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# Cooperative Communication with Different Combining Techniques in One-Dimensional Random Networks

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#### Abstract

In this paper, we investigate cooperative transmission in one-dimensional random wireless networks. In this scheme, a stationary source communicates with a stationary destination with the help of N relays, which are randomly placed in a one-dimensional network. We derive exact and approximate expressions of the average outage probability over Rayleigh fading channels. Various Monte-Carlo simulations are presented to verify the accuracy of our analyses. *Key words*: Cooperative Communication, Uniform Distribution, Maximal Ratio Combining, Selection Combining, Fad-

ing Channels.

## I. Introduction

Cooperative wireless communication has gained attention as an efficient method for mitigating the effects of fading channels. By exploiting the broadcast nature of a wireless channel, cooperative diversity allows single-antenna radios to form a virtual antenna array. Various cooperative transmission protocols have been investigated in dual-hop networks [1], [2] as well as in multi-hop networks [3], [4].

Most of the current research efforts related to cooperative communication have been based on fixed networks, in which nodes are assumed to be stationary or distances are assumed to be constant. However, in practice, nodes can move or be randomly placed, and the distance between nodes is therefore a random variable. In [5], the CDF and PDF of the distance between randomly selected nodes were derived. However, when a common terminal is involved, the link distances are no longer independent. In [6], [7], the authors proposed an alternative approach to determine the joint CDF and PDF of the distances between the nodes and the reference terminal.

One-dimensional networks, which consist of randomly placed linear nodes, appear in many real applications. Several studies have focused on the connectivity between the source and the destination in these types of networks [8] $\sim$ [10]. In these models, two nodes are able to communicate if they are within radio range of one another. In practice, the channel between the nodes can be affected due to multipath phenomena or fading. As presented in [3], [4], a receiver in a fading environment

can receive a signal transmitted by a transmitter even if it is out of the transmitter's radio range. Hence, nodes in one-dimensional networks can also exploit cooperative diversity to enhance the reliability of data transmission.

In this paper, we study cooperative transmission in one-dimensional random wireless networks. In the considered scheme, a source attempts to transmit its information to a destination with the help of relays. We assume that the position of the source and the destination is fixed, while relays are randomly located in a one-dimensional network. Each operating relay follows the selective decode-and-forward scheme [1], in which the data are forwarded to the destination if decoded successfully. At the destination, a variety of combining techniques, such as maximal ratio combining (MRC) and selection combining (SC), can be employed to obtain diversity from the multiple signal replicas available from the relays and the source. The cooperative communication (CC) and direct transmission (DT) performances are evaluated and compared via simulations and analyses.

We first derive the exact and approximate expressions of outage probability over Rayleigh fading channels when SC technique or MRC technique is employed at the destination. Second, the average outage probability in random networks is derived. Finally, Monte-Carlo simulations are presented to validate our analyses. The results show that the CC protocol outperforms the DT protocol at high SNR regimes, similar to the behaviors of fixed networks.

The rest of this paper is organized as follows: Section  ${\rm I\!I}$  describes the system model, Section  ${\rm I\!I}$  presents the

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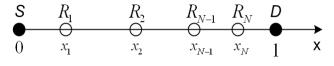


Fig. 1. Cooperative communication in one-dimensional network.

theoretical analyses of the CC and DT protocols, Section  $\mathbb{N}$  provides the Monte-Carlo simulations and theoretical results, and section  $\mathbb{V}$  presents our research conclusions.

# II. System Model

Fig. 1 presents a one-dimensional network in which a source node *S* communicates with a destination *D* with the help of *N* mobile nodes, which are called cooperative nodes or relays. In this figure, we assume that the source *S* and the destination *D* have coordinate 0 and 1, respectively while coordinates  $x_j$  ( $1 \le j \le N$ ) of relays  $R_j$  are uniformly and independently random variables in interval (0, 1). In Fig. 1, we chose these positions of the source and the destination to simplify the calculations. For other positions of the source and the destination in the network, the mathematical derivations can be determined in the same manner.

