six of them took the matriculation exam at a mathematics literacy level.

Research tools – comprised a questionnaire for determining geometric thinking levels and a reflective questionnaire.

1. Van Hiele questionnaire examining the first three thinking levels according to Van Hiele theory. The questionnaire consisted of 15 items, each 5 representing a separate comprehension level. The questions were mixed and without any level ordering (as presented in the original questionnaire, Patkin, 2010). The questions representing the first level were questions nos. 1, 3, 8, 11 and 13. The questions representing the second level were questions nos. 2, 7, 10, 12, 14. The questions representing the third level were questions nos. 4, 5, 6, 9, 15 (See Appendix A). The allocated response time was 30 minutes.
2. Reflective questionnaire – the pre-service teachers had to answer in writing the question: What was your sense when answering the Van Hiele questionnaire of space? The questionnaire was administered twice. Its objective was reflection – feelings and attitudes. It was first administered after filling in the Van Hiele questionnaire of space for the first time and then it was administered following the intervention programme and filling in the Van Hiele questionnaire for the second time.

Research procedure

The study was conducted within the framework of an annual geometry course. At the beginning of the course, prior to the intervention programme, Van Hiele questionnaires were circulated among all the pre-service teachers (pre). Once they have completed the questionnaires, they were asked to describe in writing their feelings following it (reflective questionnaire). Then, they attended six lessons which lasted 1.5 hours (intervention programme), during which they participated in activities and diversified experiences with 3D shapes, using illustration aids and actual experience of building 3D shapes in relation to the various spatial thinking levels (see Appendix B for some of the activities). At the end of the activity, the pre-service teachers were asked once more to respond to the questionnaires (post) as well as describe again in writing their feelings after filling them in.

Analysis tools

1 *Geometric thinking level* of the pre-service teachers was examined by the average of raw scores (in %), average of weighted scores and their degree of mastery of geometric thinking levels according to Van Hiele theory.
Comment: the average of weighted scores was determined according to the criterion whereby at least 4 correct answers out of 5 in each level were accredited. This criterion was found to distinguish well between thinking levels (Ussishkin, in Parkin, 1990). The weighted scores were determined according to the following formula: weighted score = meeting the criterion in level 1 + meeting the criterion in level 2 + meeting the criterion in level 3. If "a" is the variable which represents meeting the criterion in a level and it is attributed the values of 0 or 1, then the weighted score can be represented in the following way: weighted score = a * 1 + a * 2 + a * 43 (if we had related also to level 4, we would have to add also a * 8). Hence, the score range, relating to being versed in the first three geometric thinking levels, was 0–7 (Patkin, 1990).
Through the weighted scores, the pre-service teachers' thinking levels could be identified. A pre-service teacher who has not mastered the first level got a score of 0. A pre-service teacher versed in the first level got a score of 1, in the second level the score was 3 and in the third level the score was 7. The other scores represented the "inconsistent" pre-service teachers.

2 The questionnaires were analysed in a qualitative research method.

3. FINDINGS

The first research question focused on the geometric thinking level. Table 1 presents the averages (in %) and the standard deviation of the raw scores before the experiment (pre) and immediately following it (post).

Table 1. Average raw scores and standard deviation in the Van Hiele questionnaire (in %)

Test	Average	Standard deviation
Pre	49.7	28.6
Post	94	7

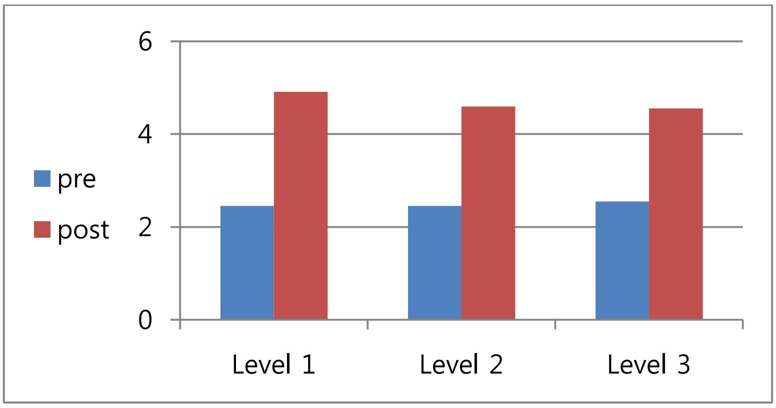
Table 1 illustrates that the raw scores average considerably increased and the standard deviation decreased considerably. Moreover, it shows an improvement in knowledge of solid geometry, improved learning being defined as the differences between the averages of the raw scores in the pre- and post-test (44.3%).

