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A STUDY ON THE TWISTED q-EULER POLYNOMIALS AND TRANSFER OPERATORS†

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Abstract. In this paper, we consider twisted q-Euler polynomials and define a p-adic q-transfer operator (see [6]). From this operator, we investigate the eigenvalues of the p-adic q-transfer operator on the space of twisted q-Euler polynomials.

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1. Introduction

Throughout this paper, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{Z}_p$, $\mathbb{Q}_p$ and $\mathbb{C}_p$ will denote the ring of rational integers, the field of rational numbers, the ring of p-adic integers, the field of p-adic rational numbers and the completion of algebraic closure of $\mathbb{Q}_p$, respectively. Let ν_p be the normalized exponential valuation of $\mathbb{C}_p$ such that $|p|_p = p^{-\nu_p(p)} = \frac{1}{p}$. Let q be regarded as either a complex number $q \in \mathbb{C}$ or a p-adic number $q \in \mathbb{C}_p$. If $q \in \mathbb{C}$, we assume $|q| < 1$, and if $q \in \mathbb{C}_p$, we normally assume $|1-q|_p < 1$. We use the notation

$$[x]_q = \frac{1-q^x}{1-q} \quad \text{and} \quad [x]_{-q} = \frac{1-(-q)^x}{1+q}$$

for all $x \in \mathbb{Z}_p$.

Let d be a fixed positive odd integer and let p be a fixed odd prime number. We now set

$$\begin{aligned} X = X_d &= \lim_{\overleftarrow{N}} \mathbb{Z}/dp^N\mathbb{Z}, \\ X^* &= \bigcup_{\substack{0<a<\overline{f}p \\ (a,p)=1}} (a + dp\,\mathbb{Z}_p), \end{aligned}$$

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$$a + dp^N\mathbb{Z}_p \quad = \quad \{x \in X \mid x \equiv a \pmod{dp^N}\},$$

where $a \in \mathbb{Z}$ lies in $0 \le a < dp^N$.

We say that f is a uniformly differentiable function at a point $a \in \mathbb{Z}_p$ and denote this property by $f \in UD(\mathbb{Z}_p)$, if the difference quotient

$$F_f(x, y) = \frac{f(x) - f(y)}{x - y}$$

has a limit $f'(a)$ as $(x, y) \to (a, a)$. For $f \in UD(\mathbb{Z}_p)$, the fermionic p-adic invariant q-integral of f on $\mathbb{Z}_p$ is defined by

$$I_{-q}(f) = \int_{\mathbb{Z}_p} f(x)d\mu_{-q}(x) = \lim_{N\to\infty} \frac{1}{[dp^N]_{-q}} \sum_{x=0}^{dp^N-1} f(x)(-q)^x, \quad \text{(see [1-14])}.$$

Let $T_p = \cup_{n\ge0} C_{p^n} = \lim_{n\to\infty} C_{p^n}$ be the locally constant space, where $C_{p^n} = \{w|w^{p^n} = 1\}$ is the cyclic group of order of p^n. For $w \in T_p$, $\phi_w : \mathbb{Z}_p \to \mathbb{C}_p$ is the locally constant function $x \mapsto w^x$ (cf. [2-4]). In [14], Kim et al defined the twisted q-Euler polynomials by using p-adic q-integral on $\mathbb{Z}_p$ as follows:

$$E^{(h)}_{n,w}(x, q) = \int_{\mathbb{Z}_p} q^{(h-1)y} w^y [x + y]^n_q d\mu_{-q}(y), \tag{1}$$

where $w \in T_p$ and $h \in \mathbb{Z}$ and $n \in \mathbb{N}$.

We note that the transfer operator encodes information about an iterated map and is frequently used to study the behavior of dynamical systems, statistical mechanics, quantum chaos and fractals. The transfer operator is defined as an operator L acting on the space of functions $\phi : X \to \mathbb{C}$ as

$$(L\phi)(x) = \sum_{y\in\phi^{-1}(x)} g(y)\phi(y)$$

where $g : X \longrightarrow \mathbb{C}$ is an auxiliary valuation function [6,15,16]. In this paper we consider the p-adic q-transfer operator on the space of all twisted q-Euler polynomials and investigate the eigenvalues of the p-adic q-transfer operators related with twisted q-Euler polynomials.