Current limitations in radio implementation preclude transmitting and receiving at the same time in the same frequency band. Due to considerable attenuation over the wireless channel and insufficient electrical isolation between transmit and receive circuitry, we adopted a Time Division Multiple Access (TDMA) technique in this paper.

In the first time slot, the source S broadcasts its signal to destination and all relay nodes. At the end of first time slot, relays will decode the received signal. If the channel between the source and the relay is good enough, this relay decodes and forwards the source information to the destination. Let us denote  $C_D$  as the decoding set, whose members are relays which decode successfully. The remaining relays are assumed to belong to set  $C_W$ . In the following N time slots, members of the set  $C_D$  repeat the source signal in a predetermined order [2].

## III. Performance Analysis

For ease of presentation, let us denote  $d_j$  and  $l_j$  $(1 \le j \le N)$  as distance between source *S* and relay  $R_j$ and distance between relay  $R_j$  and destination *D*, respectively. As mentioned above, clearly  $d_j = x_j$  and hence the distance  $d_j$  also has the uniform distribution in interval (0, 1).

Assume that the channels between two nodes are subjected to flat Rayleigh fading plus AWGN. The signal received at receiver j due to the transmission of transmitter i is given by

$$r_{i,j} = \sqrt{Ph_{i,j}s} + \eta_j \tag{1}$$

where *P* is the average transmit power of node *i* ( in this paper, it is assumed that the source and relays have same transmit power *P*),  $\eta_j$  is AWGN noise sample with variance  $N_0/2$  per dimension at receiver *j*,  $h_{i,j}$  is fading coefficient between nodes *i* and *j*, *s* is the signal transmitted by the transmitter *i*.

Because  $h_{i,j}$  has Rayleigh distribution,  $|h_{i,j}|^2$  has exponential distribution with parameter  $\lambda_{i,j}$ . To take path loss into account, we can model the variance of channel coefficient between nodes *i* and *j* as a function of distance between two nodes [10]. Therefore, the parameter  $\lambda_{i,j}$  can be expressed by

$$\lambda_{i,j} = d_{i,j}^{\beta} \tag{2}$$

where  $\beta$  is path loss exponent that varies from 2 to 6 and  $d_{i,i}$  is the distance between node *i* and node *j*.

Now, for ease of presentation, we introduce some notations used in next parts, as follows. Let us denote  $h_{1,j}$  and  $h_{2,j}$  as the channel coefficients between the source S and relay  $R_j$  and between relay  $R_j$  and the destination, respectively,  $\lambda_{1,j}$  and  $\lambda_{2,j}$  as the parameters of  $|h_{1,j}|^2$  and  $|h_{2,j}|^2$ , respectively. We also denote  $h_{2,0}$  and  $\lambda_{2,0}$  as the channel coefficient between the source S and the destination D and the parameter of  $|h_{2,0}|^2$ , respectively.

In direct transmission scheme (DT), the source transmits the signal to the destination directly without the help of any relays. The mutual information between the source S and the destination D is defined by

$$I_{S,D} = \log_2 \left( 1 + P \left| h_{2,0} \right|^2 / N_0 \right)$$
(3)

Now, we define the outage probability of the S-D link as

$$\mathbf{P}_{out}^{DT} = \Pr\left[I_{S,D} < R\right] \tag{4}$$

where R is the target rate of the system.

Using the CDF of  $|h_{2,0}|^2$ , we can easily calculate the outage probability of direct transmission as

$$P_{\rm DT}^{\rm out} = 1 - \exp\left(-\lambda_{2,0}\,SNR\right) = 1 - \exp\left(-\lambda_{2,0}\,SNR\right) \tag{5}$$

where  $SNR = P / N_0$  is the average signal to noise ratio (SNR).

For the cooperative communication, the mutual information between the source S and relay  $R_j$  is determined by

$$I_{1,j} = \frac{1}{N+1} \log_2 \left( 1 + P \left| h_{1,j} \right|^2 / N_0 \right)$$
(6)

where the factor 1/(N+1) account for the fact that the overall transmission is split into N+1 time slots.

