International Journal of Reliability and Applications
Vol. 13, No. 2, pp. 71-80, 2012

Reliability approximation for a complex system under the stress-strength model

Sadananda Nayak*
Ramakrishna Mission Vidyabhaban, Midnapore, West Bengal, India -721101

Dilip Roy
Centre for Management Studies, The University of Burdwan, West Bengal, India-713104

Abstract. This paper introduces a new approach for evaluating reliability of a complex system in terms of distributional parameters where analytical determination of reliability is intractable. The concept of discrete approximation, reported in the literature so far, fails to meet the latter requirement in terms of distributional parameters. The current work aims at offering a bound based approach where reliability planners not only get a clear idea about the extent of error but also can manipulate in terms of distributional parameters. This reliability approximation has been under taken under the Weibull frame work which is the most widely used model for reliability analysis. Numerical study has been carried out to examine the strength of our proposed reliability approximation via closeness between the two reliability bounds. This approach will be very useful during the early stages of product design as the distributional parameters can be adjusted.

Key Words*: Stress-strength analysis, Weibull distribution, reliability approximation and extent of error*

NOTATIONS

$S_X(x)$: Survival function of a random variable evaluated at the point x
$F_X(x)$: Cumulative distribution function corresponding to $S_X(x)$
W (λ, α) : Weibull distribution with scale parameter λ and shape parameter α
E(X) : Expectation of random variable X
R : System reliability
U ($\lambda_r, \lambda_C, \lambda_t, \lambda_M$, λ, α, θ, β, φ, ϒ) : Upper bound of the reliability under the Weibull Setup - W (λ, α)

* Corresponding Author.
E-mail address: *sadanandastat@gmail.com*

L ($\lambda_r, \lambda_C, \lambda_t, \lambda_M, \lambda, \alpha, \theta, \beta, \varphi, \Upsilon$) : Lower bound of the reliability under the Weibull Setup - W (λ, α)

R_{approx} : Reliability approximation under the Weibull setup

1. INTRODUCTION

The stress (S) and the strength (X) are treated as random variables in stress-strength models where the reliability of a component is measured by the probability that its strength exceeds the stress during its operation. The estimation problem of P(X>S), based on the reliability of a practical stress-strength model, has attracted the attention of many authors.

Ordinary transformation technique of Parzen (1960) can be employed to determine the system reliability when stress and strength distributions are directly known. Such analytical approaches virtually fail when stress variable is made up of multiple stochastic factors. Since the form of the stress function is generally complicated in nature, finding an exact distribution of the same is intractable in most of the cases. In the literature, some alternative approaches are available for approximating the system reliability. These are (i) Taylor-series method (ii) Monte-Carlo method (iii) Quadrature method (iv) Discretization Method and (v) Discrete approximation method. Evans (1975) has reviewed the first three methods with their relative advantages and disadvantages. The concept of discretization was imbedded in factorial experiment method, proposed by Taguchi (1978) to approximate a normal distribution by a 3-point discrete distribution. Experimental design approach was later modified by D`Errico and Zaino (1988) to approximate more closely the moments of the response function. In this sense, this method may be considered as moment equalization method. English et al (1996) used this modified approach for analyzing statistical tolerancing under the stress-strength model. A theoretical concept of discrete concentration for the univariate and bivariate setup was suggested by Roy (1993). Roy and Dasgupta (2000) presented a discretizing approach for evaluating reliability of complex systems under stress-strength model. Using the survival function of continuous random variable, Roy and Dasgupta (2001) proposed a discretization approach. They approximated the system reliability of a complex system under stress-strength model. Discrete normal distribution was separately studied by Roy (2003).The concept of linear transformation of the discretized variable for equalization of the first two moments of a continuous variable was introduced by Roy (2004). Applicability of discretization approach has been examined by studies on shear stress of different engineering items. Roy and Dasgupta (2002) studied discretization of Weibull distribution and Roy (2004) examined discretization of Rayleigh distribution to approximate the system reliability. Roy (2002) also considered discretization of the uniform distribution. Characterization of bivariate discrete distributions based on mean residual life properties has been examined by Roy (2005). Recently, Roy and Ghosh (2009) have studied discretization of continuous random variable using the failure rate function to approximate the system reliability. In particular, the discretization of the Rayleigh and Lomax distributions has been studied by them. Recently, reliability approximation under stress strength model from different consideration has been examined by Nayak (2011). Approximation of system reliability has been studied by Xie and Lai (1998) using one step conditioning. A stress-strength inference reliability model with strength degradation under the assumptions that stress-strength are statistically independent have been examined by Xue and Yang (1997). They have also presented simple formulas for estimating upper and lower bounds for stress-strength reliability. Kundu and Gupta (2005) have considered estimation of P(X>S) where X and S are treated as independent random variables. Raqab and Kundu (2005) have considered the case where X and S are treated as independent generalized Rayleigh random variables. Here we will consider the case where X and S are independent Weibull random variables.