Table 2 presents the average raw scores and standard deviation in the Van Hiele questionnaire (in %) in each level.

Table 2. The average raw scores and standard deviation in the Van Hiele questionnaire (in %)

Level	Time of test	Average of correct answers belonging to the level	Standard deviation
1	**Pre**	2.45	1.01
	Post	4.91	0.29
2	**Pre**	2.45	0.66
	Post	4.59	0.67
3	**Pre**	2.55	1.23
	Post	4.55	0.96
Total average of correct answers	**Pre**	2.48	1.45
	Post	4.68	0.6

Graph 1 describes from an additional aspect the profile of the pre-service teachers who gave several correct answers to the Van Hiele questionnaire of space, divided by levels.



Graph 1. Profile of respondents who gave several correct answers in the first three levels

Table 2 and Graph 2 show that the increase in the correct answers average was in all the three thinking levels and it was within the 2-2.46 range.

Table 3 presents the distribution of pre-service teachers who correctly answered at least 4 questions in each level (in %).

Table 3. Distribution of pre-service teachers who correctly answered at least 4 questions in each level (in %).

Level	Pre	Post
1	18.2	95.5
2	9.1	91.0
3	27.3	91.0

Table 3 illustrates that only 18.2% of the respondents answered correctly the questions related to the first thinking level which deals with the visual level. However, 27.3% of the respondents answered correctly the questions related to the third thinking level, dealing with the ordering level. That is, prior to the intervention programme, some pre-service teachers were identified as "inconsistent" in their geometric thinking levels.

For the purpose of clarification, the research findings were analysed also in relation to the index of thinking level mastery according to Van Hiele. Table 4 presents the distribution of pre-service teachers who are versed in the different thinking levels according to Van Hiele theory (in %)

Table 4. Distribution of pre-service teachers versed in the different thinking levels according to Van Hiele theory (in %)

Weighted score	Level	Pre (N = 22)	Post (N = 22)
0	0	59 (13)	0
1	1	0	4.5 (1)
3	2	0	4.5 (1)
7	3	4.5 (1)	82 (18)
Other	"Inconsistent"*	36.5 (8)	9 (2)

* Are not versed in the cognitive level according to the definition

Table 4 shows that a change has transpired in mastery of thinking levels according to Van Hiele theory, the major change, 82%, occurring in the third thinking level. Conversely, the findings indicate a considerable decrease in the percentage of pre-service teachers defined as "inconsistent": 8 (36.5%) pre-service teachers before the intervention programme versus 2 (9%) following it.

Table 5 presents the average mastery of thinking level.

Table 5. Average mastery of thinking level

Time	Weighted scores Average	Standard deviation
Pre	0.13	2.55
Post	2.59	1.53

Perusal of Table 5 shows that the average mastery of thinking level has increased and the standard deviation has decreased.

The second research question dealt with reflection. Immediately after responding to the first questionnaire, prior to the intervention programme, the participants were asked to reflect, by means of questionnaires, upon the filling-in of the questionnaire designed to determine the geometric thinking levels. All of the participants, without any exception, indicated a sense of frustration, ignorance, shame and lack of confidence in this field. The responses were divided into two: feelings and knowledge or lack thereof in the field.

There were 28 assertions associated with feelings:

- Frustration, helplessness and disappointment (7 assertions) – “an unpleasant sense of frustration...”; “I felt I am not familiar with a material which young children know...”; “a sense of failure that until now I don't know the meaning of words like ‘prism’, ‘cylinder’ and others, this being a subject learnt by elementary school
- Confusion (7 assertions) – “I felt confused, I didn't know on what to focus”; “I was not acquainted with the names and parts and failed to create a whole picture. It’s as if I know but I don’t really know”
- Embarrassment and shame (6 assertions) – “I felt uncomfortable submitting the questionnaire to you; I was ashamed of myself and embarrassed from you!”; “an unpleasant sense, I felt I was only getting more in trouble”; “It's a shame I don't remember…”
- Pressure, apprehension and panic (5 assertions) – “I felt pressure! I guessed many answers”; “there was pressure because in most questions I was not sure if I was giving the right answer or not…”; “I felt panic…”; “I was afraid to fail”.
- Lack of confidence (3 assertions) – “This lack of confidence which I had experienced, drove me crazy”.

