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Estimation of Nonlinear Impulse Responses of Stock Indices by Asset Class

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Abstract

We estimate nonlinear impulse responses of stock indices by asset class by the Local Projection method as suggested by Jorda (2005) to compute impulse responses. The method estimates impulse responses without the specification and estimation of the underlying multivariate dynamic system unlike the usual way of vector autoregression(VAR). It estimates Local Projections at each period of interest rather than extrapolating into increasingly distant horizons with the advantages of easy estimation and non-linear flexible specification. The Local Projection method adequately captures the nonlinearity and asymmetry of the impulse responses of the stock indices compared to those from VARs.

Keywords: Impulse response, Local Projection method, vector autoregression, nonlinearity.

1. Introduction

Vector autoregression(VAR) proposed by Sims (1980) is a statistical model used to capture linear interdependencies among multiple time series and has been widely used in economic analysis. However, there is a limit of the impulse responses from VARs in that they cannot reflect the nonlinear features of impulse responses. In a VAR setting, the impulse response of a variable caused by a certain shock is simply calculated as twice that of the original response (when the shock is assumed to be double). VARs cannot capture possible nonlinear effects due to the linearity of their impulse responses even if we believe there should be more (or less) effect when the shock is increased or decreased. There have been many research efforts to overcome the linearity of impulse responses of VARs such as the analysis of the impact of oil prices on macroeconomic variables. Mork (1989) and Hamilton (2000) showed that the impact of oil prices on GDP was asymmetric where rising oil prices lead to a slowdown in the economy while the effect of dropping oil prices was not statistically significant. Mork *et al.* (1994) argued that the asymmetric effect of oil price changes appeared in most OECD countries. Lee *et al.* (1995) proved that there existed asymmetric correlations between oil prices and business activities. Cunado and Gracia (2005) showed that the impact of rising oil prices on CPI was larger than that of falling oil prices. Besides the above research about the asymmetry of the effect of oil prices, there have been studies about the nonlinearity of the effects of oil

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prices on macroeconomic variables. Hamilton (1983, 2009) argued that oil shocks were responsible for recessions; however, not all changes in the price of oil have the same effect on the economy. For example, an oil price increases that simply reverses previous price decreases are unlikely to have a significant effect. He asserted that we might regard rising oil prices as an oil shock only if the prevailing price of oil is higher than it has been over the past three years. This argument implies that the impact of oil prices higher than the level would be enhanced, which can be interpreted as nonlinear effects. Jorda (2005) introduced the Local Projection method to compute impulse responses without the specification and estimation of the underlying multivariate dynamic system. Unlike VARs, the method estimates Local Projections at each period of interest rather than extrapolating into increasingly distant horizons. With the advantages such as easy estimation in the context of regression techniques, robustness to misspecification, and easy accommodation of highly non-linear and flexible specification, the method can be regarded as a good alternative to estimate impulse responses compared with VARs. Kim and Yoon (2009) applied the Flexible Local Projection method to oil prices and showed the nonlinearity and asymmetry of impulse responses. This motivates us to apply the Local Projection method to fluctuating data such as stock indices than oil prices to see whether the method could capture nonlinearity.

This paper is organized as follows. In Section 2, a brief sketch of the Local Projection method and the rationale for the usefulness of the method are presented, followed by a simulation study and real data analysis. Section 3, conclude with a summary of the results.

2. Local Projection Method

Zellner and Franz (1974), Cooley and Dwyer (1998), and Jorda (2005) argue that data generating process(DGP) does not necessarily follow the VAR process. If the real DGP does not follow the VAR process, the ordinary generating of impulse response cannot be estimated correctly due to specification errors. Jorda (2005) proposed a Local Projection method to solve the problem. The Local Projection method is a set of h regression equations that does not need the assumption of DGP as VAR(p).

Jorda (2005) considers projecting $\boldsymbol{y}_{t+s}$ onto the linear space generated by $(\boldsymbol{y}_{t-1}, \boldsymbol{y}_{t-2}, \ldots)'$ as follows;

$$\boldsymbol{y}_{t+s} = \boldsymbol{\alpha}^s + B_1^{s+1}\boldsymbol{y}_{t-1} + B_2^{s+1}\boldsymbol{y}_{t-2} + \cdots + B_p^{s+1}\boldsymbol{y}_{t-p} + \boldsymbol{u}_{t+s}^s, \quad (s = 0, 1, 2, \ldots, h) \tag{2.1}$$

$\boldsymbol{y}_{t+s}$ is a $n \times 1$ vector; $\boldsymbol{\alpha}^s$ is a constant vector; B_i^s is a coefficient matrix where i and s denote a time lag and time span respectively; and $\boldsymbol{u}_t$ is a $n \times 1$ vector of the error term.