However, these studies were restricted to the discrete concentration approach only. Bound-based reliability approximation may be a better alternative as it has wider application than the discrete concentration approach (Roy, 1993).

The purpose of this paper is to approximate reliability of complex system based on reliability bounds. Interesting feature of the proposed approach is that the reliability approximation comes out as a function of distributional parameters and it can be adjusted for designing and redesigning the system to ensure the maximum level of reliability at a given cost.

2. RELIABILITY BOUNDS

If we can find close lower and upper bounds of reliability, the evaluation of system reliability becomes considerably easier as pointed out in Gertsbakh (1989). Usually these bounds are found to be satisfactory for practical purposes. Further, the computation of actual reliabilities are difficult than the computation of reliability bounds. We have already pointed out that reliability bounds can be obtained in terms of design parameters and these can be adjusted suitably.

As reliability bounds are item dependent and setup dependent, we have studied a very common but an important engineering item for determining the reliability bounds under the Weibull setup. Weibull distribution, as pointed out in Gertsbakh (1989), is a very useful model for reliability analysis and can give rise to other well-known models under different parametric choices. Weibull distribution has wide applicability. So, a study on reliability bounds on Weibull frame work will have a wider appeal.

The I-beam is a well known engineering item (Kapur and Lamberson, 1977). The shear stress of I-beam is function external bending moment (M), distance from the neutral axis to the extreme fibers (C), outside radious (r) and wall thickness (t). The shear stress of I-beam is given by

$$S=\frac{MC}{\pi r^3 t}$$

In real world design problems, according to Kapur and Lamberson (1977) and Kececioglu (2003), design parameters M, C, r, and t are random variables.

Under this proposed work, we have assumed that M, C, r, t and X are mutually independent random variables with M following W (λ_M, Υ), C following W (λ_C, β) , r following W (λ_r, α) and t following W (λ_t, ϕ). We also assume that Strength random variable X follows W (λ, θ).

Under this setup, we propose to evaluate system reliability R, i.e.

$$R = P(X > \frac{MC}{\pi r^3 t})$$
$$= P\{r > (\frac{MC}{\pi X t})^{\frac{1}{3}}\}$$
$$= E_M E_C E_T E_X[P\{r > (\frac{MC}{\pi X t})^{\frac{1}{3}} \mid M=m, C=c, T=t \text{ and } X=x\}] = E_M E_C E_T E_X\{e^{-\lambda_r (\frac{MC}{\pi X t})^{\frac{\alpha}{3}}}\}$$

Hence, the unconditional reliability value is given by

$$R=\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty e^{-\lambda_r (\frac{MC}{\pi X t})^{\frac{\alpha}{3}}} dF_M(m)\, dF_C(c)\, dF_X(x)\, dF_T(t) \qquad (2.1)$$

Let us now consider two lemmas to arrive at upper and lower bounds for R respectively.

Lemma 1. $e^{-X} \leq \sum_{r=0}^{2k} (-1)^r \frac{X^r}{r!}$, for X>0, and k= 1, 2,....

