International Journal of Reliability and Applications
Vol. 13, No. 2, pp. 57-69, 2012

Moment inequalities of NBU_{mgf} with testing hypotheses application

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Abstract. Our goal in this paper is to establish inequalities for the moments of new better than used in the moment generating function class (NBU_{mgf}). Using these inequalities we propose a new test for exponentiality versus NBU_{mgf} class. Pitman's asymptotic relative efficiency, power and critical values of this test are calculated to assessthe performance of the test. We proposed also a new test for exponentiality versus NBU_{mgf} in the right censored data. Sets of real dataare used as an example to elucidate the use of the proposed test forpractical problems.

Key Words: *Moment inequality, NBU_{mgf}, life testing, exponentiality, efficiency, asymptotic normality*

1. INTRODUCTION

Nonparametric families of aging distributions have been the subject of investigation for more than four decades. Ahmad (2001), one of the authorspresented moments inequalities for classes of life distributions including IFR,NBU, NBUE and HNBUE. Ahmad and Mugdadi (2004) presented three other classes, IFRA, NBUC and DMRL. Also, Abu-Yousef (2002) for DMRL and Abu-Yousef (2004) for NRBU, Al-Ruzaiza et al (2003) for HNBUE, Mahmoud et al (2003) for NRBU and (2005) for RNBU, Diab et al (2009) for NBUL, and then they use these inequalities to devise new testing procedures for exponentiallity against an alternative among the classes indicated above.

Given two non-negative random variables X and Y, with survival functions F and G, respectively, X is said to be smaller than Y in the moment generating function ordering (denoted by $X \leq_{mgf} Y$) if and only if,

$$\int_0^\infty e^{\lambda x}\bar{F}(x)dx \leq \int_0^\infty e^{\lambda x}\bar{G}(x)dx \qquad \text{for all} \quad \lambda > 0.$$

Recently based on this notion, Li (2004) introduced a new aging class of life distributions. Given a non-negative random variable X, we say that X is new better than used in the moment generating function order (denoted by $X \in NBU_{mgf}$) if

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$$X_t \leq_{mgf} X \quad \text{for all} \quad t > 0.$$

Equivalently, $X \in NBU_{mgf}$ if and only if,

$$\int_0^\infty e^{\lambda x}\bar{F}(x+t)dx \leq \bar{F}(t)\int_0^\infty e^{\lambda x}\bar{F}(x)dx \quad \text{for all} \quad \lambda, t \geq 0. \tag{1.1}$$

Some properties of the NBU_{mgf} class including some preservations properties have been discussed by Li (2004), while Zhang and Li (2004) showed that the NBU_{mgf} class is preserved under both the non-homogeneous Poissonshock model and the general shock model. Some new results are given includingsome closure properties, characterizations and testing exponentially against the NBU_{mgf} class is addressed by Ahmad and Kayid (2004).

2. MOMENT INEQUALITY

In this section we derive moment inequalities for the NBU_{mgf} class. In this,as well as subsequent all moments are assumed to be exist and finite.

Theorem 2.1 *If F is NBU_{mgf}, then for all integer $r \geq 0$*

$$\frac{\mu_{r+1}}{r+1}E(e^{\lambda X}) \geq \frac{r!}{\lambda^{r+1}}E\left[e^{\lambda X} - \sum_{j=0}^{r}\frac{(\lambda X)^j}{j!}\right] \tag{2.1}$$

proof. Since F is NBU_{mgf} then from (1.1) we have,

$$\int_0^\infty\int_0^\infty t^r e^{\lambda x}\bar{F}(t)\bar{F}(x)dxdt \geq \int_0^\infty\int_0^\infty t^r e^{\lambda x}\bar{F}(x+t)dxdt$$

for all integer $r \geq 0$.

