Time-Delay Estimation in the Multi-Path Channel based on Maximum Likelihood Criterion

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Abstract

To locate an object accurately in the wireless sensor networks, the distance measure based on time-delay plays an important role. In this paper, we propose a maximum likelihood (ML) time-delay estimation algorithm in multi-path wireless propagation channel. We get the joint probability density function after sampling the frequency domain response of the multi-path channel, which could be obtained by the vector network analyzer. Based on the ML criterion, the time-delay values of different paths are estimated. Considering the ML function is non-linear with respect to the multi-path time-delays, we first obtain the coarse values of different paths using the subspace fitting algorithm, then take them as an initial point, and finally get the ML time-delay estimation values with the pattern searching optimization method. The simulation results show that although the ML estimation variance could not reach the Cramer-Rao lower bounds (CRLB), its performance is superior to that of subspace fitting algorithm, and could be seen as a fine algorithm.

Keywords: Multi-path propagation channel, time-delay estimation, maximum likelihood criterion, subspace fitting algorithm, CRLB

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1. Introduction

Time-delay estimation is an important research aspect in the signal processing domain, and it is usually used to determine the time offset of some signal compared with the referenced signal. The time-delay estimation technique could be adopted in biomedicine, geophysics, sonar and wireless communication, having wide applications. For example, one of the important applications of wireless sensor networks is localization [1], and to locate an object accurately in the wireless sensor networks, the measurement based on time-delay is the most ideal method. The lower the time resolution is, the more distance error it would make.

The main target of the time-delay estimation is to improve the time resolution. The concerned algorithms could be divided into two categories: one is used for the estimation of the wide-band signal, and the other is used for the estimation of the narrow-band signal.

The wide-band signal has the good performance to overcome the multi-path interference, so the estimation algorithms based on the wide-band signal are concentrating on the time-delay between two signals in single path. Algorithms based on the pattern matching are the most common for the time-delay estimation of wide-band signal, in which the matching degree of two signals is the function of the time-delay difference. We could obtain the time-delay between the two signals by maximizing the function of matching degree. Different matching functions correspond to different estimation algorithms, which include the cross correlation algorithms [2], minimum square error sum algorithms [3], normalized cross correlation algorithms [4], and so on. However, the time precision of all these algorithms is determined by sampling interval. In order to improve the precision, two main methods, one is on the basis of the interpolation technology [5][6][7] and the other is of the fractional delay filter technology [8][9][11], are utilized. Besides the pattern matching algorithms, there are some other kinds of algorithms, which are based upon the Hilbert transformation [10], the minimum entropy [12], the quadrature phase detection [13], and so on. Because the wide-band signal has the better capability to combat the multi-path interference, the algorithms based on wide-band signal assume the multi-path signals have been separated perfectly. And the trend of these algorithms are concentrating on how to improve precision of the time-delay further which is affected by the sampling frequency.

However, for the narrow-band signal, researchers concentrate more on the time-delays of different paths in the multi-path propagation channel. Considering that it is hard to distinguish two arrival signals in different paths with the time-delay difference less than the inverse of the source signal bandwidth [14], traditional algorithms based on Fast Fourier Transform (FFT) algorithm could not partition the signals when the difference of the arrival times is smaller, so the super-resolution of time-delay estimation of the narrow band signal is the research focus. Now the super-resolution technologies are mainly based on subspace fitting algorithms, such as Multiple Signal Classification (MUSIC) [15], Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [16] and Generalized Eigen values Utilizing Signal Subspace Eigen vectors (GEESE) [18]. It has been proved that the performance of MUSIC and ESPRIT is inferior to the maximum likelihood (ML) estimation algorithm when they are used in the arrival angle estimation in the antennal array [22]. Besides the subspace fitting algorithms, there is another super-resolution algorithm which is based on the spectral moment estimation [23], and the performance is also superior to the MUSIC and ESPRIT. However, it is only used in the arrival angle estimation [24], and has not been introduced into the application of time-delay estimation. Because the narrow band signal will result in multi-path interference easily, the trend of these concerned algorithms is to improve the multi-path resolution further.