Proof. Proof of lemma1 follows from finite form of Maclaurin`s series, expended up to fourth term, i.e.
f(x) = f(0) + X$f^{(1)}(0)$ +$\frac{X^2}{2!} f^{(2)}(0)$ +$\frac{X^3}{3!} f^{(3)}(\theta X)$, with f(x) = e^{-X}, where $f^{(i)}(.)$ is the i[th] derivatives of f(X) and where θ lies between 0 and 1. As the fourth term is negative, we have, after dropping the same, $e^{-X} \leq 1 - X + \frac{X^2}{2} = \sum_{r=0}^{2k} (-1)^r \frac{X^r}{r!}$ for X$>$ 0 and k=1. The inequality is sharp because the bound is attained for X= 0.

Lemma 2. $e^{-X} \geq \sum_{r=0}^{2k+1} (-1)^r \frac{X^r}{r!}$, for X>0, and k= 0, 1, 2,....

Proof. Proof of lemma2 follows from finite form of Maclaurin`s series, extended up to third term. As the third term is positive, we have $e^{-X} \geq 1 - X = \sum_{r=0}^{2k+1} (-1)^r \frac{X^r}{r!}$ for X$>$ 0 and k=0. The inequality is sharp because the bound is attained for X=0.

Result 1. If the strength and the stress component random variable of the I-beam follows Weibull distribution then upper bound for the system reliability, R, is given by

$$U(\lambda_r, \lambda_C, \lambda_t, \lambda_M, \lambda, \alpha, \theta, \beta, \varphi, \Upsilon) = 1 - \lambda_r \left(\frac{1}{\pi}\right)^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}} \frac{\Gamma\left(1+\frac{\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{\alpha}{3\Upsilon}}}$$
$$+\frac{\lambda_r^2}{2}\left(\frac{1}{\pi}\right)^{\frac{2\alpha}{3}} \frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{2\alpha}{3\beta}\right)}{\lambda_C^{\frac{2\alpha}{3\beta}}} \frac{\Gamma\left(1+\frac{2\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{2\alpha}{3\Upsilon}}}$$

Proof. Using lemma1, with the choice of X=$\lambda_r (\frac{MC}{\pi X t})^{\frac{\alpha}{3}}$ and k=1, on the expression of reliability, R, given at (1), we get

$$R \leq \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty [1 - \lambda_r (\frac{MC}{\pi X t})^{\frac{\alpha}{3}} + \frac{\lambda_r^2}{2} (\frac{MC}{\pi X t})^{\frac{2\alpha}{3}}]\, dF_X(x)\, dF_M(m)\, dF_T(t) dF_C(c)$$
$$= \int_0^\infty \int_0^\infty \int_0^\infty f(M,C,t)\, dF_M(m)\, dF_T(t) dF_C(c) \qquad (2.2)$$

where,

$$f(M,C,t)=\int_0^\infty [1-\lambda_r(\frac{MC}{\pi Xt})^{\frac{\alpha}{3}}+\frac{\lambda_r^2}{2}(\frac{MC}{\pi Xt})^{\frac{2\alpha}{3}}]\,dF_X(x)$$

With

$F_X(x)$=1- $e^{-\lambda X^{\theta}}$, we can evaluate $f(M,C,t)$ and obtain

$$f(M,C,t)=1-\lambda_r(\frac{MC}{\pi t})^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}+\frac{\lambda_r^2}{2}(\frac{MC}{\pi t})^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}} \quad (2.3)$$

Using (2.3) in (2.2), we get

$$R\leq\int_0^\infty\int_0^\infty\int_0^\infty[1-\lambda_r(\frac{MC}{\pi t})^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}+\frac{\lambda_r^2}{2}(\frac{MC}{\pi t})^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}}]\,dF_M(m)dF_T(t)dF_C(c) \quad (2.4)$$

$$=\int_0^\infty\int_0^\infty f(M,C)\,dF_M(m)dF_C(c) \quad (2.5)$$

$Where,$

$$f(M,C)=\int_0^\infty[1-\lambda_r(\frac{MC}{\pi t})^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}+\frac{\lambda_r^2}{2}(\frac{MC}{\pi t})^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}}]\,dF_T(t)$$

$$=1-\lambda_r(\frac{MC}{\pi})^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}}+\frac{\lambda_r^2}{2}(\frac{MC}{\pi})^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}} \quad (2.6)$$