Note that the impulse response function from usual VARs can be represented as the difference between two forecasts due to Hamilton (1994):

$$\mathrm{IR}(t, s, \boldsymbol{d}_i) = E(\boldsymbol{y}_{t+s}|\boldsymbol{v}_t = \boldsymbol{d}_i; \boldsymbol{X}_t) - E(\boldsymbol{y}_{t+s}|\boldsymbol{v}_t = \boldsymbol{0}; \boldsymbol{X}_t), \quad (s = 0, 1, 2, \ldots, h) \tag{2.2}$$

where $\boldsymbol{X}_t = (\boldsymbol{y}_{t-1}, \boldsymbol{y}_{t-2}, \ldots)'$; $\boldsymbol{v}_t$ is the $n \times 1$ vector of reduced-form disturbances; and $\boldsymbol{d}_i$ is the vector that represents the shock of the i^{th} element in $\boldsymbol{y}_t$. Therefore, the impulse response function of the local linear projection (2.1) is

$$\mathrm{IR}\widehat{(t, s, \boldsymbol{d}_i)} = \hat{B}_1^s \boldsymbol{d}_i, \quad (s = 0, 1, 2, \ldots, h). \tag{2.3}$$

There are several restrictions of impulse responses of linear models such as VARs. The impulse responses from VARs assume properties such as symmetry, shape invariance, history independence, and multidimensionality that forms a complicated calculation of standard errors. Multidimensionality can be solved with the local linear projection (2.1); however, the other restrictions cannot be handled because of the linearity of the model. To dispense with the restrictions, Jorda proposed alternatives based on Local Projections. Suppose there is a nonlinear time series process y_t that can be expressed as a generic function of past values of a white noise process v_t in the form

$$\boldsymbol{y}_t = \Phi(\boldsymbol{v}_t, \boldsymbol{v}_{t-1}, \boldsymbol{v}_{t-2}, \ldots). \tag{2.4}$$

If it can be approximated by a Taylor series expansion around $\mathbf{0} = (0, 0, \ldots)$, then the closest equivalent to the Wold representation in nonlinear time series is the Volterra series expansion as Priestley (1988) pointed out. The expansion is represented as follows:

$$\boldsymbol{y}_t = \sum_{i=0}^{\infty} \Phi_i \boldsymbol{v}_{t-i} + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \Phi_{ij} \boldsymbol{v}_{t-i}\boldsymbol{v}_{t-j} + \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty} \Phi_{ijk} \boldsymbol{v}_{t-i}\boldsymbol{v}_{t-j}\boldsymbol{v}_{t-k} + \cdots \tag{2.5}$$

which is a polynomial extension of the Wold decomposition with the constant omitted for simplicity. If we extend the Local Projection with polynomial terms on $\boldsymbol{y}_{t-1}$, we can obtain the following equations. Note that cross-product terms are omitted for parsimony.

$$\begin{aligned}\boldsymbol{y}_{t+s} = {} & \alpha^s + B_1^{s+1}\boldsymbol{y}_{t-1} + Q_1^{s+1}\boldsymbol{y}_{t-1}^2 + C_1^{s+1}\boldsymbol{y}_{t-1}^3 + B_2^{s+1}\boldsymbol{y}_{t-2} + \cdots + B_p^{s+1}\boldsymbol{y}_{t-p} + \boldsymbol{u}_{t+s}^s, \\ & (s = 0, 1, 2, \ldots, h).\end{aligned} \tag{2.6}$$

As a result, the impulse response at time s becomes,

$$\begin{aligned}&\widehat{\mathrm{IR}(t, s, \boldsymbol{d}_i)} \\ &= \left\{\hat{B}_1^s(\boldsymbol{y}_{t-1} + \boldsymbol{d}_i) + \hat{Q}_1^s(\boldsymbol{y}_{t-1} + \boldsymbol{d}_i)^2 + \hat{C}_1^s(\boldsymbol{y}_{t-1} + \boldsymbol{d}_i)^3\right\} - \left\{\hat{B}_1^s\boldsymbol{y}_{t-1} + \hat{Q}_1^s(\boldsymbol{y}_{t-1})^2 + \hat{C}_1^s(\boldsymbol{y}_{t-1})^3\right\} \\ &= \left\{\hat{B}_1^s\boldsymbol{d}_i + \hat{Q}_1^s\left(2\boldsymbol{y}_{t-1}\boldsymbol{d}_i + \boldsymbol{d}_i^2\right) + \hat{C}_1^s\left(3\boldsymbol{y}_{t-1}^2\boldsymbol{d}_i + 3\boldsymbol{y}_{t-1}\boldsymbol{d}_i^2 + \boldsymbol{d}_i^3\right)\right\}, \quad (s = 0, 1, 2, \ldots, h)\end{aligned} \tag{2.7}$$

