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A NOTE ON THE TWISTED q-GENOCCHI NUMBERS AND POLYNOMIALS WITH WEIGHT α

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Abstract. In this paper we construct a new type of twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha)}$ and polynomials $G_{n,q,w}^{(\alpha)}(x)$. Some interesting results and relationships are obtained.

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1. Introduction

The classical Genocchi numbers are defined in a number of ways. The way in which it is defined is often determined by which sorts of applications they are intended to be used for. The Genocchi numbers have wide-ranging applications from number theory and combinatorics to numerical analysis and other fields of applied mathematics. The Genocchi numbers and polynomials possess many interesting properties and arising in many areas of mathematics and physics. Recently, many mathematicians have studied in the area of the q-Genocchi numbers and polynomials (see [1-13]). In this paper, we construct a new type of twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha)}$ and polynomials $G_{n,q,w}^{(\alpha)}(x)$.

Throughout this paper we use the following notations. By $\mathbb{Z}_p$ we denote the ring of p-adic rational integers, $\mathbb{Q}_p$ denotes the field of p-adic rational numbers, $\mathbb{C}_p$ denotes the completion of algebraic closure of $\mathbb{Q}_p$, $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{Z}$ denotes the ring of rational integers, $\mathbb{Q}$ denotes the field of rational numbers, $\mathbb{C}$ denotes the set of complex numbers, and $\mathbb{Z}_+ = \mathbb{N} \cup \{0\}$. Let ν_p be the normalized exponential valuation of $\mathbb{C}_p$ with $|p|_p = p^{-\nu_p(p)} = p^{-1}$. When one talks of q-extension, q is considered in many ways such as an indeterminate, a complex number $q \in \mathbb{C}$, or p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$ one normally

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assume that $|q| < 1$. If $q \in \mathbb{C}_p$, we normally assume that $|q-1|_p < p^{-\frac{1}{p-1}}$ so that $q^x = \exp(x \log q)$ for $|x|_p \leq 1$. Throughout this paper we use the notation:

$$[x]_q = \frac{1-q^x}{1-q}, \quad [x]_{-q} = \frac{1-(-q)^x}{1+q} \text{ (cf. [1-13]) .}$$

Hence, $\lim_{q\to 1}[x] = x$ for any x with $|x|_p \leq 1$ in the present p-adic case. For

$$g \in UD(\mathbb{Z}_p) = \{g | g : \mathbb{Z}_p \to \mathbb{C}_p \text{ is uniformly differentiable function}\},$$

the fermionic p-adic q-integral on $\mathbb{Z}_p$ is defined by

$$I_{-q}(g) = \int_{\mathbb{Z}_p} g(x) d\mu_{-q}(x) = \lim_{N\to\infty} \frac{1}{[p^N]_{-q}} \sum_{x=0}^{p^N-1} g(x)(-q)^x \text{ (cf. [3-6]) .} \tag{1.1}$$

Let

$$T_p = \cup_{m\geq 1} C_{p^m} = \lim_{m\to\infty} C_{p^m},$$

where $C_{p^m} = \{w | w^{p^m} = 1\}$ is the cyclic group of order p^m. For $w \in T_p$, we denote by $\phi_w : \mathbb{Z}_p \to \mathbb{C}_p$ the locally constant function $x \longmapsto w^x$. If we take $g_1(x) = g(x+1)$ in (1.1), then we easily see that

$$qI_{-q}(g_1) + I_{-q}(g) = [2]_q g(0). \tag{1.2}$$

From (1.2), we obtain

$$q^n I_{-q}(g_n) + (-1)^{n-1} I_{-q}(g) = [2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l g(l), \tag{1.3}$$

where $g_n(x) = g(x+n)$ (see [1-13]).

As well known definition, the Genocchi polynomials are defined by

$$F(t,x) = \frac{2t}{e^t+1} e^{xt} = e^{G(x)t} = \sum_{n=0}^{\infty} G_n(x) \frac{t^n}{n!},$$

with the usual convention of replacing $G^n(x)$ by $G_n(x)$. In the special case, $x=0$, $G_n(0) = G_n$ are called the n-th Genocchi numbers (see [1-11]).