$Using$ (2.6) in (2.5), we get

$$R\leq\int_0^\infty\int_0^\infty[1-\lambda_r(\frac{MC}{\pi})^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}}+\frac{\lambda_r^2}{2}(\frac{MC}{\pi})^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}}]\,dF_M(m)dF_C(c)$$

$$=\int_0^\infty f(M)\,dF_M(m) \quad (2.7)$$

Where,

$$f(M)=\int_0^\infty[1-\lambda_r(\frac{MC}{\pi})^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}}+\frac{\lambda_r^2}{2}(\frac{MC}{\pi})^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}}]dF_C(c)$$

$$=1-\lambda_r(\frac{M}{\pi})^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}}\frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}}+\frac{\lambda_r^2}{2}(\frac{M}{\pi})^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}}\frac{\Gamma\left(1+\frac{2\alpha}{3\beta}\right)}{\lambda_C^{\frac{2\alpha}{3\beta}}} \quad (2.8)$$

Using (2.8) in (2.7), we get

$$R\leq\int_0^\infty[1-\lambda_r(\frac{M}{\pi})^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}}\frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}}+\frac{\lambda_r^2}{2}(\frac{M}{\pi})^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}}\frac{\Gamma\left(1+\frac{2\alpha}{3\beta}\right)}{\lambda_C^{\frac{2\alpha}{3\beta}}}]\,dF_M(m)$$

$$=1-\lambda_r\left(\frac{1}{\pi}\right)^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}}\frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}}\frac{\Gamma\left(1+\frac{\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{\alpha}{3\Upsilon}}}+\frac{\lambda_r^2}{2}\left(\frac{1}{\pi}\right)^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}}\frac{\Gamma\left(1+\frac{2\alpha}{3\beta}\right)}{\lambda_C^{\frac{2\alpha}{3\beta}}}\frac{\Gamma\left(1+\frac{2\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{2\alpha}{3\Upsilon}}} \quad (2.9)$$

Hence, upper bound for the system reliability, R, of the I-beam under the Weibull setup is given by

$$U(\lambda_r,\lambda_C,\lambda_t,\lambda_M,\lambda,\alpha,\theta,\beta,\varphi,\Upsilon)=1-\lambda_r\left(\frac{1}{\pi}\right)^{\frac{\alpha}{3}}\frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}}\frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}}\frac{\Gamma\left(1+\frac{\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{\alpha}{3\Upsilon}}}$$

$$+\frac{\lambda_r^2}{2}\left(\frac{1}{\pi}\right)^{\frac{2\alpha}{3}}\frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}}\frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}}\frac{\Gamma\left(1+\frac{2\alpha}{3\beta}\right)}{\lambda_C^{\frac{2\alpha}{3\beta}}}\frac{\Gamma\left(1+\frac{2\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{2\alpha}{3\Upsilon}}}$$

This completes the proof of result 1.

Result 2. If the strength and the stress component random variable of the I-beam follows Weibull distribution then lower bound for the system reliability, R, is given by

$$L(\lambda_r, \lambda_C, \lambda_t, \lambda_M, \lambda, \alpha, \theta, \beta, \varphi, \Upsilon) = 1- \lambda_r \left(\frac{1}{\pi}\right)^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}} \frac{\Gamma\left(1+\frac{\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{\alpha}{3\Upsilon}}}$$