If  $I_{1,j}$  is higher than the system rate *R*, the relay  $R_j$  successfully decodes the source information. Otherwise, the decoding status at relay  $R_j$  fails. Hence, this outage probability is formulated as

$$P_{1,j} = \Pr\left[I_{1,j} < R\right] = 1 - \exp\left(-\lambda_{1,j}\rho\right)$$
(7)

where  $\rho = (2^{(N+1)R} - 1) / \text{SNR}.$ 

Because the decoding set  $C_D$  is a random set, the number of relays in this set is also a random variable, i.e.,  $|C_D \models k$ ,  $k \in \{0, 1, ..., N\}$ . For each k, there are  $\binom{N}{k} = \frac{N!}{k!(N-k)!}$  possible subsets of size k. Assume that  $C_D = \{R_{i_1}, R_{i_2}, ..., R_{i_k}\}$  and  $C_W = \{R_{i_{k+1}}, R_{i_{k+2}}, ..., R_{i_N}\}$ , the probability for the decoding set  $C_D$  can be obtained as

$$P_{C_D} = \prod_{m=1}^{k} \left( 1 - P_{1,i_m} \right) \prod_{m=k+1}^{N} P_{1,i_m}$$
(8)

where  $P_{1,i_m}$  is calculated as in (7).

Using the total probability law, we can write the outage probability of system as

$$P_{\text{out}}^{\text{CC}} = \sum_{C_D} P_{C_D} \Pr\left[I_{C_D} < R\right]$$
(9)

where  $I_{c_{D}}$  is the mutual information of the decode and forward transmission.

If the SC technique is used at the destination,  $I_{C_D}$  is given as

$$I_{C_{D}} = \frac{1}{N+1} \log_2 \left( 1 + \frac{P}{N_0} \max_{m=1,2,\dots,k} \left( \left| h_{2,0} \right|^2, \left| h_{2,j_m} \right|^2 \right) \right)$$
(10)

From (10), the outage probability at the destination is calculated as

$$\Pr\left[I_{C_D} < R\right] = \Pr\left[\max_{m=1,2,\dots,k} \left(\left|h_{2,0}\right|^2, \left|h_{2,i_m}\right|^2\right) < \rho\right]$$

$$= \Pr\left[\left|h_{2,0}\right|^{2} < \rho\right] \prod_{m=1}^{k} \Pr\left[\left|h_{2,i_{m}}\right|^{2} < \rho\right]$$
$$= \left(1 - \exp(-\rho)\right) \prod_{m=1}^{k} \left(1 - \exp(-\lambda_{2,i_{m}}\rho)\right)$$
(11)

From (8), (9) and (11), the outage probability of system can be written as

$$P_{\text{out}}^{\text{CC}} = \sum_{C_D} \begin{bmatrix} (1 - \exp(-\rho)) \prod_{j=k+1}^{N} (1 - \exp(-\lambda_{1,i_j}\rho)) \\ \times \prod_{j=1}^{k} \exp(-\lambda_{1,i_j}\rho) (1 - \exp(-\lambda_{2,i_j}\rho)) \end{bmatrix}$$
(12)

In a one-dimensional network, the random variables  $d_{i_j}$  are uniform and independent distributions; hence, the average outage probability  $P_{out}^{CC}$  with respect to the link distances  $d_{i_j}$  is calculated as

$$\overline{P_{\text{out}}^{\text{CC}}} = \sum_{C_{D}} \begin{pmatrix} (1 - \exp(-\rho)) \times \\ \prod_{m=k+1}^{N} \int_{0}^{1} (1 - \exp(-z_{i_{m}}^{\beta} \rho)) d(z_{i_{m}}) \times \\ \prod_{m=1}^{k} \int_{0}^{1} \exp(-z_{i_{m}}^{\beta} \rho) (1 - \exp(-(1 - z_{i_{m}})^{\beta} \rho)) d(z_{i_{m}}) \end{pmatrix}$$
(13)