and $B_1^0 = I$, $Q_1^0 = 0_n$, and $C_1^0 = 0_n$ with normalizations where 0_n denotes an $n \times 1$ zero vector. The nonlinearity can be easily implemented due to the second and third order terms in the impulse response function. The method is called flexible Local Projections. The estimates are also easily calculated by least squares, equation by equation. It is clear that the impulse response function will vary according to the sign and with the size of the experimental shock defined by $\boldsymbol{d}_i$. Therefore, it dispenses with the restrictions of the symmetry and shape invariance. The restriction of history independence can be solved since the impulse response depends on the local history $\boldsymbol{y}_{t-1}$ at which it is evaluated. Jorda argued that impulse responses comparable to local-linear or VAR-based impulse responses can be achieved by evaluation at the sample mean, $\boldsymbol{y}_{t-1} = \bar{\boldsymbol{y}}_{t-1}$. The 95% confidence interval for the cubic approximation (2.6) can be easily calculated. Defining the scaling $\boldsymbol{\lambda} \equiv (\boldsymbol{d}_i, 2\boldsymbol{y}_{t-1}\boldsymbol{d}_i, 3\boldsymbol{y}_{t-1}^2\boldsymbol{d}_i + 3\boldsymbol{y}_{t-1}\boldsymbol{d}_i^2 + \boldsymbol{d}_i^3)'$ and denote $\hat{\Sigma}_C$ the HAC(Heteroskedasticity and Autocorrelation Consistent) matrix, variance-covariance matrix of the coefficients B_1^s, Q_1^s, and C_1^s in (2.6). Then a 95% confidence interval for the impulse response at time s is approximately, $1.96 \pm (\boldsymbol{\lambda}_i'\hat{\Sigma}_C\boldsymbol{\lambda}_i)$.

3. Real Data Example

Figure 3.1 shows four monthly stock indices by asset class with normalization of setting the beginning point as 100 from month 1 through 49 following the example in Kim and Chang (2002). The