There were 20 assertions associated with knowledge or lack thereof in the field:

- Lack of knowledge, ignorance (14 assertions) – "most of what I wrote I invented and guessed"; "my knowledge during the test was non-existent"; "I felt ignorant and frustrated!"; I felt I lacked certainty and orientation in the spatial knowledge of bodies".
- Lack of memory and forgetfulness (5 assertions) – "I was angry how quickly I forgot the properties, that my memory was deleted"; "all the laws and rules were mixed to-

gether and I did not remember anything. Not from high school and not from my SAT exam – although it was just a year ago"; "Why don't I remember such basic things…".

- Uncertainty – (1 assertion) – "I felt I am stupid and know nothing".

Six weeks later, after completing the intervention programme and filling-in the questionnaire for the second time, the pre-service teachers were asked to reflect once more on the process which they had experienced during that time. All the participants indicated improved feelings, a sense of confidence and improvement of their knowledge. Here, too, the responses were divided into two: feelings and knowledge in the field.

There were 14 assertions associated with feelings:

- Feeling good (5 assertions) – "I felt better, not as before, when I had no idea about the subject"; "I feel much better. From half correct answers in the first questionnaire I achieved full success in the second!!".
- Confidence (4 assertions) – "today I feel much more confident with the material and things are much easier for me"; "I have to point out that today, after learning the material, I have confidence and knowledge of the subject more than ever".
- Liking the subject (2 assertions) – "I started liking and understanding geometry".
- Sense of success (2 assertions) – "I think that my situation has really improved".
- Lack of fear (1 assertion) – "Now I can approach the subject without fear and with more will than before".

There were 18 assertions associated with knowledge:

- Knowledge (12 assertions) – "I feel that today I know much more!"; "I sense a great improvement from the beginning of the year. I am more acquainted with basic concepts and properties of the 3D shapes which we have studied. I am excited and am waiting to continue".
- Sense of mastery (3 assertions) – "I am much more versed in the material"; "I feel much better because my acquaintance with the material has improved", "I am more confident with the terms, am better versed in the subject".
- Comprehension (3 assertions) – "I sense that I understand this material"; "Now it's not only memorisation but my comprehension of the material is much better"; "today, after the learning, I recalled all the material which I had learnt years ago and that I did not remember at the beginning of the studies".

To sum up, the findings analysis shows that the intervention programme has enhanced the pre-service teachers' mastery of geometrical thinking levels. The use of visual display and concrete activities in teaching helped the pre-service teachers to understand the material and upgraded their knowledge to a level of implementation. This was supported also by analysing the reflection before and after the process.

5. DISCUSSION AND CONCLUSIONS

The research objectives were to investigate whether the intervention programme, with task-oriented activities in solid geometry enhances pre-service teachers' mastery of their geometrical thinking levels. The study also aimed to explore their feelings towards this area of knowledge before and after the intervention programme. Activities in the programme related only to the first three thinking levels according to Van Hiele theory (Van Hiele, 1987, 1999). Studies which examined pre-service teachers' capabilities indicate that most participants usually master the first two levels whereas only a few of them are versed in the third level (Gutierrez, Jaime & Fortuny, 1991). Swafford, Jones & Thornton (1997) found that participants in a short term intervention programme can improve their geometrical knowledge and that they can advance by at least one level of thinking according to Van Hiele theory. They drew the conclusion that, by means of focused teaching, adults can quickly and easily make a progress in the Van Hiele levels. Findings of the present study also support these conclusions. The pre-service teachers participating in the study graduated high school with a matriculation certificate in mathematics and were directly or indirectly exposed to solid geometry in their previous studies (depending on the level of their mathematics studies in high school). Prior to the experiment, all of them demonstrated no mastery of the thinking levels (weighted average: 0.13). Conversely, following the intervention programme, they showed progress in their mastery and most of them were versed in the third thinking level (weighted average: 2.59).

The questionnaire related to mastery of the first three levels only. Hence, it is recommended developing additional task-oriented activities which include fourth level activities in order to promote pre-service teachers' mastery of the highest thinking level.

