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Failure patterns of repairable systems and a flexible intensity function model

R. Jiang* and C. Huang
Faculty of Automotive and Mechanical Engineering
Changsha University of Science and Technology
Changsha, Hunan 410114, China

Abstract. Engineering systems are usually repairable. The reliability of a repairable system can be represented by failure intensity function. A type of shape of failure intensity function is called a failure pattern. Reliability-Centred Maintenance (RCM) presents six typical failure patterns but its definition is unclear. It is an open issue how to recognize the failure pattern of repairable systems. This paper first discusses the problems of RCM with the notion of failure pattern; then presents the method for failure pattern recognition; and finally proposes a flexible failure intensity function model. The appropriateness of the model is illustrated by a real-world example.

Key Words: *Repairable system, mean cumulative function, failure intensity function, failure pattern, parametric model*

1. INTRODUCTION

In the reliability literature there are two different notions for failure pattern. One is associated with non-repairable items and the failure pattern means the shape of failure rate function. The other is associated with repairable systems and the failure pattern means the shape of failure intensity function. Reliability-Centred Maintenance (RCM, Moubray (1997)) presents six failure patterns but its definition and the method to obtain them are unclear. This actually deals with the issue how to identify the failure pattern. Once a failure pattern is identified, another issue is how to model it. This paper addresses these issues for repairable systems.

The paper is organized as follows. We discuss the six failure patterns of RCM in Section 2, and present possible models for modelling those failure patterns in Section 3. The method for failure pattern recognition is presented in Section 4. A flexible failure intensity function model is presented and illustrated in Section 5. The paper is concluded with a brief summary in Section 6.

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* Corresponding Author.
E-mail address: *ry_jiang @163.com*