Proof. Using lemma2, with the choice of $X=\lambda_r(\frac{MC}{\pi X t})^{\frac{\alpha}{3}}$ and k=0, on the expression of reliability, R, given at (1), we get

$$R \geq \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty [1-\lambda_r(\frac{MC}{\pi X t})^{\frac{\alpha}{3}}]\, dF_X(x)\, dF_M(m)\, dF_T(t) dF_C(c) \quad (2.10)$$

$$= \int_0^\infty \int_0^\infty \int_0^\infty f^*(M,C,t)\, dF_M(m)\, dF_T(t) dF_C(c) \quad (2.11)$$

Where,

$$f^*(M,C,t) = \int_0^\infty [1-\lambda_r(\frac{MC}{\pi X t})^{\frac{\alpha}{3}}]\, dF_X(x)$$

With

$F_X(x)$=1-$e^{-\lambda X^{\theta}}$, we can evaluate $f^*(M,C,t)$ and obtain

$$f^*(M,C,t) = 1-\lambda_r(\frac{MC}{\pi t})^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \quad (2.12)$$

Using (12) in (11), we get

$$R \geq \int_0^\infty \int_0^\infty \int_0^\infty [1-\lambda_r(\frac{MC}{\pi t})^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}]\, dF_M(m) dF_T(t) dF_C(c)$$

$$= \int_0^\infty \int_0^\infty f^*(M,C)\, dF_M(m) dF_C(c) \quad (2.13)$$

Where,

$$f^*(M,C) = \int_0^\infty [1-\lambda_r(\frac{MC}{\pi t})^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}}]\, dF_T(t)$$

$$= 1-\lambda_r(\frac{MC}{\pi})^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}} \quad (2.14)$$

Using (2.14) in (2.13), we get

$$R \geq \int_0^\infty \int_0^\infty [1-\lambda_r(\frac{MC}{\pi})^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}}]\, dF_M(m) dF_C(c)$$

$$= \int_0^\infty f^*(M)\, dF_M(m) \quad 2.15)$$

Where,

$$f^*(M) = \int_0^\infty [1-\lambda_r(\frac{MC}{\pi})^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}}] dF_C(c)$$

$$= 1-\lambda_r(\frac{M}{\pi})^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}} \quad (2.16)$$

Using (2.16) in (2.15), we get

$$R \geq \int_0^\infty [1 - \lambda_r (\frac{M}{\pi})^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}}]\, dF_M(m)$$

$$=1- \lambda_r \left(\frac{1}{\pi}\right)^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}} \frac{\Gamma\left(1+\frac{\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{\alpha}{3\Upsilon}}} \qquad (2.17)$$

Hence, lower bound for the system reliability, R, of the I-beam under the Weibull setup is given by

$$L\,(\lambda_r, \lambda_C, \lambda_t, \lambda_M, \lambda, \alpha, \theta, \beta, \varphi, \Upsilon) = 1- \lambda_r \left(\frac{1}{\pi}\right)^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}} \frac{\Gamma\left(1+\frac{\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{\alpha}{3\Upsilon}}}$$

This completes the proof of result 2.

3. RELIABILITY APPROXIMATION AND EXTENT OF ERROR

Reliability approximation and extent of error will be obtained in terms of these reliability bounds. In fact we propose average of two bounds for the reliability approximation. Therefore, reliability approximation is given by

$$R_{approx} = \frac{[U\,(\lambda_r,\lambda_C,\lambda_t,\lambda_M,\lambda,\alpha,\theta,\beta,\phi,\Upsilon) + L\,(\lambda_r,\lambda_C,\lambda_t,\lambda_M,\lambda,\alpha,\theta,\beta,\phi,\Upsilon)]}{2}$$

$$= 1- \lambda_r \left(\frac{1}{\pi}\right)^{\frac{\alpha}{3}} \frac{\Gamma\left(1-\frac{\alpha}{3\theta}\right)}{\lambda^{\frac{-\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{\alpha}{3\phi}\right)}{\lambda_t^{\frac{-\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{\alpha}{3\beta}\right)}{\lambda_C^{\frac{\alpha}{3\beta}}} \frac{\Gamma\left(1+\frac{\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{\alpha}{3\Upsilon}}}$$

$$+\frac{\lambda_r^2}{4} \left(\frac{1}{\pi}\right)^{\frac{2\alpha}{3}} \frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{2\alpha}{3\beta}\right)}{\lambda_C^{\frac{2\alpha}{3\beta}}} \frac{\Gamma\left(1+\frac{2\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{2\alpha}{3\Upsilon}}}$$