In [9], we introduced analogue of Genocchi numbers and polynomials, which is called twisted Genocchi numbers and polynomials. We define the twisted Genocchi numbers $G_{n,w}$ as follows:

$$\frac{2t}{we^t+1} = \sum_{n=0}^{\infty} G_{n,w} \frac{t^n}{n!}.$$

The twisted Genocchi polynomials $G_{n,w}(x)$ are defined by means of the generating function

$$F_w(t,x) = \frac{2t}{we^t+1} e^{xt} = \sum_{n=0}^{\infty} G_{n,w}(x) \frac{t^n}{n!}.$$

In [11], by using p-adic q-integral, the weighted q-Genocchi numbers $G_{n,q}^{(\alpha)}$ and polynomials $G_{n,q}^{(\alpha)}(x)$ are defined by

$$G_{n,q}^{(\alpha)} = n\int_{\mathbb{Z}_p} [x]_{q^\alpha}^{n-1} d\mu_{-q}(x) \text{ for } n \in \mathbb{Z}_+, \alpha \in \mathbb{Z}$$

and

$$G_{n,q}^{(\alpha)}(x) = n\int_{\mathbb{Z}_p} [x+y]_{q^\alpha}^{n-1} d\mu_{-q}(y) \tag{1.4}$$

respectively. We also define twisted q-Genocchi polynomials as follows:

$$G_{n,q,w}^{(\alpha)}(x) = n\int_{\mathbb{Z}_p} \phi_w(y)[y+x]_{q^\alpha}^{n-1} d\mu_{-q}(y), \text{ for } n \in \mathbb{N}.$$

Similarly, we have the generating function of twisted q-Genocchi polynomials $G_{n,q,w}(x)$ as follows:

$$F_{q,w}(t,x) = [2]_q t\sum_{m=0}^{\infty}(-1)^m q^m w^m e^{[m+x]_q t} \text{ (cf. [1-11]).} \tag{1.5}$$

Our aim in this paper is to define twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha)}$ and polynomials $G_{n,q,w}^{(\alpha)}(x)$ with weight α. We investigate some properties which are related to twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha)}$ and polynomials $G_{n,q,w}^{(\alpha)}(x)$ with weight α. We also derive the existence of a specific interpolation function which interpolate twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha)}$ and polynomials $G_{n,q,w}^{(\alpha)}(x)$ with weight α at negative integers.

2. Twisted q-Genocchi numbers and polynomials with weight α

Our primary goal of this section is to define twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha)}$ and polynomials $G_{n,q,w}^{(\alpha)}(x)$ with weight α. We also find generating functions of twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha)}$ and polynomials $G_{n,q,w}^{(\alpha)}(x)$.

For $\alpha \in \mathbb{Z}$ and $q \in \mathbb{C}_p$ with $|1-q|_p \le 1$, twisted q-Genocchi numbers $G_{n,q,w}^{(\alpha)}$ are defined by

$$G_{n,q,w}^{(\alpha)} = n\int_{\mathbb{Z}_p} \phi_w(x)[x]_{q^\alpha}^{n-1} d\mu_{-q}(x). \tag{2.1}$$

By using p-adic q-integral on $\mathbb{Z}_p$, we obtain,

$$\begin{aligned} &n\int_{\mathbb{Z}_p} \phi_w(x)[x]_{q^\alpha}^{n-1} d\mu_{-q}(x) \\ &= n\lim_{N\to\infty}\frac{1}{[p^N]_{-q}}\sum_{x=0}^{p^N-1}[x]_{q^\alpha}^{n-1} w^x(-q)^x \\ &= n[2]_q\left(\frac{1}{1-q^\alpha}\right)^{n-1}\sum_{l=0}^{n-1}\binom{n-1}{l}(-1)^l\frac{1}{1+wq^{\alpha l+1}}. \end{aligned} \tag{2.2}$$

By (2.1), we have

$$G^{(\alpha)}_{n,q,w} = n[2]_q \left(\frac{1}{1-q^{\alpha}}\right)^{n-1} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l \frac{1}{1+wq^{\alpha l+1}}.$$

We set

$$F^{(\alpha)}_{q,w}(t) = \sum_{n=0}^{\infty} G^{(\alpha)}_{n,q,w} \frac{t^n}{n!}.$$