In this paper we propose a time-delay estimation algorithm base upon the ML criterion so as to improve the time resolution in the multi-path propagation channel. Considering the ML value is a non-linear function of the multi-path time-delays, there are multiple local optimal solutions, so we could not obtain the global extremum by the convex optimization methods. Meanwhile, it is very hard to get the derivates of the ML function with respect to the time-delay parameters, so we could barely obtain the local optimal solution through traditional non-linear optimization technique, such as steepest descent method. Therefore in this paper, we adopt the pattern searching [28] to obtain the local extremum, avoiding derivation the function. However, for the performance of the local optimal solution being determined by the initial point, we first obtain the time-delays of different paths by the MUSIC subspace fitting algorithm, and then take the values as the initial point. Since there have already been a lot of literature discussing how to obtain the time-delays by the MUSIC and its improved algorithms [19][20], we will not elaborate its details in this paper. It must be noted that the MUSIC is only the representative of the subspace fitting algorithms, and using it is only to demonstrate that the ML algorithm could improve performance of subspace fitting algorithm further. Other algorithms, such as GEESE could also provide the initial points, which could be seen in section 4. The innovation of this paper is the combination of two different kinds of super-resolution algorithms that one is based on the ML criterion and the other is the subspace fitting algorithm. The latter provides a proper initial point for the ML, and ML improves the performance of the subspace fitting algorithm further.

The rest of this paper is organized as follows: section 2 is the system model, which represents the discrete frequency domain channel response as the matrix format; section 3 includes the design of ML function of the multi-path time-delays and the corresponding optimization procedures, and the Cramer-Rao lower bounds (CRLB) [25] is also given in section 3; section 4 is the simulations and performance analysis; and the paper is concluded in section 5.

2. System Model

The time domain impulse response of a multi-path channel is [26]:

$$h(t)=\sum_{d=0}^{D-1}\alpha_d\delta(t-\tau_d) \tag{1}$$

where D is the number of paths, τ_d and α_d is the time delay and attenuation factor of the d th path respectively. Then the frequency domain channel impulse response will be:

$$h(f)=\sum_{d=0}^{D-1}\alpha_d e^{-j2\pi f\tau_d} \tag{2}$$

The frequency domain response could be measured by the vector network analyzer [18].

After getting the frequency domain response, we uniformly sample it from f_L to f_H with the sampling frequency $f_0 = (f_H - f_L)/(N-1)$, and get N samples. Then the discrete frequency domain response could be represented as:

$$\begin{aligned} R(k) &= h(k) + W[k] \\ &= \sum_{d=0}^{D-1} \alpha_d e^{-j2\pi(f_L+(k-1)f_0)\tau_d} + W(k) \end{aligned} \tag{3}$$

where $W(k)$ is additive white Gaussian noise (AWGN) with zero mean and variance δ_W^2. We rewrite (3) in a matrix format as follows:

$$R = Ta + W \tag{4}$$

$$R = [R(0) \quad R(1) \quad \cdots \quad R(N-1)]^T \tag{5}$$

$$T = \begin{bmatrix} e^{-j2\pi f_L \tau_0} & e^{-j2\pi f_L \tau_1} & \cdots & e^{-j2\pi f_L \tau_{D-1}} \\ e^{-j2\pi(f_L+f_0)\tau_0} & e^{-j2\pi(f_L+f_0)\tau_1} & \cdots & e^{-j2\pi(f_L+f_0)\tau_{D-1}} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-j2\pi(f_L+(N-1)f_0)\tau_0} & e^{-j2\pi(f_L+(N-1)f_0)\tau_1} & \cdots & e^{-j2\pi(f_L+(N-1)f_0)\tau_{D-1}} \end{bmatrix}_{N\times D} \tag{6}$$

$$\alpha = [\alpha_0 \quad \alpha_1 \quad \cdots \quad \alpha_{D-1}]^T \tag{7}$$

$$W = [W(0) \quad W(1) \quad \cdots \quad W(N-1)]^T \tag{8}$$

Because $W(k)$ is AWGN, the elements in the vector W are independent, and W satisfies $E(WW^T) = \delta_W^2 I$, where I is an identity matrix.

3. Maximum Likelihood Estimation Algorithm

In this section, we first obtain the ML function with respect to the time-delay of different paths, and then give the procedure of pattern searching method to optimize the object function so as to get the time-delay values, and finally compute the CRLB.