Since

$$\int_0^\infty t^r\bar{F}(t)dt = E\int_0^\infty t^r I(X>t)dt$$
$$= \frac{\mu_{r+1}}{r+1},$$

and

$$\int_0^\infty e^{\lambda x}\bar{F}(x)dx = E\int_0^\infty e^{\lambda x}I(X>x)dx$$
$$= \frac{1}{\lambda}E[e^{\lambda X}-1].$$

then

$$\begin{aligned} \text{L.H.S} &= \left\{\int_0^\infty t^r \bar{F}(t)dt\right\}\left\{\int_0^\infty e^{\lambda x}\bar{F}(x)dx\right\} \\ &= \frac{\mu_{r+1}}{\lambda(r+1)}[E(e^{\lambda X})-1]. \end{aligned}$$

It is easy to show that,

$$\begin{aligned} \text{R.H.S} &= \int_0^\infty \int_0^\infty t^r e^{\lambda x}\bar{F}(x+t)dxdt \\ &= E\int_0^\infty \int_0^\infty t^r e^{\lambda x} I(X > x+t)dxdt \\ &= \frac{1}{\lambda}E\left\{e^{\lambda X}\int_0^X t^r e^{-\lambda t}dt - \int_0^X t^r dt\right\} \end{aligned}$$

Upon using the integral by parts we get

$$\text{R.H.S} = \frac{r!}{\lambda^{r+2}}E\left[e^{\lambda X} - \sum_{j=0}^{r}\frac{(\lambda X)^j}{j!}\right] - \frac{\mu_{r+1}}{\lambda(r+1)}.$$

The theorem is proved.

3. HYPOTHESIS TESTING PROBLEM

Here a test statistic based on the moment inequality for testing H_0 : F is exponential against an alternative that H_1: F is belongs to NBU_{mgf} class and not exponential is proposed as follows.Using Theorem (2.1) we may use δ as a measure of departure from exponentiality where

$$\delta = \frac{1}{\lambda}\left\{\frac{\mu_{r+1}}{r+1}E(e^{\lambda X}) - \frac{r!}{\lambda^{r+1}}E\left[e^{\lambda X} - \sum_{j=0}^{r}\frac{(\lambda X)^j}{j!}\right]\right\} \tag{3.1}$$

Note that under H_0 : $\delta = 0$, while under H_1: $\delta > 0$.

To estimate δ, let $X_1, X_2, \cdots, X_n$ be a random sample from F, so the empirical form of δ in (3.1) can be written as follows

$$\hat{\delta}_{n,r} = \frac{1}{n^2}\sum_{k=1}^{n}\sum_{i=1}^{n}\left\{\frac{X_k^{r+1}}{\lambda(r+1)}e^{\lambda X_i} - \frac{r!}{\lambda^{r+2}}\left[e^{\lambda X_i} - \sum_{j=0}^{r}\frac{(\lambda X_i)^j}{j!}\right]\right\} \tag{3.2}$$

To find the limiting distribution of $\delta_{n,r}$ we resort to the *U*-statistic theory.

Let

$$\phi(X_1, X_2) = \frac{e^{\lambda X_1}X_2^{r+1}}{\lambda(r+1)} - \frac{r!}{\lambda^{r+2}}\left[e^{\lambda X_1} - \sum_{j=0}^{r}\frac{(\lambda X_1)^j}{j!}\right]$$

and define the symmetric kernel

$$\psi(X_1\,,X_2)=\frac{1}{2!}\sum_R \phi(X_i\,,X_k)$$

where the sum is over all arrangements of X_i and X_k, this leads that $\delta_{n,r}$ in (3.2) is equivalent to U- statistic given by

$$U_n=\frac{1}{\binom{n}{2}}\sum_R \phi(X_i\,,X_k)$$

The next result summarizes the asymptotic properties of $\delta_{n,r}$

Theorem 3.1 (i) As $n\rightarrow\infty$, $\sqrt{n}(\delta_{n,r}-\delta)$ is asymptotically normal with mean 0 and variance σ^2 as given in (3.6).
(ii) Under H_0, the variance is reduced to σ_0^2 in (3.7).