2. Main results

In this section, we consider the twisted q-Euler polynomials, $E^{(h)}_{n,w}(x, q)$ as follows:

$$[2]_q \sum_{n=0}^{\infty} (-1)^n w^n q^{(1-h)n} e^{[n+x]_q t} = \sum_{n=0}^{\infty} E^{(h)}_{n,w}(x, q) \frac{t^n}{n!}. \tag{2}$$

Note that if $w = 1$ and $h = 1$, then $E^{(1)}_{n,1}(x, q) = E_n(x, q)$ is called q-Euler polynomial. From (1), we derive

$$E^{(h)}_{n,w}(x, q) = \frac{[2]_q}{(1 - q)^n} \sum_{j=0}^{n} \binom{n}{j} (-1)^j q^{xj} \frac{1}{1 + q^{h+j}w}, \tag{3}$$

where $E_{n,w}^{(h)}(x,q)$ are twisted q-Euler polynomials. By (1), we see that

$$\begin{aligned}
&E_{n,w}^{(h)}(x,q)\\
&=\int_{\mathbb{Z}_p} q^{(h-1)y}w^y[x+y]_q^n d\mu_{-q}(y)\\
&=\lim_{N\to\infty}\frac{1}{[dp^N]_{-q}}\sum_{k=0}^{dp^N-1} q^{(h-1)k}w^k[x+k]_q^n(-q)^k\\
&=\lim_{N\to\infty}\frac{1}{[d]_{-q}}\frac{1}{[p^N]_{-q^d}}\sum_{i=0}^{d-1}\sum_{k=0}^{p^N-1} w^{i+dk}[x+i+dk]_q^n(-q)^{h(i+dk)}(-1)^{i+k}\\
&=\frac{[d]_q^n}{[d]_{-q}}\sum_{i=0}^{d-1}(-1)^i q^{hi}w^i\lim_{N\to\infty}\frac{1}{[p^N]_{-q^d}}\sum_{k=0}^{p^N-1} w^{dk}\left[\frac{x+i}{d}+k\right]_{q^d}^n(-q^{dh})^k\\
&=\frac{[d]_q^n}{[d]_{-q}}\sum_{i=0}^{d-1}(-1)^i q^{hi}w^i\int_{\mathbb{Z}_p} q^{d(h-1)y}(w^d)^y\left[\frac{x+i}{d}+y\right]_{q^d}^n d\mu_{-q^d}(y). \qquad (4)
\end{aligned}$$

Now we define p-adic q-transfer operator as follows:

$$(L_{p,q}f)(x,q)=\frac{1}{[d]_{-q}}\sum_{i=0}^{d-1}(-q)^i f\left(\frac{x+i}{d}:q^d\right). \qquad (5)$$

If we take $f(x,q)=E_{n,w}^{(h)}(x,q)$, then we have

$$\begin{aligned}
&(L_{p,q}E_{n,w}^{(h)})(x,q)\\
&=\frac{1}{[d]_{-q}}\sum_{i=0}^{d-1}(-q)^i E_{n,w}^{(h)}\left(\frac{x+i}{d}:q^d\right)\\
&=\frac{1}{[d]_q^n}\frac{[d]_q^n}{[d]_{-q}}\sum_{i=0}^{d-1}(-q)^i\int_{\mathbb{Z}_p} q^{d(h-1)y}w^y\left[\frac{x+k}{d}+y\right]_{q^d}^n d\mu_{-q^d}(y). \qquad (6)
\end{aligned}$$

By (3) and (5), we have

$$(L_{p,q}E_{n,w}^{(h)})(x,q)=\frac{1}{[d]_q^n}E_{n,w}^{(h)}(x,q). \qquad (7)$$

Therefore we obtain the following theorem.

Theorem 2.1. *For $w\in T_p$ and $n,h\in\mathbb{Z}_+$, the eigenvalues of the p-adic q-transfer operator $L_{p,q}$ related with twisted q-Euler polynomials are $\frac{1}{[d]_q^n}$. That is,*

$$(L_{p,q}E_{n,w}^{(h)})(x,q)=\tfrac{1}{[d]_q^n}E_{n,w}^{(h)}(x,q).$$

Remark 2.1. In the spacial case $w=1$ and $h=1$, the eigenvalues of the p-adic q-transfer operator $L_{p,q}$ related with q-Euler polynomials $E_{n,1}^{(1)}(x,q)=E_n(x,q)$ are $\frac{1}{[d]_q^n}$ (see [2]).

Remark 2.2. We consider the twisted q-Genocchi polynomials, $G^{(h)}_{n,w}(x,q)$ as follows (see [8]):

$$t[2]_q \sum_{n=0}^{\infty} (-1)^n w^n q^{(1-h)n} e^{[n+x]_q t} = \sum_{n=0}^{\infty} G^{(h)}_{n,w}(x,q) \frac{t^n}{n!}. \tag{8}$$

Note that if $w = 1$ and $h = 1$, then $G^{(1)}_{n,1}(x,q) = G_n(x,q)$ is called q-Genocchi polynomial. By Theorem 5 in [8], we can get

$$G^{(h)}_{n,w}(x,q) = nE^{(h)}_{n,w}(x,q), \tag{9}$$

where $E^{(h)}_{n,w}(x,q)$ are twisted q-Euler polynomials. Thus the eigenvalues of the p-adic q-transfer operator $L_{p,q}$ related with twisted q-genocchi polynomials $G^{(h)}_{n,w}(x,q)$ are $\frac{n}{[d]_q^n}$.

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