3.1 Maximum Likelihood Function

Because the elements in column vector W are independent and identically distributed, the joint probability density function $f(R)$ of all the elements in column vector R is:

$$f(R) = \frac{1}{(\sqrt{2\pi\delta_W^2})^N} \times \exp(-\frac{1}{2\delta_W^2} | R - Ta |^2) \tag{9}$$

We simplify the logarithm of $f(R)$ and get its equivalent $L(\tau_0, \tau_1, \cdots, \tau_{D-1})$ as follow:

$$L(\tau_0,\tau_1,\cdots,\tau_{D-1}) = |R - Ta|^2 \tag{10}$$

Maximizing the ML function $f(R)$ is equivalent to minimizing the object function $L(\tau_0,\tau_1,\cdots,\tau_{D-1})$. Because T and a are both unknown in (10), and our task is to get T, we first fix R, then use the least square method, and finally get the solution of a:

$$a = (T^H T)^{-1} T^H R \tag{11}$$

Substituting (11) into (10) we get:

$$L(\tau_0,\tau_1,\cdots,\tau_{D-1}) = |R - T(T^H T)^{-1} T^H R|^2 \tag{12}$$

Let $P_T = T(T^H T)^{-1} T^H$, and then P_T could be seen as a projection matrix, which projects the vector in space C^N onto the space spanned by the column vectors of matrix T. Owing to R being a measured vector and fixed, minimizing (12) is equivalent to maximizing (13).

$$L(\tau_0,\tau_1,\cdots,\tau_{D-1}) = |P_T R|^2 \tag{13}$$

Considering the projection matrix has the attribution $P_T^H P = P$, we can rewrite (13) as

$$\begin{aligned} L(\tau_0,\tau_1,\cdots,\tau_{D-1}) &= (P_T R)^H (P_T R) \\ &= tr(P_T C) \end{aligned} \tag{14}$$

where $C = RR^H$. We write $C = \sum_{i=1}^{N} \delta_i \mu_i \mu_i^H$, where δ_i is the Eigen value of matrix C and μ_i is the corresponding eigenvector. So (14) becomes

$$\begin{aligned} L(\tau_0,\tau_1,\cdots,\tau_{D-1}) &= tr(P_T \sum_{i=1}^{N} \delta_i \mu_i \mu_i^H) \\ &= \sum_{i=1}^{N} \delta_i \, |P_T \mu_i|^2 \end{aligned} \tag{15}$$

Then the time-delay values $\{\hat{\tau}_i \mid i = 0,\cdots,D-1\}$ could be obtained by maximizing (15).

3.2 Optimization Procedures

From (6) we could see that the matrix T is relative to the multi-path time-delay values $\{\tau_i \mid i = 0,\cdots,D-1\}$. The ith column of the matrix T is determined by the time-delay τ_i, and once the time-delay values are determined, the matrix T and the projection matrix P_T are accordingly determined. We could get the time-delays of signal in different paths by optimizing the object function (15). However, it is a tough task to obtain the optimal value of (15): on the one hand, the object is a non-linear function of the time-delays, and there are a lot of local extrema, so we could hardly obtain the global optimal result by convex optimization method; on the other hand, it is not easy to obtain the derivate of the object function with respect to the time-delays, so we could barely find the local optimal result by traditional non-linear optimization techniques such as the steepest descent method.

Next, we utilize the pattern searching method, which avoid derivating the objection function, to maximize (15) so as to obtain the time-delay values of different paths. The pseudo codes of the whole optimization procedures are as follows:

Pseudo codes of the optimization procedures:

Selects an initial point $x_0 = \{\tau_d \mid d = 0, \cdots, D-1\}$, initial step δ_0 and scaling factor $\alpha \in (0,1)$, gives an permitted error $\varepsilon > 0$;

Lets $y_{temp} = x_{old} = x_0$; $\delta = \delta_0$ and $d = 0$;

```
While δ > ε
        While d < D − 1
                If    L(y_temp + δe_d) > L(y_temp)
                          y_temp = y_temp + δe_d ;
                Else If  L(y_temp − δe_d) > L(y_temp)
                          y_temp = y_temp − δe_d ;
                End If;
                d = d + 1 ;
        End While;
        x_new = y_temp ;
        If  L(x_new) > L(x_old)
                y_temp = 2x_new − x_old ;
                x_old = x_new ;
        Else    If x_old = x_new
                          δ = δ × α ;
                Else
                          x_old = x_new ;
                End If
                y_temp = x_new ;
        End if;
End While;
Output x_new
```