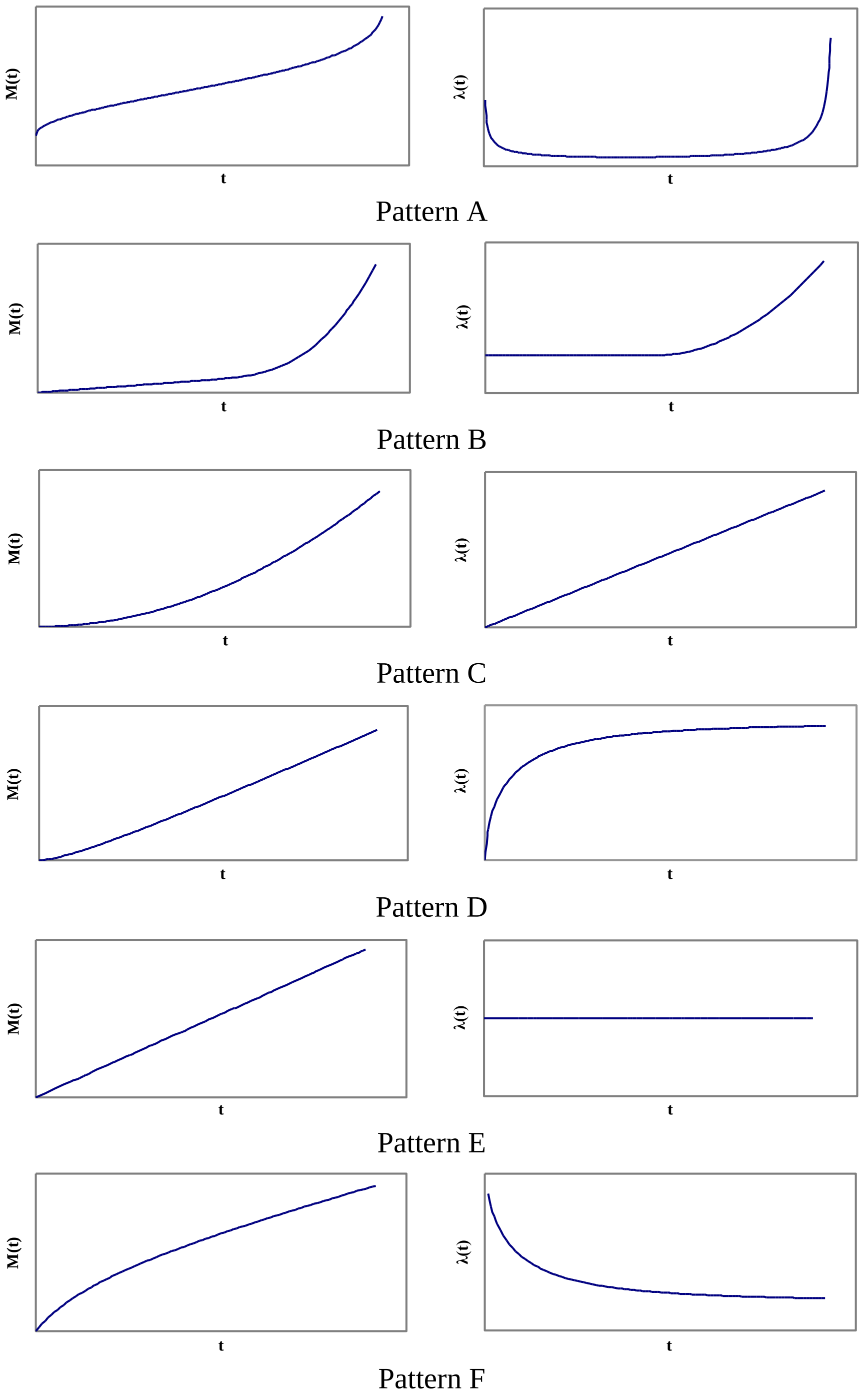

Figure 1. Six failure patterns

2. FAILURE PATTERNS OF RCM

Let $M(t)$ denote the cumulative hazard function for a non-repairable component and the mean cumulative function for a repairable system; and let $\lambda(t) = dM(t)/dt$, which is the failure rate function for a non-repairable component and the failure intensity function for a repairable system. Typical plots of $M(t)$ are shown in the left hand side of Figure 1; and the corresponding plots of $\lambda(t)$ are shown in the right hand side of Figure 1.

The plots of $\lambda(t)$ are the same as the six failure patterns of RCM in shape. They are called as Pattern *A* (Bathtub shaped), Pattern *B* (constant followed by increasing), Pattern *C* (increasing), Pattern *D* (increasing and approaching a constant), Pattern *E* (constant) and Pattern *F* (decreasing and approaching a constant), respectively. Some remarks for the failure patterns of RCM are as follows (also see Sherwin, D. (2000)):

(a) The vertical axis of the failure pattern plots is marked as "conditional probability of failure", which is neither failure rate function nor failure intensity function. According to the context, it seemingly deals with both. For example, it repeatedly mentions "probability of failure" or "failure probability", "failure rate", "infant mortality" and "wear-out". On the other hand, RCM (Moubray (1997)) gives the proportions of each failure pattern based on "studies done on civil aircraft", which is a repairable system.

(b) It is unclear how those patterns are obtained and whether other patterns exist.

(c) RCM stresses that most of failure patterns (larger than 80%) do not have aging so that the age-based preventive maintenance is ineffective. This statement is misleading since for a repairable system a constant failure intensity does not imply that the components of the system are not aging and the age-based preventive maintenance focuses on key components rather than the whole system.

3. POTENTIAL MODELS FOR MODELING THE SIX FAILURE PATTERNS

For a repairable system, let $N(t)$ denote the cumulative failure number of the system in $(0, t]$. For a given t, $N(t)$ is a random variable. Let

$$M(t) = E[N(t)] \tag{1}$$

denote the mean cumulative function (MCF), where $E(.)$ is the expected value operator. The failure intensity function is defined as Meeker and Escobar (1998)

$$\lambda(t) = dM(t)/dt\,. \tag{2}$$

When the failure intensity function is known, the MCF is given by

$$M(t) = \int_0^t \lambda(x)dx\,. \tag{3}$$

As such, the reliability evolution trend of a repairable system can be represented by either MCF or intensity function. Possible models associated with the six failure patterns for repairable systems are discussed as follows.

Possible models associated with Pattern *A* must have bathtub-shaped intensity functions. Any bathtub failure rate model can be viewed as a bathtub intensity model by revising the failure rate as the intensity function. Such a model will be presented in Section 5.

A piecewise model is a natural choice for modeling Pattern *B*. An approximate model for modeling Pattern *B* can be obtained through redefining the failure rate of the truncated exponential distribution (with a time upper limit) as the intensity function. Figure 2 illustrates the appropriateness.