Proof. Let

$$\begin{aligned}\eta_1(X_1)&=E[\phi(X_1\,,X_2)\,|X_1]\\&=\frac{1}{\lambda}\left\{(r!)e^{\lambda X_1}-\frac{r!}{\lambda^{r+1}}\left[e^{\lambda X_1}-\sum_{j=0}^{r}\frac{(\lambda X_1)^j}{j!}\right]\right\}\end{aligned}\tag{3.3}$$

and

$$\begin{aligned}\eta_2(X_1)&=E[\phi(X_2\,,X_1)\,|X_1]\\&=\frac{1}{\lambda}\left\{\frac{X_1^{r+1}}{(r+1)(1-\lambda)}-\frac{r!}{(1-\lambda)}\right\}\end{aligned}\tag{3.4}$$

Making use of (3.3) and (3.4) yields.Define $\eta(X)=\eta_1(X_1)+\eta_2(X_1)$,

$$\eta(X)=\frac{1}{\lambda}\left\{\frac{X^{r+1}}{(r+1)(1-\lambda)}+(r!)e^{\lambda X}-\frac{r!}{\lambda^{r+1}}\left[e^{\lambda X}-\sum_{j=0}^{r}\frac{(\lambda X)^j}{j!}\right]-\frac{r!}{(1-\lambda)}\right\}\tag{3.5}$$

In view of (3.5), the variance is

$$\sigma^2=Var\left\{\frac{X^{r+1}}{\lambda(1+r)(1-\lambda)}+\frac{(r!)}{\lambda}e^{\lambda X}-\frac{r!}{\lambda^{r+2}}\left[e^{\lambda X}-\sum_{j=0}^{r}\frac{(\lambda X)^j}{j!}\right]-\frac{r!}{\lambda(1-\lambda)}\right\}\tag{3.6}$$

Under H_0 it is easy to prove that $\mu_0=E[\eta(X)]=0$ and the variance σ_0^2 reduce to

$$\sigma_0^2 = \frac{(r!)^2}{\lambda^{2r+4}}\left[\frac{(1-\lambda^{r+1})^2}{(1-2\lambda)} + \sum_{i=0}^{r}\sum_{j=0}^{r}\frac{\lambda^{i+j}(i+j)!}{i!j!} - 2(1-\lambda^{r+1})\sum_{j=0}^{r}\frac{\lambda^j}{(1-\lambda)^{j+1}}\right]$$
$$+\frac{2\,r!}{\lambda^2(1-\lambda)^2}\left[\frac{1}{(r+1)} - \frac{(r!)(1-\lambda^{r+1})}{\lambda^{r+1}(1-\lambda)^{r+1}}\right]$$
$$+\frac{2\,r!}{\lambda^{r+3}(r+1)(1-\lambda)}\sum_{j=0}^{r}\frac{\lambda^j(r+j+1)!}{j!} - \frac{(r!)^2}{\lambda^2(1-\lambda)^2}, \tag{3.7}$$

for all λs make σ_0^2 positive.

4. THE PITMAN ASYMPTOTIC EFFICIENCY (PAE) FOR NBU_{mgf}

This section includes the calculations of the asymptotic efficiencies of the NBU_{mgf} test statistic. This calculation are done using the following alternativefamilies

(i) Linear failure rate family (LFR): $\overline{F}_1(x) = \exp\left(-x - \frac{\theta}{2}x^2\right)$, $x > 0$, $\theta \geq 0$.

(ii) Weibull family: $\overline{F}_2(x) = \exp(-x^{\theta})$, $x > 0$, $\theta > 0$.

Consider

$$\delta_\theta = \int_0^\infty\int_0^\infty t^r e^{\lambda x}\bar{F}_\theta(x)\bar{F}_\theta(t)dxdt - \int_0^\infty\int_0^\infty t^r e^{\lambda x}\bar{F}_\theta(x+t)dxdt,$$

then

$$\frac{\partial}{\partial\theta}\delta_\theta = \int_0^\infty\int_0^\infty t^r e^{\lambda x}\bar{F}_\theta(t)\bar{F}'_\theta(x)dxdt + \int_0^\infty\int_0^\infty t^r e^{\lambda x}\bar{F}'_\theta(t)\bar{F}_\theta(x)dxdt$$
$$-\int_0^\infty\int_0^\infty t^r e^{\lambda x}\bar{F}'_\theta(x+t)dxdt$$