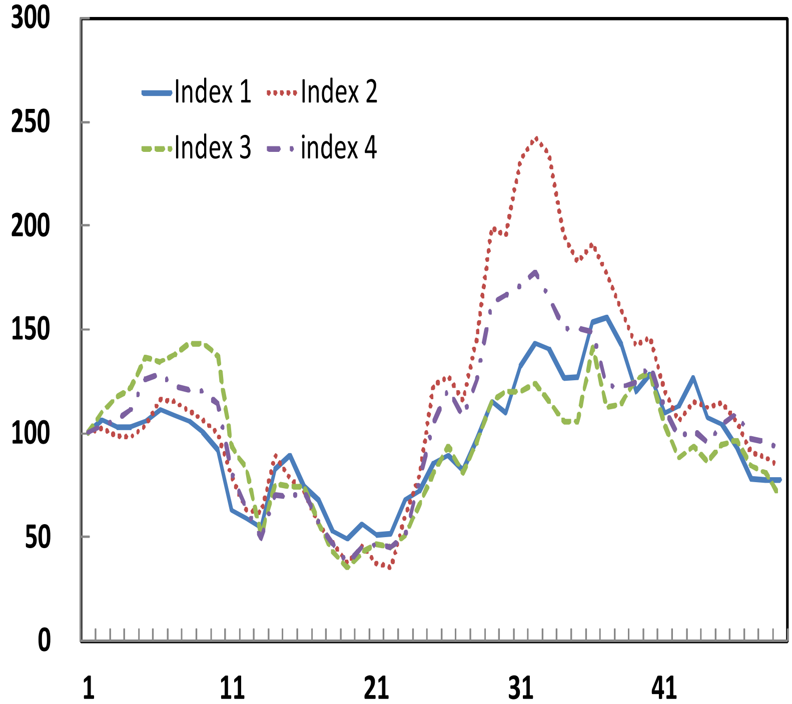

Figure 3.1. Stock indices by asset class

stock universe is ordered by market capitalization to tell large and small classes. Each stock in the classes is also ordered by the book-to-market ratio to distinguish the value ones and growth ones. Finally, whole stocks are divided into large market capitalization(large cap) growth stocks(Index 1), large cap value stocks(Index 2), small cap growth stocks(Index 3), and small cap value stocks(Index 4). Figure 3.1 shows that those four classes do not move independently and it seems that there is some correlation between them. Shocks to the stock market a directly influence a certain stock class as well as others according to the correlation between the classes. Note that every Index in the model is in the form of year-on-year growth rate.

Figure 3.2 represents the different movements of impulse responses from VARs and Local Projection models. The response from the VARs decreases in a short period while that of Local Projection shows fluctuation compared to the VAR model. This can be easily understood if we think about the difference in the response functions of each method introduced in the previous section. Second and third order terms in the impulse response function of the Local Projection model create this kind of fluctuation. Consequently, this rebounding pattern grows as the shock increases. We also have to be cautious about the interpretation of the analysis results. Even though we see the relatively large fluctuation and rebounding effects, they tend to be moving within the confidence intervals after month 5 or 6, which means responses still remain close to zero after some amount of time. Confidence intervals are omitted in the figures to avoid complexity.

The impulse responses of each Index by a plus shock to Index 1 are shown in Figure 3.3–Figure 3.6. The magnitude of the shocks increased from one and a half times, two times, and four times that

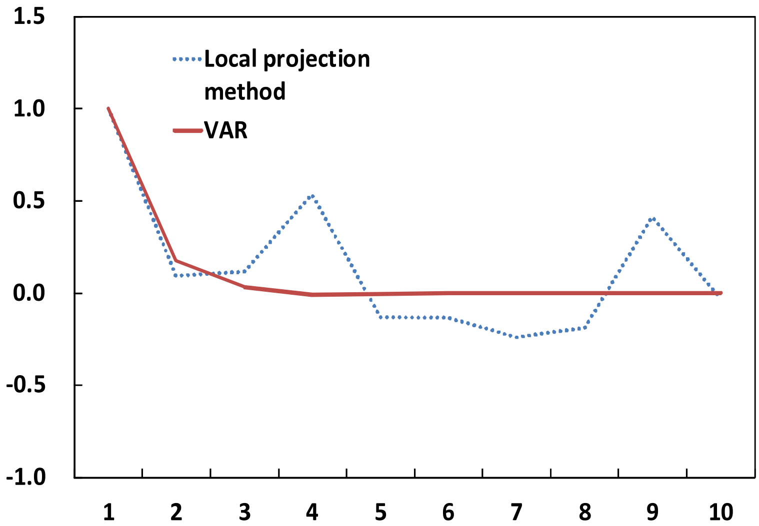

Figure 3.2. Impulse responses by Local Projection and VAR

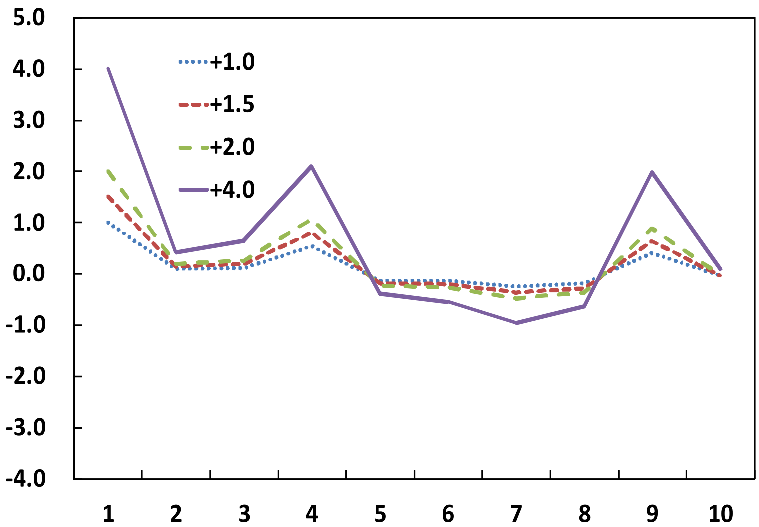

Figure 3.3. Impulse responses of Index 1 by a plus shock to Index 1

of the original shock. Overall, the responses show nonlinear patterns as the magnitude of the shock increased.