In the above optimization procedures, $y_{temp} = y_{temp} + \delta e_d$ represents the time-delay of the d th path increases by δ units, and the time-delays of the other paths keep unchanged. For the optimization procedures, the optimal solution is related with the initial point, and different initial points may result in different local extrema. Considering that many literatures [15][17][18] have applied the MUSIC to estimating the time-delay values in the multi-path propagation channel, and acquired good performance, so we decide to take the x_0 resulted from MUSIC as the initial point. However, it must be noted that other subspace fitting algorithms could also provide the initial points, and the corresponding performances of the ML will not be the same. Although in this paper the initial point is provided by MUSIC, it could easily extend to the other algorithms, such as ESPRIT and GEESE.

When using the MUSIC to estimate the time-delays, we need to make the cross correlation matrix, which is formed by the frequency domain samples, non-singular. This could be implemented by the forward-backward spatial smoothing technology [17]. We first divide the N samples into $M = \lceil N/K \rceil$ subsets, where $\lceil \mathrm{x} \rceil$ denotes the maximum integer that does not larger than x, and every subset includes $N-M+1$ elements, then we compute the cross correlation matrix $\sum_m$ of each subset, and finally we get the non-singular cross correlation matrix $\sum_{US}$ using (16) :

$$\sum_{US} = \frac{1}{2M}\sum_{m=0}^{M-1}(\sum_m + J\sum_m^H J) \tag{16}$$

where J is a matrix with the anti-diagonal elements equal to 1 and other elements equal to 0, and the superscript H denotes conjugate. Next we use the matrix $\sum_{US}$ to estimate time-delay values of different paths and take them as an initial point for the optimization procedure. We will not elaborate the details of the MUSIC algorithm and its application to time-delay estimation, and interesting readers could refer to [15][18][21].

3.3 Cramer Rao Lower Bounds

In the parameter estimation, we usually compare the variance of the estimator with the CRLB. CRLB is the minimum variance of the estimator that could be reached, which equals to the inverse of the Fisher information, and the variance of any unbiased estimator will not be less than the CRLB. Although the CRLB has been given in [15], it is based on the continuous function, i.e, the number of sampling points is infinite. However, in this paper, the number of sampling points is finite. The $D \times D$ Fisher information matrix J is determined by the time-delays of the D paths, and the (i, j) th element of J is

$$\begin{aligned} J_{i,j} &= -E\left(\frac{\partial^2 \ln f}{\partial \tau_i \partial \tau_j}\right) \\ &= \frac{2\pi^2}{\delta_w^2}\sum_{n=0}^{N-1}\left(\alpha_i\alpha_k^*(f_L+nf_0)^2 e^{j2\pi(f_L+nf_0)(\tau_k-\tau_i)} + \alpha_i^*\alpha_k(f_L+nf_0)^2 e^{j2\pi(f_L+nf_0)(\tau_i-\tau_k)}\right) \end{aligned} \tag{17}$$

Thus the sum of the variances of the time-delay values in every path satisfies the following inequation:

$$\sum_{d=0}^{D-1}(\hat{\tau}_d-\tau_d)^2 \geq tr(J^{-1}) \tag{18}$$

where $\hat{\tau}_d$ and τ_d are the estimated value and real value of the d th path respectively. Obviously, when N tends to infinity, our CRLB is consistent with that in [15].

4. Performance Analysis

The simulation runs on a notebook PC whose CPU is Duo8400 and memory capacity is 2G. The simulation software is Matlab 7.0. In order to evaluate the performance of our ML estimation algorithm, we compare it with the subspace fitting algorithms and the CRLB. The main simulation parameters are as follows: the sample start frequency f_L is 5GHz, the frequency domain sampling interval f_0 is 2MHz, and the received signal strength of every path is followed uniform distribution between 0 and 1. Because the number of multi-path components could be obtained by the minimum descriptive length criteria (MDL) [27], we assume that D is 6. The standard variance of the time-delay is computed by $\sqrt{\sum_{d=0}^{D-1}(\hat{\tau}_d-\tau_d)^2}$ where $\hat{\tau}_d$ and τ_d are the estimation value and the real value of the d th path respectively.