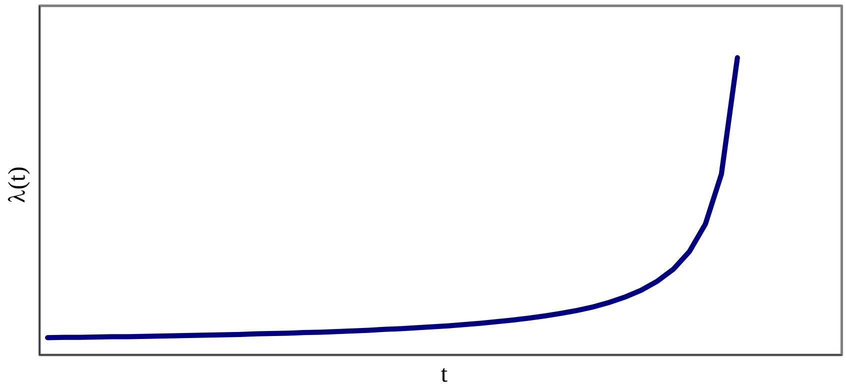

Figure 2. Failure rate of the truncated exponential distribution

A potential model associated with Pattern *C* is the power-law model given by

$$M(t) = (t/\eta)^{\beta}, \ \beta, \eta > 0. \quad (4)$$

It is also appropriate for modeling Pattern *E* with $\beta = 1$.

Finally, the five extended models developed by Jiang (2011) can be appropriate for modeling Patterns *D* and F.

4. FAILURE PATTERN RECOGNITION

Failure pattern recognition requires field failure data from one or more identical items. It involves a three-step procedure: data collection, non-parametric estimation of MCF, and fitting an appropriate parametric model to the collected data using the least squared method. We discuss the last two steps as follows.

Suppose that a population consists of n items. The failure point process of the i-th item is given by $(t_{ij}, 1 \le j \le n_i)$, where t_{ij} is the time of the j-th failure of the i-th item and n_i the total number of the failures observed for the i-th item. The total number of failure events is $m = \sum_{i=1}^{n} n_i$.

The failure process of the i-th item can be described by cumulative number of failures $N_i(t)$, which is a staircase function. For example, if a failure occurs at t_j, then $N_i(t_j^-) = N_i(t_j) - 1$.

For a given time t, $N_i(t)$ is a random variable and can be represented by a distribution. This distribution can be different at different time t. Let $\mu(t)$ denote the point-wise average of $N_i(t)$'s ($1 \le i \le n$) for the population. It is a non-parametric estimate of $M(t)$. As an example, Figure 3 shows the plot of MCF for a set of jet engines obtained from the non-parametric estimation.

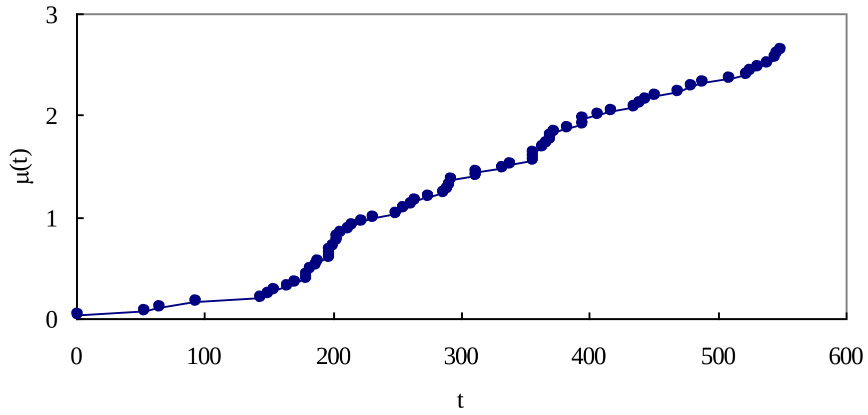

Figure 3: Non-parametric estimation of MCF for jet engines

It is noted that the non-parametric estimate of MCF at t_j is not continuous with $\mu(t_j^-) = \mu(t_{j-1}) < \mu(t_j)$. To facilitate the use of least square method, we define the representative value of MCF at t_j as below:

$$\mu_0(t_j) = [\mu(t_{j-1}) + \mu(t_j)]/2 . \quad (5)$$

Let $M(t;\theta)$ be a parametric model of MCF. The parameter set θ can be estimated by minimizing the sum of squared errors given by:

$$SSE = \sum_{j=1}^{m} [M(t_j;\theta) - \mu_0(t_j)]^2 . \quad (6)$$

Once $M(t;\theta)$ is obtained, the intensity function can be derived from (2) and then we can examine the shape of $\lambda(t)$, i.e., failure pattern.