Table 3.1 shows the impulse responses according to a shock to Index 1. Note that the responses are normalized, that is divided by the magnitude of each shock. We can see that the absolute value of the responses decreases at the maximum point (month 4) except for Index 4. The response of Index 4 at month 1 is smallest of all the indices, but it becomes second biggest one among all the indices at month 4 with the impulse enlarged four times. Therefore, we can say that value stocks

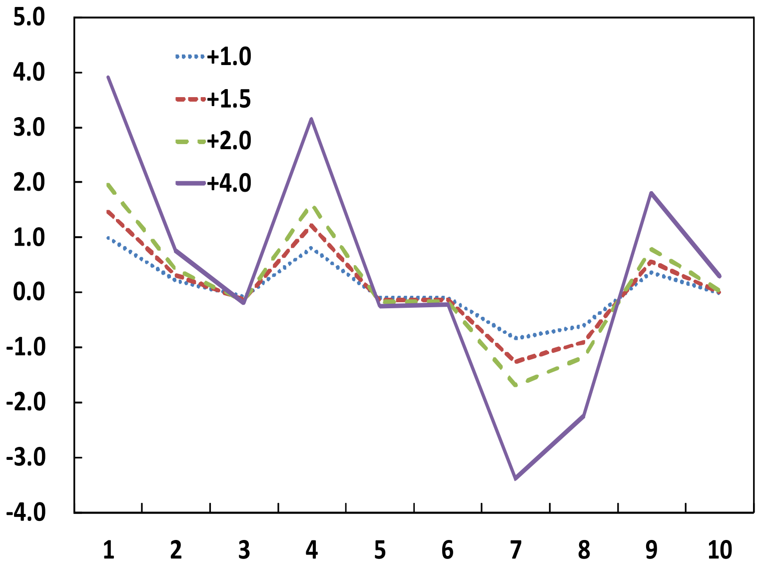

Figure 3.4. Impulse responses of Index 2 by a plus shock to Index 1

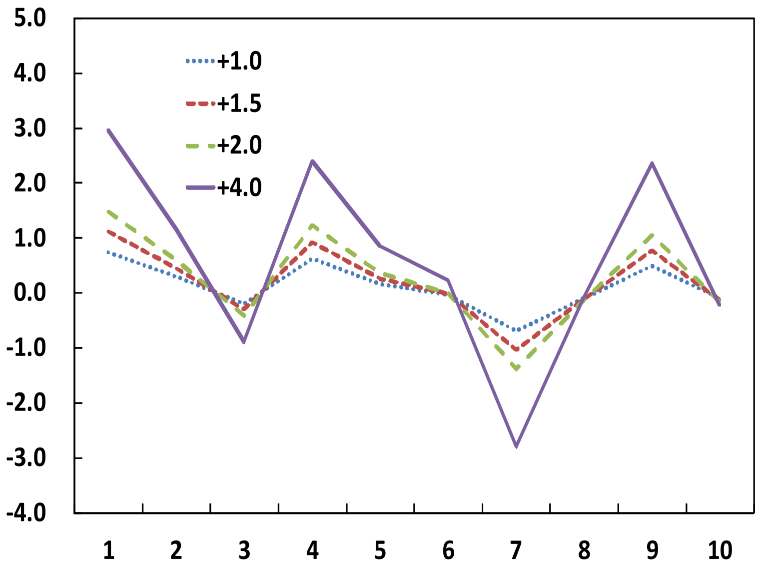

Figure 3.5. Impulse responses of Index 3 by a plus shock to Index 1

such as Index 2 and Index 4 tend to be sensitive to the enlarged shocks to Index 1.

The impulse responses of each Index by a minus shock to Index 1 are also considered. Similar to the previous case, the magnitude of the shock is increased to 4 times the original shock. The nonlinearity of the impulse responses is shown in this case. The asymmetry of impulse responses is also discovered through comparison of the results with the previous case. We can see that a plus