It is easy to prove that

$$\mathrm{PAE}(\hat{\delta}_{n,r},F) = \frac{1}{\sigma_0}\left|\frac{1}{1-\lambda}\int_0^\infty x^r\bar{F}'_\theta(x)dx + \frac{r!}{\lambda^{r+1}}\int_0^\infty \bar{F}'_\theta(x)\sum_{i=0}^{r}\frac{(\lambda x)^i}{i!}dx\right.$$
$$\left.-\frac{r!(1-\lambda^{r+1})}{\lambda^{r+1}}\int_0^\infty e^{\lambda x}\bar{F}'_\theta(x)dx\right|,$$

$$\mathrm{PAE}(\hat{\delta}_{n,r},LFR) = \frac{1}{\sigma_0}\left|\frac{r!(1-\lambda^{r+1})}{(1-\lambda)^3\lambda^{r+1}} - \frac{r!}{2\lambda^{r+1}}\sum_{i=0}^{r}(i+2)(i+1)\lambda^i - \frac{(r+2)!}{2(1-\lambda)}\right|$$

$$\text{PAE}(\hat{\delta}_{n,r}, Weibull) = \frac{1}{\sigma_0}\left|\frac{r!(\lambda^{r+1}-1)[\ln(1-\lambda)-1+\gamma]}{\lambda^{r+1}(1-\lambda)^2} - \frac{r!}{\lambda^{r+1}}\sum_{i=1}^{r}\lambda^i\{1+(i+1)[\sum_{j=0}^{i-1}\frac{1}{i-j}-\gamma]\} - \frac{r!}{(1-\lambda)}\{1+(r+1)[\sum_{k=0}^{r-1}\frac{1}{r-k}-\gamma]\}\right|$$

where γ = 0.57720⋯ (Eular constant).

Table 4.1 gives the efficiencies of our proposed test δ_n comparing with the tests given by Kango (1993)(U_n) and Mugdadi and Ahmad (2005)(δ_3).

Table 4.1. Pitman asymptotic efficiencies for various values of λ and r

	$\hat{\delta}_n$				U_n	δ_3
	λ	$r=1$	$r=2$	$r=3$		
LFR	0.1	0.8518	0.6394	0.4414	0.433	0.408
	0.2	0.7906	0.5766	0.3914		
	0.3	0.6974	0.4917	0.3268		
	0.4	0.5384	0.3643	0.2355		
	1.6	2.07	4.855	1.44		
	2.1	2.003	2.1927	1.305		
	3.0	2.236	1.7928	1.1547		
	4.0	3.0551	1.732	1.0935		
	4.9	9.5003	1.7592	1.071		
Weibull	0.1	15.1957	73.3678	378.129	0.132	0.170
	0.2	3.2098	6.8224	16.7385		
	0.3	1.2465	1.5159	2.2374		
	0.4	0.5742	0.4675	0.4498		
	1.6	1.9922	3.9107	0.9983		
	2.1	2.5358	2.2211	1.0989		
	3.0	3.5287	2.1517	1.121		
	4.0	5.4535	2.2791	1.1502		
	4.9	18.1952	2.4407	1.1813		

Note that our test outperforms the others for most values of λ and r.

5. MONTE CARLO NULL DISTRIBUTION CRITICAL POINTS

Many practitioners, such as applied statisticians, and reliability analysts areinterested in simulated percentiles. Table (5.1) gives these percentile pointsof the statistic $\delta_{n,r}$ given in (3.2) at r = 1 and the calculations are based on1000 simulated samples of sizes n = 2(1)50.

Table 5.1. Critical values of statistic $\hat{\delta}_n$ at $\lambda = 0.1$ & r = 1