We also propose half of the absolute deviation between the two bounds as the extent of error. Therefore, extent of error is given by

$$\text{Error} \leq \left| \frac{[U\,(\lambda_r,\lambda_C,\lambda_t,\lambda_M,\lambda,\alpha,\theta,\beta,\phi,\Upsilon) - L\,(\lambda_r,\lambda_C,\lambda_t,\lambda_M,\lambda,\alpha,\theta,\beta,\phi,\Upsilon)]}{2} \right|$$

$$=\frac{\lambda_r^2}{4} \left(\frac{1}{\pi}\right)^{\frac{2\alpha}{3}} \frac{\Gamma\left(1-\frac{2\alpha}{3\theta}\right)}{\lambda^{\frac{-2\alpha}{3\theta}}} \frac{\Gamma\left(1-\frac{2\alpha}{3\phi}\right)}{\lambda_t^{\frac{-2\alpha}{3\phi}}} \frac{\Gamma\left(1+\frac{2\alpha}{3\beta}\right)}{\lambda_C^{\frac{2\alpha}{3\beta}}} \frac{\Gamma\left(1+\frac{2\alpha}{3\Upsilon}\right)}{\lambda_M^{\frac{2\alpha}{3\Upsilon}}}$$

4. NUMERICAL STUDY

We next cover a numerical study of the reliability approximation along with the extent of error. For this purpose, we have calculated lower and upper reliability bounds for some specific choices of the distributional parameters of an I-beam. The specific choices of distributional parameters, considered here in, are α=6, $\lambda_r = 12, \lambda_t = .04$, $\lambda = .3$, $\lambda_C = 6$, θ =7, ϕ =8, β=5, ϒ=4. We have allowed the other parameter, $\lambda_{M,}$ to vary so that it can cover a wide range of reliability values. The corresponding reliability approximation and extent of error have been shown in the table1. It may be observed from the given table that error term sharply decreases as reliability increases.

Table4.1. Reliability approximation and extent of error under the Weibull setup

Sl. No.	Stress parameter(λ_M)	Upper bound	Lower bound	Reliability approximation	Extent of error
1	10000	0.866822	0.866634	0.866728	9.37E-05
2	9500	0.873481	0.873303	0.873392	8.90E-05
3	9000	0.880140	0.879971	0.880055	8.43E-05
4	8500	0.886799	0.886639	0.886719	7.96E-05
5	8000	0.893457	0.893308	0.893383	7.50E-05
6	7500	0.900116	0.899976	0.900046	7.03E-05
7	7000	0.906775	0.906644	0.906710	6.56E-05
8	6500	0.913434	0.913312	0.913373	6.09E-05
9	6000	0.920093	0.919981	0.920037	5.62E-05
10	5500	0.926752	0.926649	0.926700	5.15E-05
11	5000	0.933411	0.933317	0.933364	4.69E-05
12	4500	0.940070	0.939986	0.940028	4.22E-05
13	4000	0.946729	0.946654	0.946691	3.75E-05
14	3500	0.953388	0.953322	0.953355	3.28E-05
15	3000	0.960047	0.959990	0.960018	2.81E-05
16	2500	0.966705	0.966659	0.966682	2.34E-05
17	2000	0.973364	0.973327	0.973346	1.87E-05
18	1500	0.980023	0.979995	0.980009	1.41E-05
19	1000	0.986682	0.986663	0.986673	9.37E-06
20	500	0.993341	0.993332	0.993336	4.69E-06

5. CONCLUSIONS AND DISCUSSIONS

Analytical determination of reliability of a complex system is setup dependent and item dependent and a very difficult task. From the literature, we observe that for the intractable cases different author`s have suggested different procedures numerically to approximate system reliability. But there is no work in the literature for reliability approximation with the idea of extent of error in terms of design parameters. Further manipulation of design parameters can`t be under taken under their approach.