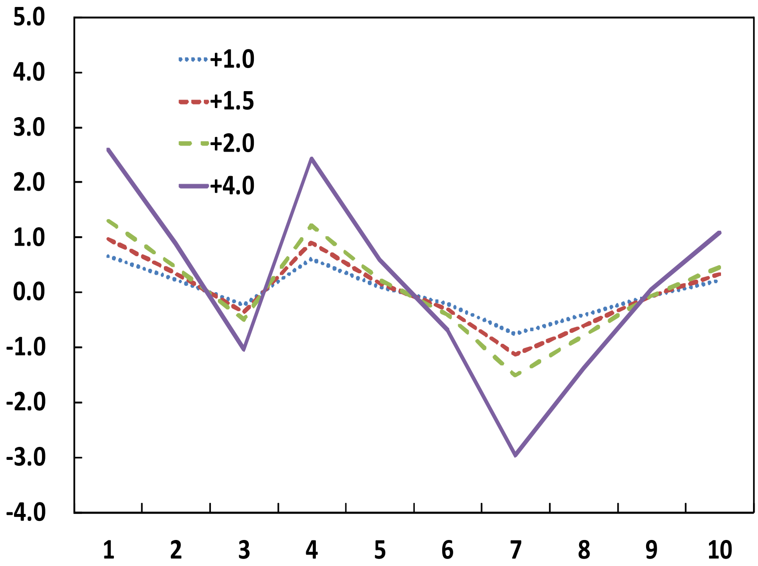

Figure 3.6. Impulse responses of Index 4 by a plus shock to Index 1

Table 3.1. Impulse responses of indices by a plus shock to Index 1 (normalized by multiples)

Index	Impulse multiples	Month 1	2	3	4	5	6	7	8	9	10
Index 1	1	1.000	0.093	0.116	0.532	−0.132	−0.135	−0.242	−0.190	0.411	−0.032
	1.5	1.000	0.095	0.124	0.532	−0.125	−0.135	−0.242	−0.184	0.426	−0.022
	2	1.000	0.097	0.132	0.531	−0.118	−0.135	−0.241	−0.178	0.441	−0.013
	4	1.000	0.106	0.162	0.526	−0.095	−0.136	−0.238	−0.158	0.495	0.023
Index 2	1	0.976	0.215	−0.093	0.806	−0.106	−0.096	−0.849	−0.611	0.357	−0.020
	1.5	0.976	0.210	−0.086	0.804	−0.098	−0.089	−0.849	−0.603	0.374	−0.005
	2	0.976	0.205	−0.079	0.802	−0.090	−0.081	−0.849	−0.594	0.391	0.011
	4	0.976	0.188	−0.051	0.787	−0.064	−0.054	−0.845	−0.564	0.451	0.072
Index 3	1	0.738	0.292	−0.200	0.615	0.158	−0.032	−0.691	−0.095	0.489	−0.103
	1.5	0.738	0.292	−0.204	0.613	0.169	−0.016	−0.693	−0.082	0.506	−0.095
	2	0.738	0.291	−0.208	0.611	0.179	−0.001	−0.695	−0.070	0.524	−0.087
	4	0.738	0.288	−0.221	0.598	0.213	0.055	−0.700	−0.021	0.588	−0.055
Index 4	1	0.647	0.227	−0.241	0.600	0.097	−0.222	−0.760	−0.420	−0.062	0.210
	1.5	0.647	0.226	−0.245	0.602	0.107	−0.213	−0.757	−0.408	−0.049	0.221
	2	0.647	0.225	−0.248	0.604	0.116	−0.204	−0.755	−0.395	−0.036	0.231
	4	0.647	0.222	−0.261	0.607	0.148	−0.173	−0.741	−0.348	0.012	0.269

shock to the Index 1 causes a relatively large responses of the other indices at month 2, while a minus shock to the Index 1 brings about the largest impacts on the other indices at month 4.

Table 3.2 shows the impulse responses caused by a minus shock to Index 1. The smallest values of the responses are obtained at month 4 if we set aside month 1. The response of Index 4 at month 1 shows smallest absolute value and it remains close to the smallest ones at month 4, which means small value stocks are relatively robust against a minus shock to large cap stocks unlike the previous case with a plus shock.