By using above equation and (2.2), we have

$$\begin{aligned} F^{(\alpha)}_{q,w}(t) &= \sum_{n=0}^{\infty} G^{(\alpha)}_{n,q,w} \frac{t^n}{n!} \\ &= [2]_q \sum_{n=0}^{\infty} \left(n \left(\frac{1}{1-q^{\alpha}}\right)^{n-1} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l \frac{1}{1+wq^{\alpha l+1}} \right) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \left(n[2]_q \left(\frac{1}{1-q^{\alpha}}\right)^{n-1} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l \left(\sum_{m=0}^{\infty} (-wq)^m q^{\alpha l m} \right) \right) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \left(n[2]_q \left(\frac{1}{1-q^{\alpha}}\right)^{n-1} \sum_{m=0}^{\infty} (-wq)^m (1-q^{\alpha m})^{n-1} \right) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} \left(\sum_{m=0}^{\infty} n[2]_q (-wq)^m [m]^{n-1}_{q^{\alpha}} \right) \frac{t^n}{n!}. \end{aligned} \tag{2.3}$$

Hence, we get the following form

$$G^{(\alpha)}_{n,q,w} = n[2]_q \sum_{m=0}^{\infty} (-1)^m w^m q^m [m]^{n-1}_{q^{\alpha}}.$$

Also, since

$$\sum_{n=0}^{\infty} n[m]^{n-1}_{q^{\alpha}} \frac{t^n}{n!} = te^{[m]_{q^{\alpha}} t}.$$

$$F^{(\alpha)}_{q,w}(t) = [2]_q t \sum_{m=0}^{\infty} (-1)^m w^m q^m e^{[m]_{q^{\alpha}} t}.$$

Thus twisted q-Genocchi numbers $G^{(\alpha)}_{n,q,w}$ are defined by means of the generating function

$$F^{(\alpha)}_{q,w}(t) = [2]_q t \sum_{n=0}^{\infty} (-1)^n w^n q^n e^{[n]_{q^{\alpha}} t}. \tag{2.4}$$

By using (2.1), we have

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,q,w}^{(\alpha)} \frac{t^n}{n!} &= \sum_{n=0}^{\infty} n \int_{\mathbb{Z}_p} \phi_w(x)[x]_{q^\alpha}^{n-1} d\mu_{-q}(x) \frac{t^n}{n!} \\ &= \int_{\mathbb{Z}_p} \phi_w(x) \sum_{n=0}^{\infty} n[x]_{q^\alpha}^{n-1} \frac{t^n}{n!} d\mu_{-q}(x) \\ &= t \int_{\mathbb{Z}_p} \phi_w(x) e^{[x]_{q^\alpha} t} d\mu_{-q}(x).\end{aligned} \tag{2.5}$$

From (2.3) and (2.5), we have

$$\begin{aligned}\sum_{n=0}^{\infty} G_{n,q,w}^{(\alpha)} \frac{t^n}{n!} &= t \int_{\mathbb{Z}_p} \phi_w(x) e^{[x]_{q^\alpha} t} d\mu_{-q}(x) \\ &= [2]_q t \sum_{m=0}^{\infty} (-1)^m w^m q^m e^{[m]_{q^\alpha} t}.\end{aligned}$$

Next, we introduce twisted q-Genocchi polynomials $G_{n,q,w}^{(\alpha)}(x)$ with weight α. The twisted q-Genocchi polynomials $G_{n,q,w}^{(\alpha)}(x)$ are defined by

$$\frac{G_{n,q,w}^{(\alpha)}(x)}{n} = \int_{\mathbb{Z}_p} \phi_w(y)[y+x]_{q^\alpha}^{n-1} d\mu_{-q}(y). \tag{2.6}$$

When $w = 1$, above (2.1) and (2.6) will become the corresponding definitions of the weighted Genocchi numbers $G_{n,q}^{(\alpha)}$ and polynomials $G_{n,q}^{(\alpha)}(x)$.