n	0.01	0.05	0.10	0.90	0.95	0.99
2	0.000	0.001	0.006	2.981	6.228	14.965
3	0.000	0.005	0.015	2.080	3.686	9.229
4	-0.090	0.004	0.018	1.709	2.527	6.365
5	-0.369	-0.018	0.012	1.392	2.042	4.913
6	-0.675	-0.075	0.002	1.197	1.841	3.669
7	-0.729	-0.114	0.003	1.043	1.632	3.295
8	-0.901	-0.174	-0.013	0.978	1.341	2.214
9	-0.929	-0.227	-0.019	0.847	1.244	2.459
10	-0.964	-0.202	-0.034	0.809	1.164	2.030
11	-1.128	-0.226	-0.035	0.819	1.150	1.893
12	-1.223	-0.267	-0.059	0.706	0.936	1.896
13	-1.701	-0.269	-0.084	0.667	0.909	1.559
14	-1.113	-0.271	-0.094	0.624	0.895	1.440
15	-1.053	-0.343	-0.126	0.649	0.837	1.225
16	-1.284	-0.360	-0.104	0.622	0.828	1.440
17	-1.534	-0.402	-0.131	0.613	0.791	1.263
18	-1.483	-0.338	-0.140	0.605	0.732	1.307
19	-1.319	-0.369	-0.158	0.551	0.717	1.184
20	-1.278	-0.340	-0.125	0.537	0.692	1.085
21	-1.339	-0.350	-0.127	0.479	0.670	1.006
22	-1.403	-0.307	-0.174	0.521	0.634	0.912
23	-1.423	-0.320	-0.122	0.483	0.631	0.906
24	-1.367	-0.363	-0.154	0.492	0.615	0.947
25	-1.032	-0.332	-0.152	0.466	0.622	0.918
26	-1.452	-0.405	-0.169	0.458	0.573	0.890
27	-1.283	-0.390	-0.165	0.455	0.575	0.847
28	-1.224	-0.423	-0.174	0.425	0.554	0.854
29	-1.294	-0.382	-0.162	0.434	0.544	0.784
30	-1.153	-0.430	-0.182	0.428	0.537	0.845
31	-1.184	-0.444	-0.201	0.422	0.522	0.842
32	-1.267	-0.408	-0.173	0.423	0.512	0.723
33	-1.221	-0.428	-0.156	0.404	0.497	0.731
34	-1.249	-0.417	-0.172	0.402	0.495	0.696
35	-1.290	-0.445	-0.194	0.385	0.511	0.718
36	-1.163	-0.446	-0.212	0.380	0.488	0.720
37	-1.326	-0.404	-0.198	0.386	0.449	0.665
38	-1.199	-0.453	-0.186	0.379	0.480	0.690
39	-1.217	-0.424	-0.200	0.369	0.438	0.689
40	-1.140	-0.436	-0.188	0.364	0.447	0.695
41	-1.275	-0.448	-0.208	0.359	0.423	0.624
42	-1.245	-0.441	-0.208	0.350	0.457	0.632
43	-1.136	-0.432	-0.196	0.347	0.439	0.592
44	-1.170	-0.395	-0.188	0.334	0.413	0.634
45	-1.048	-0.414	-0.208	0.346	0.451	0.652
46	-1.155	-0.443	-0.215	0.338	0.422	0.634
47	-1.128	-0.430	-0.209	0.344	0.404	0.622
48	-1.058	-0.433	-0.200	0.333	0.414	0.562
49	-1.229	-0.461	-0.236	0.330	0.416	0.587
50	-1.092	-0.470	-0.218	0.318	0.413	0.580

6. THE POWER ESTIMATES

Table 6.1 shows the power estimate of the test statistic $\delta_{n,r}$ given in (3.2) at the significant level 0.05 using LFR and Weibull distributions. The estimateare based on 1000 simulated samples for sizes *n* = 10, 20 and 30.

Table 6.1. Power estimates using α = 0.05 at r = 1

	n	Powers			
		$\theta = 1$	$\theta = 2$	$\theta = 3$	$\theta = 4$
LFR	10	1.000	1.000	1.000	1.000
	20	1.000	1.000	1.000	1.000
	30	1.000	1.000	1.000	1.000
Weibull	10	0.951	1.000	1.000	1.000
	20	0.950	1.000	1.000	1.000
	30	0.950	1.000	1.000	1.000

From Table 6.1 we can show that our test has very good power.

7. TESTING HYPOTHESIS AGINST *NBU$_{mgf}$* ALTERNATIVE FOR CENSORED DATA

In this section a test statistic is proposed to test H_0 versus H_1 with randomly right-censored samples. Such a censored data is usually the only data availablein life testing model where patients may be lost before the completing of a study. Using the censored data (Z_i,δ_i), *i*= 1, 2, 3, $\cdots$, *n* then the product limit estimator of survival function is given by

$$\bar{F}_n(x) = 1 - F_n(x) = \prod_{i<Z_i<k} \left[\frac{n-i}{n-i+1}\right]^{\delta_i}, x \in [0\ , Z_{(n)},]$$