From **Fig. 1** to **Fig. 3** the time-delays of different paths are 20ns, 30ns, 40ns, 50ns, 60ns and 70ns; and in **Fig. 4**, the time-delay of the first path follows uniform distribution between [5], and the values of the other five paths all follow uniform distribution between [5] compared to the former path. The curve composed of the results of MUSIC is labeled as MUSIC and the curve composed of the results of ML with the initial point provided by MUSIC is labeled as ML-MUISIC in all figures. Although our initial point is provided by the MUSIC, it could also come from other subspace fitting algorithms. So we add the other two initial points which are obtained by the coarse time-delay estimation algorithm [18] labeled as GEESE-COARSE in **Fig.1** and the fine time-delay estimation algorithm [18] labeled as GEESE-FINE in **Fig.2** for comparison. And the corresponding ML results are also provided in **Fig.1** which is labeled as ML-GEESE-COARSE and **Fig.2** which is labeled as ML-GEESE-FINE. From these figures, we find that although the ML algorithm could not reach the CRLB, its performance is far superior to that of corresponding subspace fitting algorithms. We can see the MUSIC for the time-delay estimation as a coarse algorithm and the corresponding ML as a fine one. And it is also for the GEESE algorithm.

The results with different ratios of signal to noise (SNR) are shown in **Fig.1** when the whole frequency domain sampled bandwidth is 300MHz and $K=3$, and the SNR changes from 5dB to 50dB. As is shown in **Fig.1**, although the GEESE-COARSE is superior to the ML-MUSIC, it is inferior to the ML-GEESE-COARSE. The results illuminate that on the one hand, ML estimation result is affected by the initial point, and good initial point will produce good ML result; on the other hand, ML could improve the performance of subspace fitting algorithms further.

The performance under different sampled bandwidth are shown in **Fig.2** when the SNR equals to 30dB and $K=3$, and the sampled bandwidth changes from 250MHz to 450MHz.

Fig.2 shows that the performance of MUSIC is far inferior to that of ML-MUSIC when the bandwidth is low. And when the bandwidth increases, the performance of the MUSIC is improved. Just like **Fig.1**, the performance of ML-MUSIC is less than GEESE-FINE due to MUSIC being inferior to GEESE. Although the GEESE-FINE has improved the performance of the GEESE-COARSE [18], it could still be improved by the ML further.

Now we analyze the performance of the two algorithms when we use the forward backward spatial smoothing method to make the covariance matrix of the samples non-singular. **Fig.3** shows the results when the sampled bandwidth is 300MHz and the SNR is 30dB, and the value of K changes from 2 to 4. From **Fig.3** we could see that the two algorithms are almost free from the influence of K. In fact, to make the covariance matrix non-singular is only needed for the MUSIC, rather than for the ML. When the total sampled bandwidth and sampling frequency are fixed, the effect is not obvious to make the matrix non-singular under different K. So the performance of MUSIC will not be changed obviously when K changes, and accordingly the performance of ML algorithm will also have little change.