By using p-adic q-integral, we obtain

$$\frac{G_{n,q,w}^{(\alpha)}(x)}{n} = \sum_{l=0}^{n-1} \binom{n-1}{l} [x]_{q^\alpha}^{n-1-l} q^{\alpha l x} \frac{G_{l+1,q,w}^{(\alpha)}}{l+1}. \tag{2.7}$$

Observe that, if $q \to 1, w = 1$, then $G_{n,q,w}^{(\alpha)} \to G_n$ and $G_{n,q,w}^{(\alpha)}(x) \to G_n(x)$. Note that, if $q \to 1$, then $G_{n,q,w}^{(\alpha)} \to G_{n,w}$ and $G_{n,q,w}^{(\alpha)}(x) \to G_{n,w}(x)$. From(2.6), we note that

$$\begin{aligned}F_{q,w}^{(\alpha)}(t,x) &= \sum_{n=0}^{\infty} G_{n,q,w}^{(\alpha)}(x) \frac{t^n}{n!} \\ &= \sum_{n=0}^{\infty} n \int_{\mathbb{Z}_p} w^y [x+y]_{q^\alpha}^{n-1} d_{\mu_{-q}}(y) \frac{t^n}{n!} \\ &= [2]_q \sum_{n=0}^{\infty} n \left(\frac{1}{1-q^\alpha}\right)^{n-1} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l q^{\alpha x l} \frac{1}{1+wq^{\alpha l+1}} \frac{t^n}{n!} \\ &= [2]_q \sum_{m=0}^{\infty} (-1)^m w^m q^m \sum_{n=0}^{\infty} n[x+m]_{q^\alpha}^{n-1} \frac{t^n}{n!} \\ &= [2]_q t \sum_{m=0}^{\infty} (-1)^m w^m q^m e^{[x+m]_{q^\alpha} t}.\end{aligned} \tag{2.8}$$

Therefore, we obtain the following theorem.

Theorem 2.1. *For $\alpha \in \mathbb{Q}$, we have*

$$F_{q,w}^{(\alpha)}(t,x) = [2]_q t \sum_{m=0}^{\infty} (-1)^m w^m q^m e^{[x+m]_{q^\alpha} t}$$

and

$$\begin{aligned} G_{n,q,w}^{(\alpha)}(x) &= n[2]_q \left(\frac{1}{1-q^\alpha} \right)^{n-1} \sum_{l=0}^{n-1} \binom{n-1}{l} (-1)^l q^{\alpha x l} \sum_{m=0}^{\infty} (-wq^{\alpha l+1})^m \\ &= n[2]_q \sum_{m=0}^{\infty} (-1)^m w^m q^m [x+m]_{q^\alpha}^{n-1}. \end{aligned} \tag{2.9}$$

By(2.9), we have the following distribution relation:

Theorem 2.2. *For any positive integer $m \equiv 1 \pmod 2$, we have*

$$G_{n,q,w}^{(\alpha)}(x) = \frac{[2]_q}{[2]_{q^m}} [m]_{q^\alpha}^n \sum_{i=0}^{m-1} (-1)^i q^i w^i G_{n+1,q^m,w^m}^{(\alpha)} \left(\frac{i+x}{m} \right), \quad n \in \mathbb{Z}_+.$$

Let $g(x) = t\phi_w(x) e^{[x]_{q^\alpha} t}$. By (1.3), (2.1) , and (2.6), left-hand side is the following form

$$\begin{aligned} & q^n I_{-q}(g_n) + (-1)^{n-1} I_{-q}(g) \\ &= q^n \int_{\mathbb{Z}_p} t\phi_w(x+n) e^{[x+n]_{q^\alpha} t} d\mu_{-q}(x) \\ &\qquad + (-1)^{n-1} \int_{\mathbb{Z}_p} t\phi_w(x) e^{[x]_{q^\alpha} t} d\mu_{-q}(x) \\ &= q^n w^n \sum_{m=0}^{\infty} m \int_{\mathbb{Z}_p} \phi_w(x) [x+n]_{q^\alpha}^{m-1} d\mu_{-q}(x) \frac{t^m}{m!} \\ &\qquad + (-1)^{n-1} \sum_{m=0}^{\infty} m \int_{\mathbb{Z}_p} \phi_w(x) [x]_{q^\alpha}^{m-1} d\mu_{-q}(x) \frac{t^m}{m!} \\ &= \sum_{m=0}^{\infty} \left(q^n w^n G_{m,q,w}^{(\alpha)}(n) + (-1)^{n-1} G_{m,q,w}^{(\alpha)} \right) \frac{t^m}{m!}. \end{aligned} \tag{2.10}$$

and right-hand side is the following form

$$\begin{aligned} {}_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l t w^l e^{[l]_{q^\alpha} t} &= [2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l t w^l \sum_{k=0}^{\infty} [l]_{q^\alpha}^k \frac{t^k}{k!} \\ &= \sum_{m=0}^{\infty} [2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} q^l w^l m [l]_{q^\alpha}^{m-1} \frac{t^m}{m!}. \end{aligned} \tag{2.11}$$

By comparison between (2.10) and (2.11), we have the following form.

$$m[2]_q \sum_{l=0}^{n-1} (-1)^{n-1-l} w^l q^l [l]_{q^\alpha}^{m-1} = w^n q^n G_{m,q,w}^{(\alpha)}(n) + (-1)^{n-1} G_{m,q,w}^{(\alpha)}.$$

Hence, we have the following theorem.