Kaplan and Meier (1958).
In this case the proposed test statistic is

$$\hat{\delta}_n^c = \sum_{i=1}^{n}\sum_{j=1}^{n} t_j^r e^{\lambda x_i}\left\{\prod_{m=1}^{i-1} c_m^{\delta_m}\prod_{m=1}^{j-1} c_m^{\delta_m} - \prod_{s=1}^{l-1} c_s^{\delta_s}\right\}(x_i - x_{i-1})(t_j - t_{j-1}), \tag{7.1}$$

where $c_k = \frac{n-k}{n-k+1}$, and

$$l = \begin{cases} z's \le z_i + z_j & if \;\; z_i + z_j < z_n \\ n & if \;\; z_i + z_j \ge z_n \end{cases}$$

Table 7.1 gives the critical values percentiles of δ_n^c test for sample sizes$n$ = 5(1)50 based on 1000 replications.

Table 7.1. Critical values of statistic $\hat{\delta}_n^c$ at $\lambda = 0.1$ & $r = 1$

n	0.01	0.05	0.10	0.90	0.95	0.99
5	-3.186	-1.210	-0.537	0.111	0.249	0.911
6	-4.225	-1.335	-0.655	0.130	0.323	1.287
7	-4.567	-1.510	-0.802	0.179	0.364	1.024
8	-4.843	-1.738	-0.857	0.154	0.398	1.064
9	-4.573	-1.723	-0.891	0.150	0.305	1.151
10	-4.983	-2.026	-0.988	0.156	0.323	1.053
11	-4.758	-1.963	-1.020	0.122	0.293	0.871
12	-5.420	-2.112	-1.218	0.124	0.277	0.908
13	-4.742	-2.152	-1.260	0.097	0.252	0.683
14	-4.516	-2.107	-1.291	0.074	0.210	0.592
15	-4.905	-2.219	-1.334	0.068	0.170	0.592
16	-4.814	-2.218	-1.317	0.054	0.147	0.488
17	-5.086	-2.300	-1.512	0.031	0.114	0.409
18	-5.510	-2.327	-1.556	0.017	0.096	0.356
19	-5.316	-2.557	-1.654	0.009	0.072	0.365
20	-4.942	-2.747	-1.693	-0.007	0.048	0.298
21	-5.020	-2.590	-1.696	-0.018	0.056	0.284
22	-5.097	-2.605	-1.756	-0.014	0.037	0.282
23	-4.829	-2.657	-1.861	-0.029	0.024	0.185
24	-4.975	-2.832	-1.866	-0.027	0.030	0.197
25	-5.457	-2.816	-1.916	-0.054	-0.006	0.193
26	-5.299	-2.795	-2.007	-0.060	-0.003	0.126
27	-5.376	-3.026	-2.083	-0.059	-0.002	0.161
28	-5.187	-2.862	-2.006	-0.077	-0.006	0.150
29	-5.148	-2.857	-2.140	-0.087	-0.027	0.128
30	-5.556	-2.951	-2.113	-0.101	-0.041	0.062
31	-5.551	-3.192	-2.274	-0.096	-0.024	0.091
32	-5.704	-3.086	-2.282	-0.117	-0.052	0.081
33	-5.729	-2.913	-2.329	-0.137	-0.062	0.060
34	-5.721	-3.143	-2.367	-0.130	-0.063	0.039
35	-5.602	-3.225	-2.348	-0.163	-0.091	0.034
36	-5.381	-3.252	-2.482	-0.156	-0.078	0.040
37	-5.607	-3.213	-2.426	-0.174	-0.096	-0.017
38	-6.182	-3.355	-2.532	-0.183	-0.100	0.012
39	-6.137	-3.303	-2.529	-0.196	-0.126	-0.007
40	-5.868	-3.322	-2.637	-0.205	-0.112	0.006
41	-6.460	-3.402	-2.570	-0.194	-0.120	-0.000
42	-6.838	-3.591	-2.680	-0.228	-0.139	-0.036
43	-6.463	-3.474	-2.746	-0.244	-0.155	-0.057
44	-6.691	-3.585	-2.769	-0.249	-0.153	-0.045
45	-7.058	-3.693	-2.803	-0.269	-0.176	-0.059
46	-6.731	-3.610	-2.839	-0.264	-0.177	-0.073
47	-6.681	-3.896	-2.983	-0.283	-0.172	-0.068
48	-7.196	-3.934	-2.949	-0.294	-0.199	-0.080
49	-7.646	-3.926	-2.923	-0.292	-0.200	-0.068
50	-7.475	-4.044	-3.103	-0.316	-0.215	-0.102
80	-7.906	-5.134	-3.890	-0.307	0.000	0.000
81	-10.270	-6.401	-5.116	-0.613	-0.450	-0.258