In (13), we can calculate the integral  $\int_{0}^{1} \exp(-z^{\beta}) dz$ 

as 
$$\int_0^1 \exp(-z^\beta \rho) dz = \frac{\Gamma(1/\beta) - \Gamma(1/\beta, 1)}{\beta \sqrt{\rho}}$$
 where  $\Gamma(.)$ 

is the complete gamma function and  $\Gamma(.,.)$  is the incomplete gamma function [11]. However, it is impossible to calculate the integral  $\int_0^1 \exp\left(-\left(z^\beta + (1-z)^\beta\right)\rho\right) dz$  for an arbitrary value real  $\beta$ . In case that  $\beta = 2$  and  $\beta = 3$ , we can respectively obtain

$$\int_{0}^{1} \exp\left(-\left(z^{2} + (1-z)^{2}\right)\rho\right) dz = \frac{\sqrt{\pi}}{\sqrt{2\rho}} \frac{Erf\left(\sqrt{\rho/2}\right)}{\exp\left(\rho/2\right)}$$
(13.1)  
$$\int_{0}^{1} \exp\left(-\left(z^{3} + (1-z)^{3}\right)\rho\right) dz = \frac{\sqrt{\pi}}{\sqrt{3\rho}} \frac{Erf\left(\sqrt{3\rho/4}\right)}{\exp\left(\rho/4\right)}$$
(13.2)

where  $\text{Erf}(\cdot)$  is the error function [11].

Therefore, for an arbitrary value real  $\beta$  ( $\beta \neq 2, \beta \neq 3$ ), we calculate the exact value of (13) by numerical results. However, when the number of relays *N* is large, the numerical calculation takes much time. Therefore, we propose a simple method to calculate approximately  $\overline{P_{\text{out}}^{\text{CC}}}$  as follows.

At high SNR value, equations (8) and (11) can be approximated as

$$P_{C_D} \approx \prod_{m=k+1}^{N} \left( \lambda_{1,i_m} \rho \right) = \rho^{N-k} \prod_{m=k+1}^{N} \lambda_{1,i_m}$$
(14)

$$\Pr\left[I_{C_D} < R\right] \approx \rho \prod_{m=1}^{k} \left(\lambda_{2,i_m} \rho\right) = \rho^{k+1} \prod_{m=1}^{k} \lambda_{2,i_m}$$
(15)

Substituting (14) and (15) into (9), we obtain (16) as

$$P_{\text{out}}^{\text{CC}} \approx \sum_{C_D} \rho^{N+1} \prod_{m=k+1}^N \lambda_{1,i_m} \prod_{m=1}^k \lambda_{2,i_m}$$
(16)

From (16),  $\overline{P_{out}^{CC}}$  in (13) can be approximated simply as follows:

$$\overline{P_{\text{out}}^{\text{CC}}} \approx \sum_{C_D} \rho^{N+1} \prod_{m=k+1}^N \int_0^1 (z_{i_m})^\beta \, dz_{i_m} \prod_{m=1}^k \int_0^1 (1 - z_{i_m})^\beta \, dz_{i_m} \\\approx \frac{2^N \rho^{N+1}}{(1 + \beta)^N}$$
(17)

If the destination employs MRC technique,  $I_{C_D}$  can be determined by

$$I_{C_{D}} = \frac{1}{N+1} \log_2 \left( 1 + \frac{P}{N_0} \left( \left| h_{2,0} \right|^2 + \sum_{j=1}^{k} \left| h_{2,i_j} \right|^2 \right) \right)$$
(18)

From (18), we have

$$\Pr\left[I_{C_{D}} < R\right] = \Pr\left[\left|h_{2,0}\right|^{2} + \sum_{j=1}^{k} \left|h_{2,i_{j}}\right|^{2} < \rho\right]$$
(19)