Theorem 2.3. *Let $m \in \mathbb{Z}_+$. If $n \equiv 0 \pmod 2$, then*

$$w^n q^n G_{m,q,w}^{(\alpha)}(n) - G_{m,q,w}^{(\alpha)} = m[2]_q \sum_{l=0}^{n-1} (-1)^{l+1} w^l q^l [l]_{q^\alpha}^{m-1}.$$

If $n \equiv 1 \pmod 2$, then

$$w^n q^n G_{m,q,w}^{(\alpha)}(n) + G_{m,q,w}^{(\alpha)} = m[2]_q \sum_{l=0}^{n-1} (-1)^{l} w^l q^l [l]_{q^\alpha}^{m-1}.$$

From (1.2), we note that

$$\begin{aligned}
[2]_q t &= q \int_{\mathbb{Z}_p} t\phi_w(x+1) e^{[x+1]_{q^\alpha} t} d\mu_{-q}(x) + \int_{\mathbb{Z}_p} t\phi_w(x) e^{[x]_{q^\alpha} t} d\mu_{-q}(x) \\
&= \sum_{n=0}^{\infty} \left(nqw \int_{\mathbb{Z}_p} \phi_w(x)[x+1]_{q^\alpha}^{n-1} d\mu_{-q}(x) + n \int_{\mathbb{Z}_p} \phi_w(x)[x]_{q^\alpha}^{n-1} d\mu_{-q}(x) \right) \frac{t^n}{n!} \\
&= \sum_{n=0}^{\infty} \left(qwG_{n,q,w}^{(\alpha)}(1) + G_{n,q,w}^{(\alpha)} \right) \frac{t^n}{n!}.
\end{aligned}$$

Therefore, we obtain the following theorem.

Theorem 2.4. *For $n \in \mathbb{Z}_+$, we have*

$$G_{0,q,w}^{(\alpha)} = 0 \quad \textit{and} \quad qwG_{n,q,w}^{(\alpha)}(1) + G_{n,q,w}^{(\alpha)} = \begin{cases} [2]_q, & \textit{if } n = 1, \\ 0, & \textit{if } n > 1. \end{cases}$$

From (2.6) and (2.7), we derive the following binomial form.

$$\begin{aligned}
\frac{G_{n+1,q,w}^{(\alpha)}(x)}{n+1} &= \int_{\mathbb{Z}_p} w^y [x+y]_{q^\alpha}^n d_{\mu_{-q}}(y) \\
&= \sum_{l=0}^{n} \binom{n}{l} [x]_{q^\alpha}^{n-l} q^{\alpha l x} \frac{G_{l+1,q,w}^{(\alpha)}}{l+1} \\
&= \frac{q^{-\alpha x}}{n+1} \left([x]_{q^\alpha} + q^{\alpha x} G_{q,w}^{(\alpha)} \right)^{n+1}.
\end{aligned}$$

Hence, we have the following form.

$$q^{\alpha x} G_{n+1,q,w}^{(\alpha)}(x) = \left([x]_{q^\alpha} + q^{\alpha x} G_{q,w}^{(\alpha)} \right)^{n+1}. \tag{2.12}$$

Also by Theorem 2.4 and (2.12), we have the following corollary.

Corollary 2.5. *For $n \in \mathbb{Z}_+$, we have*

$$G^{(\alpha)}_{0,q,w} = 0 \quad \textit{and} \quad qw(1 + q^{\alpha} G^{(\alpha)}_{q,w})^n + q^{\alpha} G^{(\alpha)}_{n,q,w} = \begin{cases} q^{\alpha}[2]_q, & \textit{if } n = 1, \\ 0, & \textit{if } n > 1, \end{cases}$$

with the usual convention of replacing $(G^{(\alpha)}_{q,w})^n$ by $G^{(\alpha)}_{n,q,w}$.