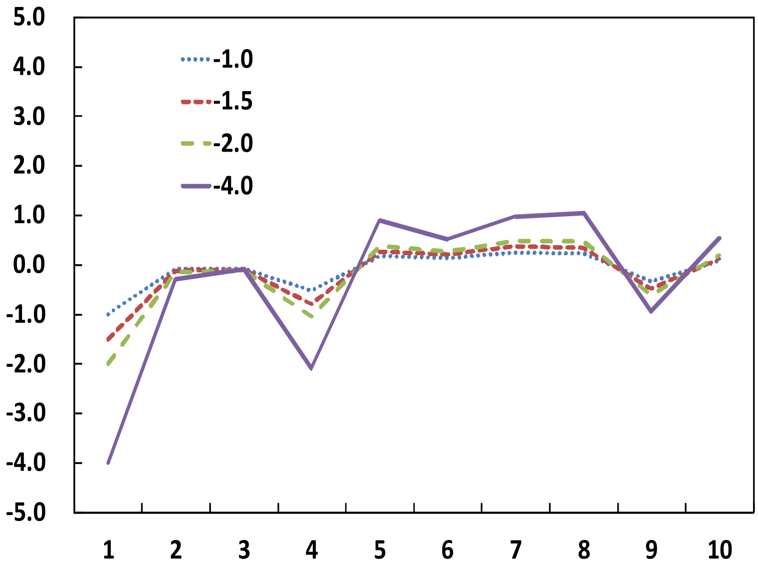

Figure 3.7. Impulse responses of Index 1 by a minus shock to Index 1

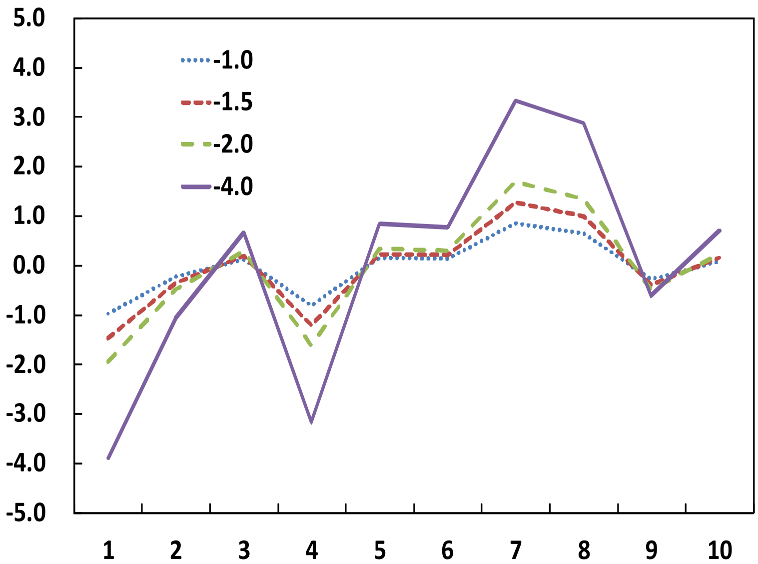

Figure 3.8. Impulse responses of Index 2 by a minus shock to Index 1

4. Conclusion and Future Work

We did the analysis with real data to see how well the nonlinear impulse responses could be captured by the Local Projection method. The nonlinearity of impulse responses is detected well by the Local projection method. Especially, the asymmetry of impulse responses caused by a plus and minus shock to a certain variable is also captured by the method. We can see that value stocks such as