7.1 The power estimates

In this section we present an estimation of the power for testing exponentiality Versus NBU_{mgf}. Using significance level α = 0.05with suitable parameter values of θ at n=10, 20 and 30, and for commonly used distributions in reliabilitysuch as LFR and Weibull alternatives which include in Table 7.2.

Table 7.2. Power estimates

	n	Powers			
		$\theta = 1$	$\theta = 2$	$\theta = 3$	$\theta = 4$
LFR	10	0.999	1.000	1.000	1.000
	20	1.000	1.000	1.000	1.000
	30	1.000	1.000	1.000	1.000
Weibull	10	0.951	1.000	1.000	1.000
	20	0.950	1.000	1.000	1.000
	30	0.951	1.000	1.000	1.000

It is clear from Table 7.2 that our test has excellent power.

8. APPLICATIONS

In this section we calculate the $\delta_{n,r}$ test statistic for real examples to illustratethe application of our test.

Example 8.1. The data set of 40 patients suffering from blood cancer (Leukemia)from one of ministry of health hospitals in Saudi Arabia sees Abouammoh et al. (1994). The ordered life times (in years) are

0.315	0.496	0.616	1.145	1.208	1.263	1.414	2.025
2.036	2.162	2.211	2.370	2.532	2.693	2.805	2.910
2.912	3.192	3.263	3.348	3.348	3.427	3.499	3.534
3.767	3.751	3.858	3.986	4.049	4.244	4.323	4.381
4.392	4.397	4.647	4.753	4.929	4.973	5.074	4.381

It was found that the value of test statistic for the data set by formula (3.2) $\delta_{n,1}$=13.156, which is greater than the critical value in Table (5.1). Then wereject the null hypothesis of exponentially and accept H1which states that the data set has NBU_{mgf} property.

Example 8.2. Let us consider the data of Susarla and Vanryzin (1978),which represent 81 survival times (in months) of patients melanoma. Out ofthese 4^6 represents non-censored data and the ordered values are

3.25	3.5	4.75	4.75	5	5.25	5.75	5.75	6.25	6.5
6.5	6.75	6.75	7.78	8	8.5	8.5	9.25	9.5	9.5
10	11.5	12.5	13.25	13.5	14.25	14.5	14.75	15	16.25
16.25	16.5	17.5	21.75	22.5	24.5	25.5	25.75	27.5	29.5
31	32.5	34	34.5	35.25	58.5				

The ordered censored data are

4	5.25	11	12.5	13.75	16.75	18.25	19	20
20.25	21.5	23.25	25	27	28.5	30	31	31.25
32.25	32.5	33	33.5	35	36.75	37	37.75	38
38	39.5	45.25	47.5	48.25	48.5	53.25	53.75	

Now taking into account the whole set of survival data (both censored anduncensored).It was found that the value of test statistic for the data set by formula (7.1) is given byδ_n^c=175.29 and this value greater than the tabulated criticalvalue (see Table 7.1). There is enough evidence to accept H1which states thatthe data set has NBU_{mgf} property.

9. CONCLUSIONS

Moments inequalities of NBU_{mgf} class of life distributions are deduced.Based on these inequalities a new test for exponentiality versus NBU_{mgf} class is constructed. The PAREs, powers and critical values of this test arecalculated. Based on PAEs comparison between our test and tests of Kango (1993)(*Un*) and Mugdadi and Ahmad (2005)($\delta 3$) are given. Our study showedthat our test performs higher PAE with respect to *Un*and $\delta 3$ tests for all valuesof λ and r. Our test gives high powers for LFR and Weibull alternatives. Based on right censored data a test for exponentiality versus NBU_{mgf} class is also given. Finally setsof real data are used to elucidate the proposed testfor practical problems.

ACKNOWLEDGMENTS

The authors are thankful to the referees and the editor in chief for valuable comments which enhanced the presentation of our paper.

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