The importance and contribution of such an intervention programme are supported by the reflection questionnaires. Before the intervention programme, the reports were about frustration, confusion, lack of knowledge and confidence. Conversely, following the activities, the pre-service teachers indicated success, sense of confidence and wish to continue learning and improving in this field.

In light of the above, it is recommended adopting this approach when dealing with additional geometric contents.

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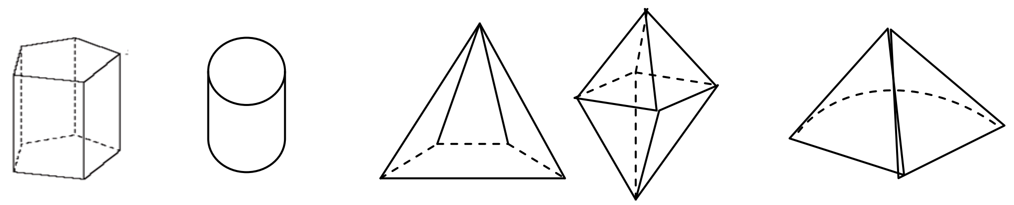
APPENDIX A

1. Circle the **prism** among the following objects



2. Which of the following statements is always true?

a. If an object has two bases then it must be a rectangular parallelepiped.

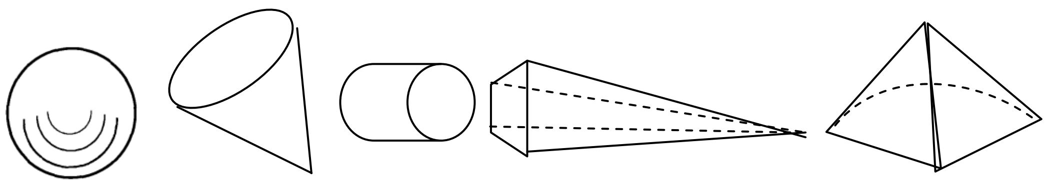
b. If an object has two bases then it must be a polyhedron.

c. If an object has two bases then it must be a cylinder.

d. If an object has two bases then it must be a regular polyhedron.

e. Statements a-d are false.

3. Circle the object shaped like a **cone**



4. Which of the following statements is not true for a prism?

a. The prism has two congruent and parallel bases.

b. The lateral surface of the prism is constructed of rectangles or parallelograms.

c. All the faces of the prism are rectangles or parallelograms.

d. The prism is a 3-dimensional object.

e. All the faces of the prism are polygons.

5. Which of the following statements is true?

a. If an object is a polyhedron then it is also a prism.

b. If an object is a prism then it is also a polyhedron.

c. If an object is not a polyhedron, then it is a prism.

d. If an object is not a prism, it is not a polyhedron.

e. If an object is a prism, it is not a polyhedron.

6. Which of the following statements is true?

a. If an object has 8 vertices it must be a rectangular parallelepiped.

b. If an object has 8 vertices it must be a cube.

c. If an object has 8 vertices it must be a pyramid.

d. If an object has 8 vertices it must be a regular polyhedron.

e. Statements a-d are false.

7. Which of the following statements is true for every cylinder?

a. The bases of a cylinder are circular.

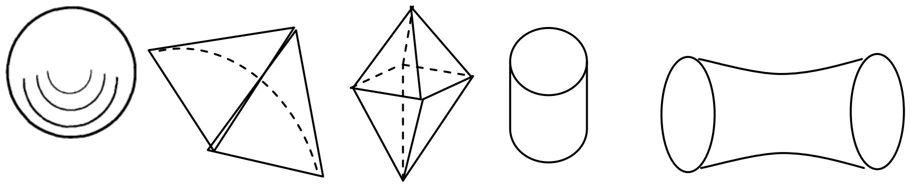
b. The bases of the cylinder are congruent and parallel squares

c. The bases of the cylinder are constructed of regular polygons.

d. The bases of the cylinder are pentagons.

e. The bases of the cylinder are triangles.

8. Circle the **polyhedron** among the following objects



9. Which of the following statements is true?

a. If an object is a regular polyhedron, it is also a cube.

b. If an object is a cube, it is also a regular polyhedron.

c. If an object is not a regular polyhedron, it is a cube.

d. If an object is not a cube, it is not a regular polyhedron.

e. Statements a-d are true.

10. Which of the following statements is true for every polyhedron?

a. In a polyhedron all faces are congruent.