So, we have tackled analytic hard reliability determination problem to bridge this gap in the literature. Our proposed method for approximating reliability of complex system for the Weibull framework under the stress-strength model has greater applicability as it has been presented in terms of design parameters and can be adjusted. This current work is an important one for product planning when the actual reliability is not available. So, we recommend this method for approximating the reliability of complex system for intractable cases under the stress-strength model.

REFERENCES

D`Errico, J.R. and Zaino, N.A Jr. (1988). Statistical tolerancing using a modification of Taguchi's method, *Technometrics*, **30**, 397-405.

Evans, D.H. (1975). Statistical tolerancing: The state of the art: Part II, Methods for estimating moments, *Journal of Quality Technology*, **7**, 1-12.

English, J.R. Sargent, T.and Landers, T.L. (1996). A discretizing approach for stress\ strength analysis, *IEEE Trans. Reliability*, **45**, 84-89.

Gertsbakh, I.B. (1989). *Statistical Reliability Theory*, Marcel Dekker Inc., New York and Basel.

Kapur, K.C. and Lamberson, L.R. (1977). *Reliability in Engineering Design*, John Wiley and Sons.

Kundu, D. and Gupta, R. D. (2005). Estimation of P(Y<X) for Generalized Exponential Distributions, *Metrika*, **61**(3), 291-308.

Kececioglu, D. B. (2003). Robust Engineering Design-By-Reliability, *DEStech* Publications, 1148 Elizabeth Ave., #2, Lancaster, PA 17601-4359, 919.

Nayak, S. (2011). Reliability approximation for an engineering item under a Weibull framework, Journal *of Management Research in Emerging Economics*, **1**, 70-80.

Parzen, E. (1960). *Modern probability theory and its application*, John Wiley and Sons.

Roy, D. (1993). Reliability measures in the discrete bivariate setup and related characterization results for a bivariate geometric distribution, *J. Multivariate Analysis*, **46**, 362-373.

Roy, D. (2002). Discretization of continuous distributions, *Calcutta Statistical Association Bulletin*, **52**, 297-313.

Roy, D. (2003). The discrete normal distribution, *Communications in Statistics-Theory and Methods*, **32**, 1871-1883.

Roy, D. (2004). Discrete Rayleigh Distribution, *IEEE Trans. Reliability*, **53**, 255-260.

Roy, D. (2005). Characterization of bivariate discrete distributions based on residual life properties, Brazilian Journal of Probability and Statistics, **19**, 53-64.

Roy, D. and Dasgupta, T. A. (2000). Continuous approximation for evaluating reliability of a complex system under stress\strength model, *Communications in statistics - Simulation and Computation*, **29**, 811-821.

Roy, D. and Dasgupta, T. (2001). A discretizing approach for evaluating reliability of complex systems under stress\strength model, *IEEE Trans. Reliability*, **50**, 145-150.

Roy, D. and Dasgupta, T. (2002). Evaluation of reliability of complex systems by means of a discretizing approach: Weibull setup, *International Journal for Quality and Reliability Management*, **19**, 792-801.

Roy, D. and Ghosh, T. (2009). A new discretization approach with application in reliability estimation, IEEE Transactions on Reliability, Vol.58, pp456-461.

Taguchi, G. (1978). Performance Analysis Design, *International Journal of production Research*, **16**, 521-530.

Raqab, M.Z. and Kundu, D. (2005). Comparison of different estimators of P(Y<X) for Scaled Burr Type distribution, *Communication in Statistics-Computations and Simulations*, **34**, 465-483.

Xie, M. and Lai, C.D. (1998). Reliability bounds via conditional inequalities, Journal of applied probability, **35**, 104-114.

Xue, J. and Yang, K. (1997). Upper and Lower bounds of stress-strength inference reliability with random strength-degradation, **46**, 142-145.