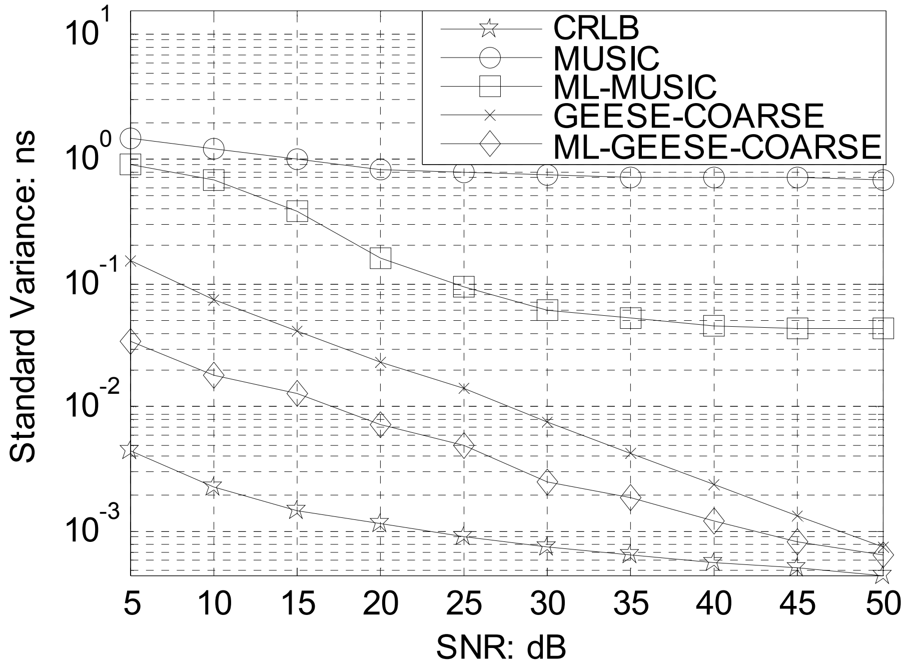

Fig. 1. Logarithm of Standard Variance under different SNR

Finally, we analyze the performance of the two algorithms under different time-delay values. **Fig.4** shows the results of the standard variances of time-delay under different values of Dt when SNR is 30dB and K=3. As is shown in **Fig.4**, the ML and the MUSIC are both free from the influence of the value of Dt. The results indicate that the two algorithms are both robust to the time-delay values.