Table 1. Failure point processes of 25 jet engines in (0, 550)

Engine	t_1	t_2	t_3	t_4	t^+
1	150	407	526		550
2	291	439			550
3	93	179	357	547	550
4	53	203	275	395	550
5	2	188	265	364	550
6	65	250	370	550	550
7	183	290	545		550
8	144	338	523		550
9	223	531			550
10	197	367			550
11	187	215	357		550
12	197	356			550
13	213	370			550
14	171	332	539		550
15	197	312	435		550
16	200	312			550
17	262	509			550
18	255	395			550
19	286	452			550
20	206	383	479		550
21	179	444			550
22	232	488			550
23	165	417			550
24	155	373			550
25	203	292	469		550

5. A FLEXBLE MODEL

5.1 An illustration

We illustrate the failure pattern recognition approach outlined in the above section using a real-world example. The data shown in Table 1 deal with removal data for 25 jet engines operating for 550 flying hours reported in Weckmana, Shell and Marvel (2001). Here, t^+ indicates a right-censoring observation.

The non-parametric estimation of MCF has been shown in Figure 3, and the non-parametric estimation of failure intensity is shown in Figure 4. As seen from Figures, the failure pattern is different from those shown in Figure 1.

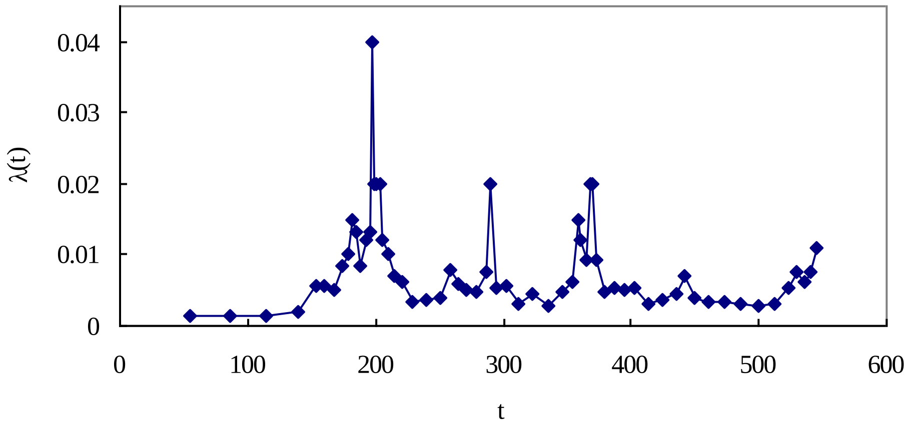

Figure 4. Non-parametric estimation of failure intensity for jet engines

A careful examination for Figures 3 and 4 suggests that the MCF plot may be divided into three parts with the partition points at about 203 and 367 hours. Let t_p denote the partition point. Then the plot after t_p can be vertically moved toward to the bottom using the transformation $\mu^*(t) = \mu(t) - \mu(t_p)$, and horizontally moved toward to the left using the transformation $\mu^{**}(t - t_p) = \mu^*(t)$. The transformed MCF plot is shown in Figure 5. Now, it is clear that the first part has Pattern *C* and the second and third parts have Pattern *A*.