For calculating the term  $\Pr[I_{C_D} < R]$  in (19), we use moment generating function (MGF) to find the CDF of  $S_{\Sigma} = |h_{2,0}|^2 + \sum_{j=1}^{k} |h_{2,i_j}|^2$ . The MGF of  $|h_0|^2$  and  $|h_{2,i_j}|^2$ are expressed as MGF(s) =  $\lambda_{2,i_m} / (\lambda_{2,i_m} + s)$ . Furthermore, due to the independence of  $|h_{2,0}|^2$  and  $|h_{2,i_j}|^2$ , the MGF of  $S_{\Sigma}$  is given by

$$MGF_{S_{\Sigma}}(s) = MGF_{|h_{2,i}|^{2}}(s) \prod_{j=1} MGF_{|h_{2,i_{j}}|^{2}}(s)$$
$$= \prod_{j=0}^{k} \frac{\lambda_{2,i_{j}}}{\lambda_{2,i_{j}} + s}$$
(20)

Note that in (20),  $i_0 = 0$ .

Furthermore, we can easily see that the probability that  $l_{i_n} \neq l_{i_m}$  if  $i_n \neq i_m$   $(1 \le i_n, i_m \le N \text{ and } i_n \neq i_m)$  equals to zero. Therefore, the link distances  $l_{i_n}$  and  $l_{i_m}$  are di-

fferent with each other. Using partial fraction expansion, equation (20) can be decomposed into

$$\mathrm{MGF}_{S_{\Sigma}}(s) = \sum_{m=0}^{k} \frac{\eta_m}{\lambda_{2,m} + s}$$
(21)

where  $\eta_m = \lambda_{2,m} \prod_{n=0,n\neq m}^{k} \lambda_{2,n} / (\lambda_{2,n} - \lambda_{2,m}).$ 

Now, using the inverse Laplace transform for (21), we obtain the PDF of  $S_{\Sigma}$  as

$$f_{S_{\Sigma}}\left(x\right) = \sum_{j=0}^{k} \eta_{j} \exp\left(-\lambda_{2,j}x\right)$$
(22)

From (22), the term  $\Pr[I_{C_D} < R]$  in (19) can be given as

$$\Pr\left[I_{C_{D}} < R\right] = \int_{0}^{\rho} f_{S_{2}}(x) dx = \sum_{j=0}^{k} \frac{\eta_{j}}{\lambda_{2,i_{j}}} \left(1 - \exp(-\rho\lambda_{2,i_{j}})\right)$$
(23)

Now, substituting (8) and (23) into (9), we obtain the outage probability over Rayleigh fading channel as

$$P_{\text{out}}^{\text{CC}} = \sum_{C_D} \begin{bmatrix} \prod_{j=1}^{k} \exp\left(-\lambda_{1,i_j}\rho\right) \prod_{j=k+1}^{N} \left(1 - \exp\left(-\lambda_{1,i_j}\rho\right)\right) \\ \times \sum_{j=0}^{k} \frac{\eta_j}{\lambda_{2,i_j}} \left(1 - \exp\left(-\rho\lambda_{2,i_j}\right)\right) \end{bmatrix}$$
(24)

Similar to (13), the average outage probability  $P_{out}^{CC}$  with respect to the link distances  $d_{i_j}$  is calculated as

$$\overline{P_{\text{out}}^{\text{CC}}} = \sum_{C_{D}} \left( \prod_{m=k+1}^{N} \int_{0}^{1} \left( 1 - \exp\left(-z_{i_{m}}^{\beta}\rho\right) \right) d\left(z_{i_{m}}\right) \times \prod_{m=1}^{k} \int_{0}^{1} \exp\left(-z_{i_{m}}^{\beta}\rho\right) d\left(z_{i_{m}}\right) \times \int_{0}^{1} \dots \int_{0}^{1} \sum_{j=0}^{k} \frac{\eta_{j}}{z_{i_{j}}^{\beta}} \left( 1 - \exp\left(-z_{i_{j}}^{\beta}\rho\right) \right) d\left(z_{i_{0}}\right) d\left(z_{i_{1}}\right) \dots d\left(z_{i_{k}}\right) \right)$$
(25)