3. The analogue of the Genocchi zeta function

By using twisted q-Genocchi numbers and polynomials, twisted q-Genocchi zeta function and twisted Hurwitz q-Genocchi zeta functions are defined. These functions interpolate the twisted q-Genocchi numbers and twisted q-Genocchi polynomials, respectively. In this section we assume that $q \in \mathbb{C}$ with $|q| < 1$. Let ω be the p^N-th root of unity. From (2.4), we note that

$$\begin{aligned} \frac{d^{k+1}}{dt^{k+1}} F^{(\alpha)}_{q,w}(t)\bigg|_{t=0} &= (k+1)[2]_q \sum_{n=1}^{\infty} (-1)^n w^n q^n [n]^k_{q^{\alpha}} \\ &= G^{(\alpha)}_{k+1,q,w}, (k \in \mathbb{N}). \end{aligned}$$

By using the above equation, we are now ready to define twisted q-Genocchi zeta functions.

Definition 3.1. Let $s \in \mathbb{C}$.

$$\zeta^{(\alpha)}_{q,w}(s) = [2]_q \sum_{n=1}^{\infty} \frac{(-1)^n w^n q^n}{[n]^s_{q^{\alpha}}}. \tag{3.1}$$

Note that $\zeta^{(\alpha)}_{q,w}(s)$ is a meromorphic function on $\mathbb{C}$. Note that, if $q \to 1$, then $\zeta^{(\alpha)}_{q,w}(s) = \zeta(s)$ which is the Genocchi zeta functions. Relation between $\zeta^{(\alpha)}_{q,w}(s)$ and $G^{(\alpha)}_{k,q,w}$ is given by the following theorem.

Theorem 3.2. *For $k \in \mathbb{N}$, we have*

$$\zeta^{(\alpha)}_{q,w}(-k) = \frac{G^{(\alpha)}_{k+1,q,w}}{k+1}.$$

Observe that $\zeta^{(\alpha)}_{q,w}(s)$ function interpolates $G^{(\alpha)}_{k,q,w}$ numbers at non-negative integers. By using (2.9), we note that for $k \in \mathbb{N}$

$$\begin{aligned} \frac{d^{k+1}}{dt^{k+1}} F^{(\alpha)}_{q,w}(t,x)\bigg|_{t=0} &= (k+1)[2]_q \sum_{n=0}^{\infty} (-1)^n w^n q^n [n+x]^k_{q^{\alpha}} \\ &= G^{(\alpha)}_{k+1,q,w}(x). \end{aligned} \tag{3.2}$$

and by (3.2),(2.9)

$$\left(\frac{d}{dt}\right)^{k+1} \left(\sum_{n=0}^{\infty} G^{(\alpha)}_{n,q,w}(x) \frac{t^n}{n!} \right)\Bigg|_{t=0} = G^{(\alpha)}_{k+1,q,w}(x), \text{ for } k \in \mathbb{N}. \tag{3.3}$$

By (3.2) and (3.3), we are now ready to define the twisted Hurwitz q-Genocchi zeta functions.

Definition 3.3. Let $s \in \mathbb{C}$.

$$\zeta_{q,w}^{(\alpha)}(s,x) = [2]_q \sum_{n=0}^{\infty} \frac{(-1)^n w^n q^n}{[n+x]_{q^\alpha}^s}. \tag{3.4}$$

Note that $\zeta_{q,w}^{(\alpha)}(s,x)$ is a meromorphic function on $\mathbb{C}$. Obverse that, if $q \to 1$, then $\zeta_{q,w}^{(\alpha)}(s,x) = \zeta(s,x)$ which is the Hurwitz Genocchi zeta functions. Relation between $\zeta_{q,w}^{(\alpha)}(s,x)$ and $G_{k,q,w}^{(\alpha)}(x)$ is given by the following theorem.

Theorem 3.4. *For $k \in \mathbb{N}$, we have*

$$\zeta_{q,w}^{(\alpha)}(-k,x) = \frac{G_{k,q,w}^{(\alpha)}(x)}{k+1}.$$

Observe that $\zeta_{q,w}^{(\alpha)}(-k,x)$ function interpolates $G_{k,q,w}^{(\alpha)}(x)$ numbers at non-negative integers.

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