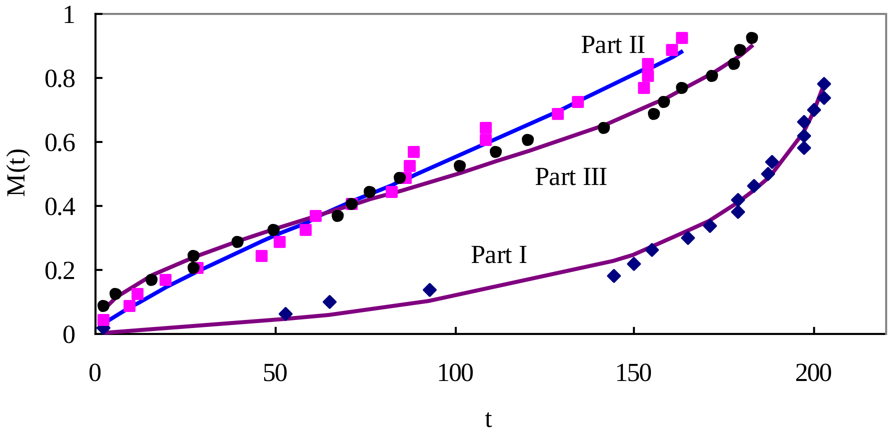

Figure 5. Transformed MCF

5.2 A flexible model

Jiang (2012) presents two bathtub failure rate models. One of them has the cumulative hazard function given by

$$H(t) = a[-\ln(1-t/\gamma)]^b, t \in (0,\gamma), a, b, \gamma > 0\,. \tag{7}$$

Letting $M(t) = H(t)$, we have the failure intensity function given by

$$\lambda(t) = ab[-\ln(1-t/\gamma)]^{b-1}/(\gamma - t)\,. \tag{8}$$

For $b < 1$, when $t \to 0$, $\lambda(t) \to abt^{b-1}/\gamma^b \to \infty$; and when $t \to \gamma$, $\lambda(t) \to \infty$, implying that $\lambda(t)$ is bathtub-shaped. For $b \geq 1$, $\lambda(t)$ is increasing. As such, the failure intensity function can be bathtub-shaped or increasing so that the model can be appropriate for fitting all the three parts shown in Figure 5.

The parameters estimated by the least squared method are shown in Table 2. The fitted MCFs are displayed in Figure 5. As seen from the figure, the fitted models are in good agreement with the observed data points.

From the fitted models we obtain the intensity function shown in Figure 6, which is similar to Figure 4 in shape. This confirms the appropriateness of the pattern recognition approach and the proposed model.

Table 2. Model parameters

	a	γ	b	Shape
Part I	0.1935	213.2	1.2372	Increasing
Part II	1.6080	437.5	0.7956	Bathtub
Part III	0.5966	202.8	0.4836	Bathtub

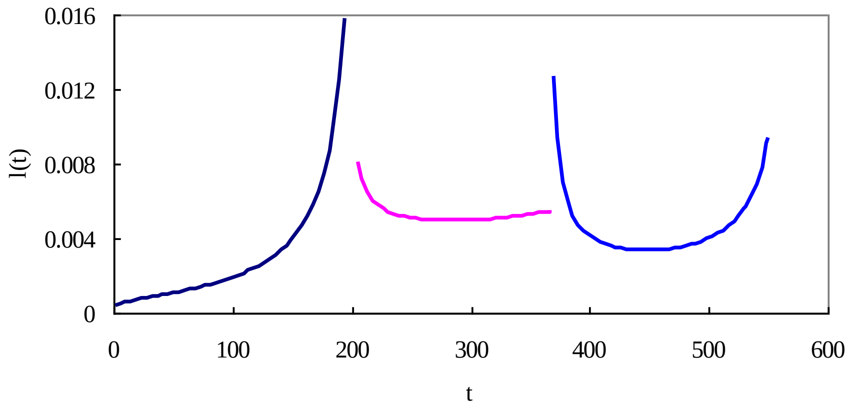

Figure 6. The intensity function derived from the fitted models

An explanation for the plot of Figure 6 is as follows. The system begins with low failure intensity and then the deterioration rate quickly increases probably due to wear-out failures and inappropriate preventive maintenance. The reliability in the second and third stages gets improved probably through implementing a better preventive maintenance policy. The failure intensity decreases at the beginning of the last two stages probably due to poor maintenance quality. These two factors eventually result in a complex failure pattern.

6. CONCLUSIONS

In this paper we have pointed out the problems of RCM with the six failure patterns; identified the potential models for modeling those failure patterns; proposed a three-step procedure to identify the failure pattern of a specific repairable system based on field data; and presented a flexible model for modeling the mean cumulative function. The usefulness and appropriateness of the model have been illustrated by a real-word example. The example showed that the real-world failure pattern can be much more complex than the six failure patterns of RCM.

There are a number of topics for the future study in this direction. These include:

(a) To identify typical failure patterns for various engineering systems based on field data

(b) To develop new models for modeling new failure patterns, and

(c) To identify appropriate activities for a specific failure pattern.

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