Now, we find a simple expression to approximately calculate  $\overline{P_{out}^{CC}}$ . Indeed, at high SNR, we can approximate  $\Pr[I_{C_D} < R]$  by (see the proof at Appendix A)

$$\Pr\left[I_{C_D} < R\right] \approx \frac{\rho^{k+1}}{(k+1)!} \prod_{j=0}^k \lambda_{2,j_j}$$
(26)

Similar to SC technique, from (24), we also obtain

$$P_{\text{out}}^{\text{CC}} \approx \sum_{C_D} \frac{\rho^{N+1}}{(k+1)!} \prod_{m=k+1}^{N} \lambda_{1,i_m} \prod_{m=1}^{k} \lambda_{2,i_m}$$
(27)

Then, we calculate  $P_{out}^{CC}$  approximately with respect to the link distances as

$$\overline{P_{\text{out}}^{\text{CC}}} \approx \sum_{C_{D}} \frac{\rho^{N+1}}{(k+1)!} \prod_{m=k+1}^{N} \int_{0}^{1} (z_{i_{m}})^{\beta} dz_{i_{m}} \prod_{m=1}^{k} \int_{0}^{1} (1-z_{i_{m}})^{\beta} dz_{i_{m}}$$
$$\approx \frac{\rho^{N+1}}{(1+\beta)^{N}} \sum_{k=1}^{N} \frac{N!}{k!(N-k)!(k+1)!}$$
(28)

Now, we investigate the diversity order of cooperative communication in random networks via the following proposition:

**Proposition 1**: The obtained diversity order of cooperative communication when the destination uses SC or MRC technique is N+1.

Proof

From the definition of diversity order stated in [12], relying on (17) and (26), we can easily prove the proposition 1 as follows:

Diversity = 
$$-\lim_{SNR \to +\infty} \frac{\log(P_{out}^{CC})}{\log(SNR)} = N + 1$$
 (29)

# IV. Simulation Results

In this section, we provide some numerical results of the outage probabilities that have been developed in Section III and verify these results with simulations. Fig. 2 shows the outage probability as a function of SNR  $(P/N_0)$  in dB in one-dimensional network. In this figure, we choose target rate R = 1 bit/sec/Hz, the path loss exponent  $\beta = 3$  and the number of cooperative

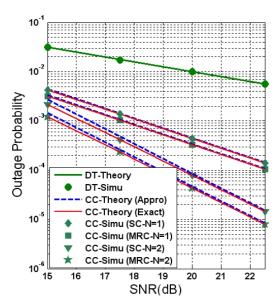


Fig. 2. Outage probability as a function of SNR in dB when R=1 and  $\beta = 3$ .

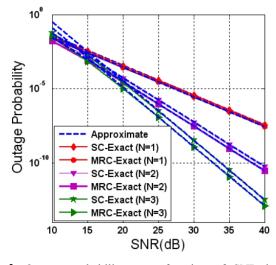


Fig. 3. Outage probability as a function of SNR in dB when R=1 and  $\beta = 4$ .

nodes *N* varying from 0 to 2. In case of N=0 (DT), the source transmits the signal to the destination directly without the help of relays. Note that the simulation and numerical results are in good agreement and the approximate results (dotted lines) are matched very well with the exact results (solid lines) at high SNR region. Furthermore, for all values *N*, the cooperative transmission (CC) outperforms direct transmission (DT). This is due to the fact that cooperative communication achieves higher diversity order than that of the direct communication.

In Fig. 3, the path loss exponent is set to 4, target rate R is set to 1 bit/sec/Hz and the results are plotted by theoretical calculations. We can see from these figures that the performance of systems employing MRC receiver is always better as compared to equivalent systems using SC. Furthermore, the system can obtain full diversity order in the sense that the diversity order is the total number of cooperative nodes N in the network plus 1. This validates the statement of Proposition 1.

# V. Conclusion

In this paper, we evaluated the performance of cooperative communication when relays were placed randomly in one-dimensional networks. We proposed a simple approximation of the average outage probability for the decode-and-forward system to reduce the numerical computations. The numerical and simulated results were presented to demonstrate the validity of the analytical results. The approximate results are also fixed very well with the exact results at high SNR region. Similar to deterministic networks, the cooperative transmission in random networks not only outperforms direct transmission but also achieves higher diversity gain.