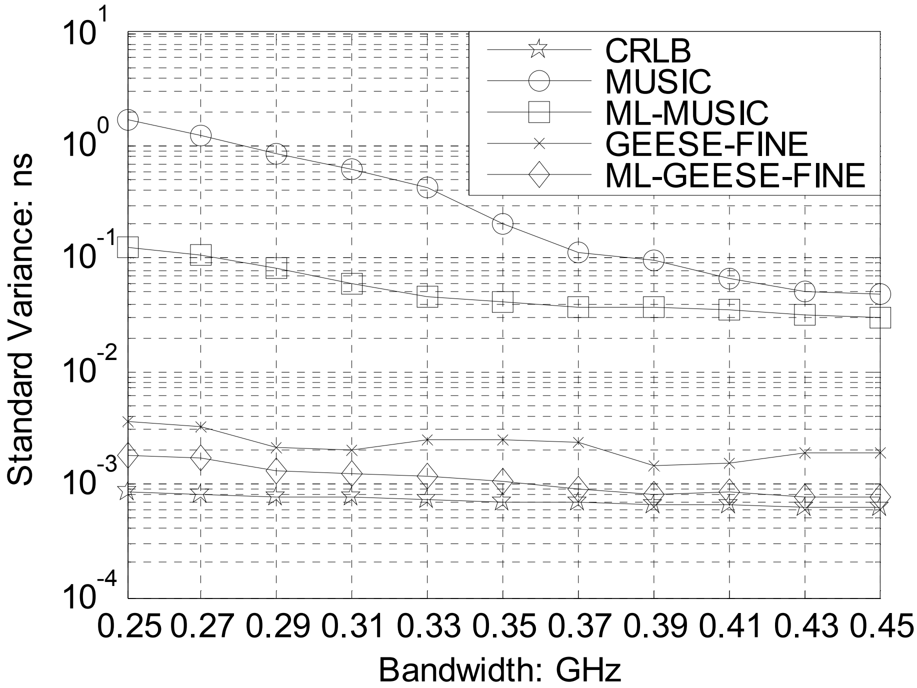

Fig. 2. Standard Variance under different Bandwidth

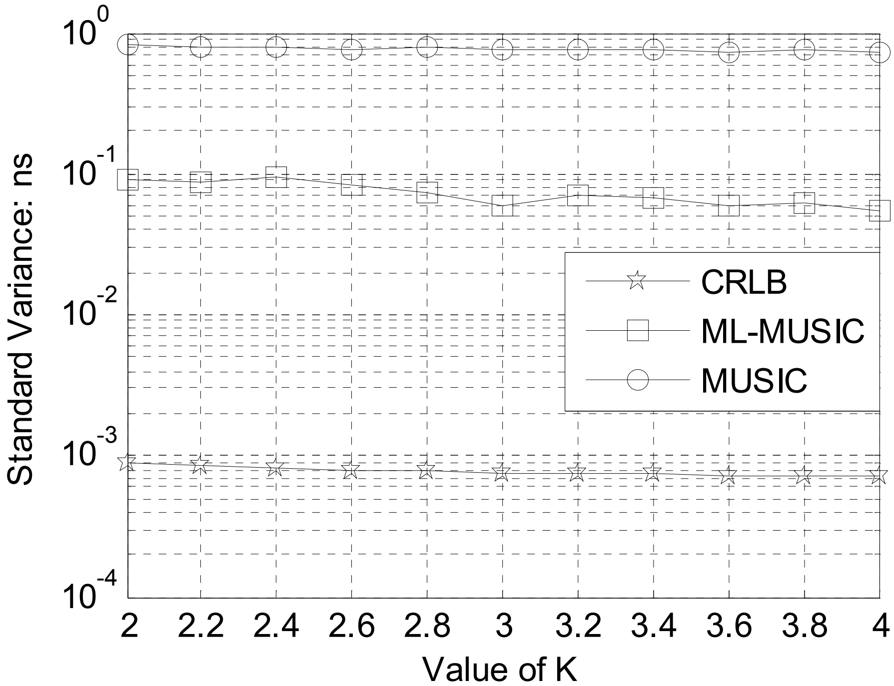

Fig. 3. Standard Variance under different Value of K

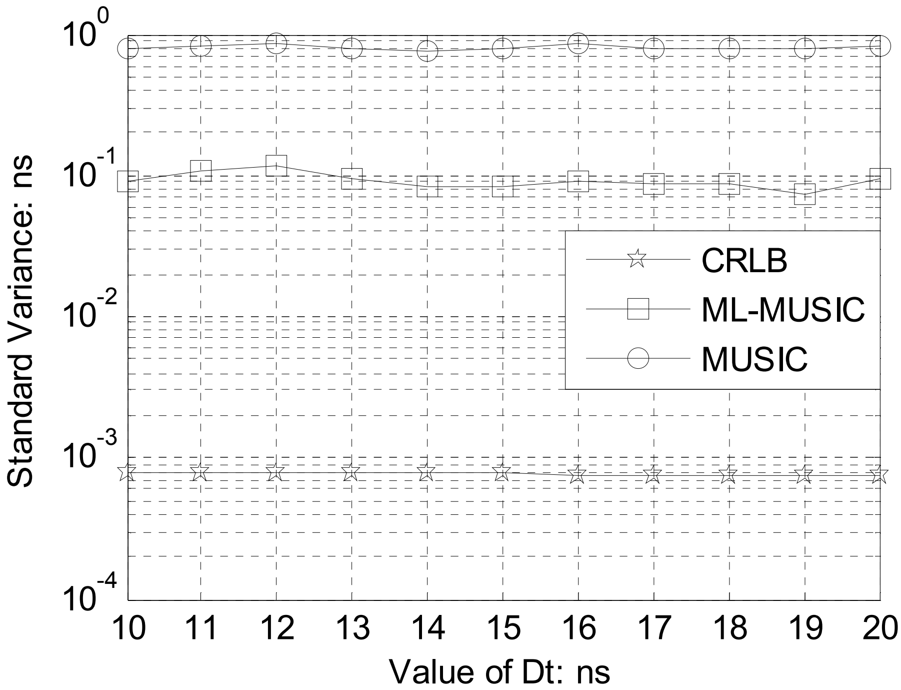

Fig. 4. Standard Variance under different Value of Dt

5. Conclusions

In this paper, we propose a ML algorithm to estimate the time-delay of different paths in the wireless multi-path propagation channel. After uniformly sampling the frequency domain response of the multi-path channel, which could be obtained by adopting the vector network analyzer, we get the joint probability density function, and then we estimate the time-delay values of different paths based on the ML criterion. Because the ML value is a non-linear function of the time-delay, we could hardly obtain the global optimal result by the convex optimization methods and also hardly obtain the derivate of the ML function with respect to the time-delay parameters, we make use of the pattern searching method to obtain the local optimal result and take it as the time-delay values of different paths. Because the performance of the local optimal result is related with the initial point, we obtain it by the subspace fitting algorithm. Simulation results show that although the ML estimation variance could not reach the CRLB, its performance is superior to that of subspace fitting algorithm, and it could be seen as a fine one based on the coarse result of the subspace fitting algorithm.

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