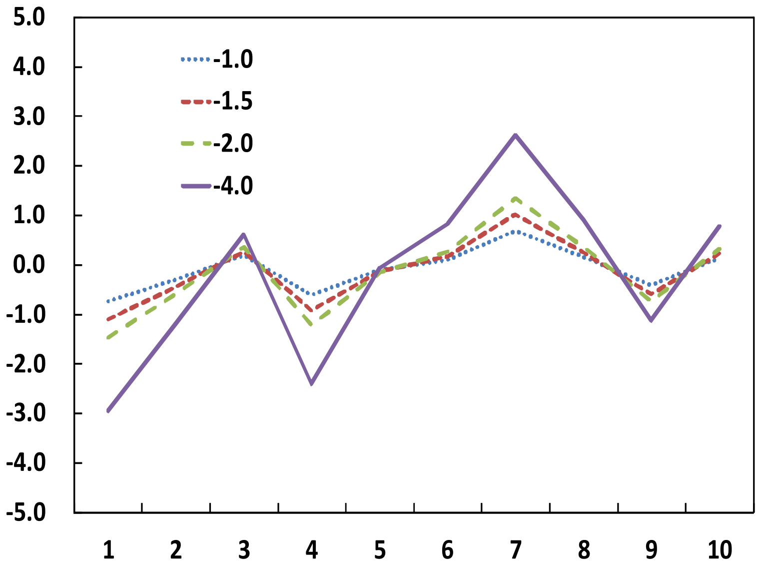

Figure 3.9. Impulse responses of Index 3 by a minus shock to Index 1

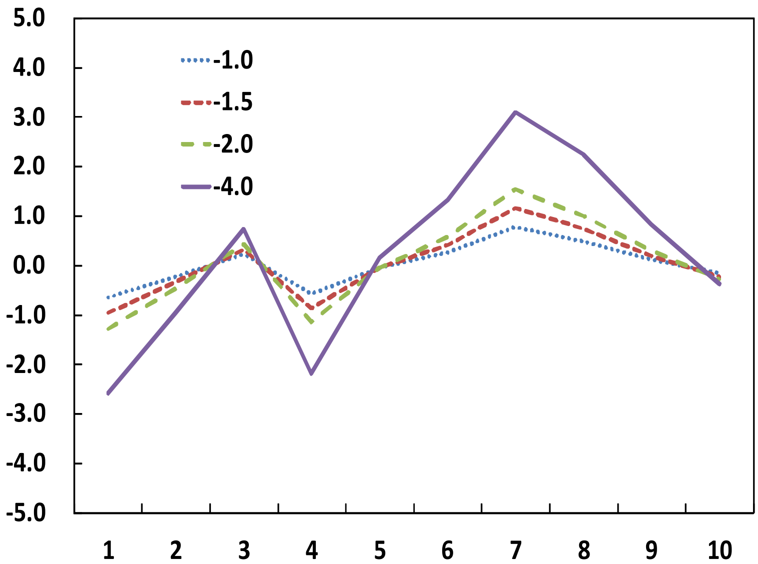

Figure 3.10. Impulse responses of Index 4 by a minus shock to Index 1

Index 2 and Index 4 (no matter if they are large or small) tend to be sensitive to the enlarged shocks to Index 1, while small value stocks are relatively robust against a minus shock to large cap stocks. We may assert that the limitation of VARs could be overcome by this method from the analysis in this paper. However, we should be cautious about the interpretation of the analysis results. Even though we see a relatively large fluctuation and rebounding effects, they tend to

Table 3.2. Impulse responses of indices by a minus shock to Index 1 (normalized by multiples)

Index	Impulse multiples	Month 1	2	3	4	5	6	7	8	9	10
Index 1	1	−1.000	−0.084	−0.081	−0.531	0.165	0.133	0.243	0.216	−0.346	0.071
	1.5	−1.000	−0.082	−0.072	−0.531	0.174	0.132	0.243	0.223	−0.329	0.081
	2	−1.000	−0.080	−0.063	−0.530	0.183	0.131	0.243	0.230	−0.311	0.091
	4	−1.000	−0.071	−0.024	−0.523	0.226	0.128	0.242	0.261	−0.235	0.132
Index 2	1	−0.976	−0.234	0.122	−0.808	0.143	0.131	0.846	0.649	−0.282	0.082
	1.5	−0.976	−0.239	0.129	−0.807	0.153	0.141	0.844	0.660	−0.262	0.098
	2	−0.976	−0.245	0.137	−0.805	0.164	0.151	0.842	0.670	−0.242	0.113
	4	−0.976	−0.267	0.167	−0.794	0.212	0.194	0.832	0.716	−0.154	0.177
Index 3	1	−0.738	−0.296	0.183	−0.615	−0.109	0.097	0.680	0.146	−0.412	0.138
	1.5	−0.738	−0.296	0.178	−0.614	−0.095	0.114	0.677	0.159	−0.392	0.147
	2	−0.738	−0.297	0.173	−0.613	−0.080	0.132	0.673	0.172	−0.371	0.156
	4	−0.738	−0.301	0.152	−0.601	−0.015	0.207	0.656	0.226	−0.283	0.195
Index 4	1	−0.647	−0.232	0.222	−0.585	−0.050	0.261	0.768	0.472	0.117	−0.166
	1.5	−0.647	−0.233	0.216	−0.580	−0.037	0.271	0.769	0.486	0.131	−0.154
	2	−0.647	−0.235	0.211	−0.574	−0.024	0.282	0.771	0.499	0.146	−0.143
	4	−0.647	−0.242	0.186	−0.547	0.036	0.328	0.774	0.556	0.208	−0.093

be moving within the confidence intervals after month 7 (or later), which means responses still remains not far away from zero after some time. Based on our analysis, we can think about more useful applications of the Local Projection method and a large study for the comparison of the performance with other nonlinear approaches such as LSTAR and threshold VARs. We can also extend the Local Projection algorithm to find a threshold of the impulse responses. There would be significant interest in finding a certain level of shock at which the impulses are enlarged or shrunk abruptly, because of their nonlinear movement. It could be possible by a mixture with any method dealing with nonlinear regression models like regression trees as illustrated in Chang and Kim (2011).

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