## Appendix A

Proof of Equation (26)

At high SNR value, we can approximate  $\exp(-\rho\lambda_{2,i_m})$ in (23) as

$$\exp(-\rho\lambda_{2,i_j}) \approx \sum_{t=0}^{k+1} (-1)^t \frac{\left(\rho\lambda_{2,i_j}\right)^t}{t!}$$
(A.1)

Substituting (A.1) into (23), we have

$$\Pr\left[I_{C_{D}} < R\right] \approx \sum_{m=0}^{k} \sum_{t=1}^{k+1} \frac{\eta_{m}}{\lambda_{2,m}} (-1)^{t-1} \frac{\left(\rho \lambda_{2,i_{j}}\right)^{t}}{t!}$$
$$\approx \sum_{t=1}^{k+1} \frac{\left(-1\right)^{t-1} \rho^{t}}{t!} S(t)$$
(A.2)

where  $S(t) = \sum_{m=0}^{k} \eta_m \left(\lambda_{2,i_m}\right)^t$ . Using  $\eta_m = \lambda_{2,m} \prod_{n=0,n\neq m}^{k} \lambda_{2,n} / \left(\lambda_{2,n} - \lambda_{2,m}\right)$ , (we) rewrite

S(t) as follows:

$$S(t) = \sum_{m=0}^{k} \left( \prod_{n=0,n\neq m}^{k} \frac{\lambda_{2,i_n}}{\lambda_{2,i_n} - \lambda_{2,i_m}} \right) (\lambda_{2,i_m})^{t}$$
  
=  $\frac{\prod_{m=0}^{k} \lambda_{2,i_m}}{\prod_{w=0}^{k-1} \left( \prod_{u=w+1}^{k} \lambda_{2,i_w} - \lambda_{2,i_u} \right)} \sum_{n=0}^{k} (\lambda_{2,i_n})^{t-1} \frac{\prod_{w=0}^{k-1} \left( \prod_{u=w+1}^{k} \lambda_{2,i_w} - \lambda_{2,i_u} \right)}{\prod_{l=0,l\neq n}^{N} \lambda_{2,i_l} - \lambda_{2,i_n}}$   
(A.3)  
In (A.3) considering  $L = \sum_{n=0}^{k} (\lambda_{2,i_n})^{t-1} \frac{\prod_{w=0}^{k-1} \left( \prod_{u=w+1}^{k} \lambda_{2,i_w} - \lambda_{2,i_u} \right)}{\prod_{u=w+1}^{k-1} \left( \prod_{w=w+1}^{k} \lambda_{2,i_w} - \lambda_{2,i_w} \right)}$ 

In (A.3), considering 
$$L = \sum_{n=0}^{\infty} (\lambda_{2,i_n})^{1-1} \frac{w=0}{\sum_{l=0,l\neq n}^{N} \lambda_{2,i_l} - \lambda_{2,i_n}}$$

It is obvious that *L* is a polynomial of  $\lambda_{2,i_j}$  and the order of this polynomial can be determined by  $n = \max(t-1,k-1)$ . We should note that the equation  $L(\lambda_{2,i_j}) = 0$  has k-1 roots  $\lambda_{2,i_j}$ , where  $0 \le i_n, i_m \le k$  and  $i_n \ne i_m$ . Thus, if  $t \le k$ ,  $N(\lambda_{2,i_j}) = 0$  for  $\forall \lambda_{i_j}$ .

If t = k + 1, after some manipulation, we obtain  $S(k+1) = (-1)^k \prod_{k=1}^k \lambda_{2,i_m}$ .

Therefore, from (A.2) and the remarks mentioned above, we have  $\Pr[I_{C_D} < R] \approx \frac{\rho^{k+1}}{(k+1)!} S(k+1) = \frac{\rho^{k+1}}{(k+1)!}$  $\prod_{m=0}^k \lambda_{2,i_m}, \text{ which proves (26).}$  This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2011-0006043).

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