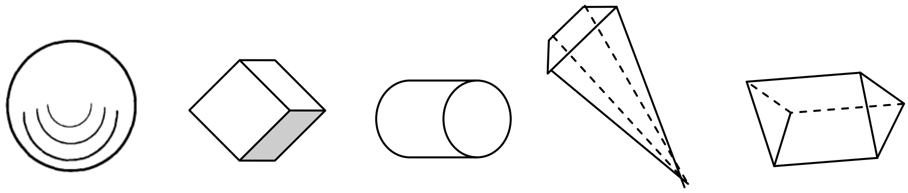
b. All polyhedrons are constructed of pairs of parallel faces.

c. A polyhedron does not have curved surfaces.

d. Each vertex in a polyhedron intersects a number of other faces.

e. Statements a-d are true.

11. Among the following objects, circle the one shaped like a **cylinder**



12. A cube has

a. 12 faces

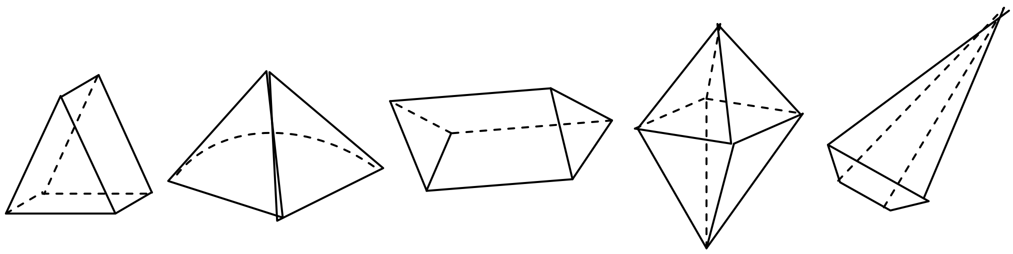
b. 4 faces

c. 6 faces

d. 8 faces

e. 2 faces

13. Which of the following shapes is **a pyramid**?



14. Which of the following statements is true for every prism?

a. The lateral surface of the prism is constructed entirely of triangles.

b. The prism had two parallel bases.

c. The base of the prism is rectangular.

d. The lateral surface of the prism is constructed of regular polygons.

e. Four faces intersect every vertex of the prism.

15. All pyramids have in common:

a. All pyramids have a triangular-shaped base.

b. The lateral surface is constructed of triangles.

c. The base is quadrangle and the lateral surface is constructed of triangles.

d. All faces of the pyramid are triangles.

e. All statements a-d are false.

APPENDIX B

Example of activities from the intervention programme:

1. The inquiry stage: whereby learners lead themselves individually to the discovery of their own knowledge. This stage consists of three sub-stages:

1.1 Individual activity, in the framework of which each learner received an assignment related to the learnt concept as well as reflected and invoked his or her current knowledge about the learnt subject.

For example: in the first lesson the learners had to allocate a name to a group of given bodies and then to classify those 3D shapes according to criteria chosen by them. At the end of this stage, the written information was gathered.

1.2 Group activity, in the framework of which the learners were divided into work group of 4-5 members. Each group chose one member, who was asked to document the entire group process which took place, including the discourse. At the beginning, all the learners presented to the group their way of solving the assignment (given at the first stage). Thus, they shared their individual knowledge with the group and became acquainted with the individual knowledge of their peers. Then, the group members decided which information they wanted to present to the entire class and explained the reasons for this choice.

For example: the learners reported to their group about the classifications and criteria according to which they classified individually. Then they decided on accepted classifications and criteria which they would present to the entire class as a group. At the end of this stage, the written information was gathered.

1.1 Class activity. The group heads reported about the group discussion and its outcomes. For example: each group head reported about the classifications and criteria chosen in the group for classifying the 3D shapes as well as about mistakes and misconceptions raised by each of the group members.

2.The explanation stage: whereby the lecturer presented terminology of the concepts, encouraging the learners to use it in their discourse and work.

2.1 Summary of the class activity and inculcation of concepts by the lecturer while addressing the mistakes and misconceptions.

2.2 Free orientation. Giving to the learners assignments which manifested (directly or indirectly) the use of those acquired concepts in order to enhance the learners' competences and use of those acquired concepts.

2.3 Integration. The lecturer reviewed everything which the learners had investigated and made a synthesis of the entire learnt subject.