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Decode-and-Forward Relaying Systems with N^{th} Best-Relay Selection over Rayleigh Fading Channels

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Abstract

In this paper, we evaluate performances of dual-hop decode-and-forward relaying systems with the N^{th} best-relay selection scheme. In some schemes, such as scheduling or load balancing schemes, the best relay is unavailable and hence the system must resort the second best, third best, or generally the N^{th} best relay. We derive the expressions of the outage probability and symbol error rate (SER) for this scenario over Rayleigh fading channels. Monte-Carlo simulations are presented to verify the analytical results.

Key words: Cooperative Communication, Decode-and-Forward Relaying, Outage Probability, Rayleigh Fading Channel.

Ⅰ. Introduction

Cooperative communication is well known to improve channel capacity and achieve higher diversity gain under fading environments when compared to direct transmission [1], [2]. Cooperative diversity protocols utilize dual-hop relaying scheme when the source has no direct link to the destination. For amplify-and-forward relaying schemes [3], [4], the relay just amplifies the signal received from the source and forwards the amplified signal to the destination. Decode-and-forward relaying schemes can use either the fixed relaying scheme [5] or the adaptive relaying scheme [6]. In the first strategy, the relay always detects and forwards the received signal to the destination. In contrast, the relay in the second scheme just sends the signal to the destination if it decodes it successfully; otherwise, it keeps silent.

Relay selection is an important issue for the improvement of system performance. Therefore, most of the methods related to relay selection have mainly focused on the best-relay selection. However, in some cases, the system cannot choose the best relay due to some scheduling or load balancing conditions. In [7], [8], S.S. Ikki *et al* have proposed cooperative schemes in which the N^{th} best relay would be used to forward the source's information to the destination.

In this paper, we investigate dual-hop decode-and-forward relaying systems using the N^{th} best-relay selection scheme over Rayleigh fading channels. We first derive the cumulative density function (CDF) and probability density function (PDF) of the N^{th} best order statistics of the equivalent instantaneous signal to noise ratio (SNR). Relying on these distributions, we derive the exact expression of the outage probability for the adaptive relay scheme and the approximate expressions of symbol error rate (SER) for the fixed relay scheme. Monte-Carlo simulations are presented to confirm the derived expressions.

The rest of the paper is organized as follows. The system model description is described in Section Ⅱ and performance analysis is discussed in Section Ⅲ. In Section Ⅳ, we will show the simulation results and Section Ⅴ concludes the paper.

Ⅱ. System Model Description

As shown in Fig. 1, we assume that no direct link exists between the source and the destination; hence the source's data must be sent to the destination via a relay. We also assume that M relays are available for cooperation. We further assume that all nodes are equipped with a single antenna and operate in a half-duplex mode.

The data received by relay R_i, $i \in \{1,2,...,M\}$, and the destination D due to the transmission of the source and relay R_i can be respectively presented as follows:

$$y_{SR_i} = \sqrt{P} h_{SR_i} s_1 + n_{R_i}, \tag{1}$$

$$y_{R_iD} = \sqrt{P} h_{R_iD} s_2 + n_D. \tag{2}$$

where s_1 and s_2 are signals transmitted by S and relay

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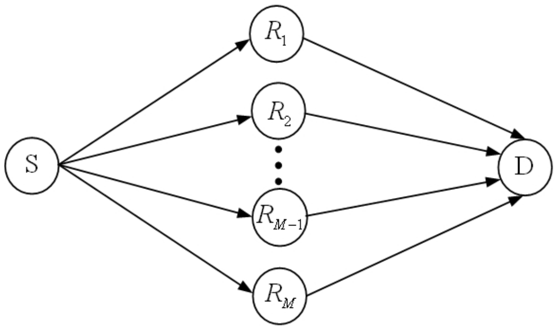

Fig. 1. Dual-hop decode-and-forward relaying scheme with M relays.

R_i, respectively, P is transmit power of the source and relay R_i, h_{SR_i} and h_{R_iD} are Rayleigh channel coefficients between the source and relay R_i and between relay R_i and the destination, respectively, n_{R_i} and n_D are zero-mean additive white Gaussian noise (AWGN) with variance σ_0^2 at relay R_i and the destination, respectively.

From (1), (2), the instantaneous received SNR at relay R_i and the destination are respectively calculated by

$$y_{1i} = \overline{\gamma} \, | h_{SR_i} |^2 , \tag{3}$$

$$y_{2i} = \overline{\gamma} \, | h_{R_iD} |^2 . \tag{4}$$

where $\overline{\gamma} = P / \sigma_0^2$ is the transmit SNR.

Since h_{SR_i} and h_{R_iD} are identical independent Rayleigh distributed random variables (RVs), the cumulative density function (CDF) of $| h_{SR_i} |^2$ and $| h_{R_iD} |^2$ can be respectively expressed as

$$F_{|h_{SR_i}|^2}(x) = 1 - \exp\left(-\Omega_1 x\right), \tag{5}$$

$$F_{|h_{R_iD}|^2}(x) = 1 - \exp\left(-\Omega_2 x\right). \tag{6}$$

where Ω_1 and Ω_2 are expected values of $| h_{SR_i} |^2$ and $| h_{R_iD} |^2$, respectively.

From (3)~(6), the CDF of γ_{1i} and γ_{2i} can be respectively given by

$$F_{\gamma_{1i}}(x) = 1 - \exp\left(-\lambda_1 x\right), \tag{7}$$

$$F_{\gamma_{2i}}(x) = 1 - \exp\left(-\lambda_2 x\right). \tag{8}$$

where $\lambda_1 = \Omega_1 / \overline{\gamma}$ and $\lambda_2 = \Omega_2 / \overline{\gamma}$.

Setting $\gamma_i = \min\left(\gamma_{1i}, \gamma_{2i}\right)$, due to the independence of γ_{1i} and γ_{2i}, the CDF of γ_i is given by [5]

$$\begin{aligned} F_{\gamma_i}\left(x\right) &= \Pr\left[\min\left(\gamma_{1i}, \gamma_{2i}\right) < x\right] \\ &= 1 - \left(1 - F_{\gamma_{1i}}\left(x\right)\right)\left(1 - F_{\gamma_{2i}}\left(x\right)\right). \\ &= 1 - \exp\left(-\left(\lambda_1 + \lambda_2\right) x\right) \end{aligned} \tag{9}$$

In this paper, the relay having the N^{th} largest value of γ_i would be chosen to transmit the data. The corresponding order statistics are then obtained by arranging random variables γ_i in a non-increasing order, denoted by $X_1, X_2, ..., X_{M-1}, X_M$, where X_1 has the largest value, X_M has the smallest value, and X_N has the N^{th} largest value. Assuming that γ_1, γ_2, $\cdots$, γ_M are independent and identical random variables, the CDF of the N^{th} order statistic is given by [7, eq.(22)]:

$$F_{X_N}\left(x\right) = \sum_{j=1}^{N} C_M^{j-1} \left[F_{\gamma_i}\left(x\right)\right]^{M-j+1} \left[1 - F_{\gamma_i}\left(x\right)\right]^{j-1}, \tag{10}$$

where $C_b^a = \dfrac{b!}{a!\left(b-a\right)!}$ with a and b to be non-negative integers and $a \le b$.

Substituting (9) into (10), we get the CDF $F_{X_N}\left(x\right)$ as follows:

$$F_{X_N}\left(x\right) = \sum_{j=1}^{N} \sum_{i=0}^{M-j+1} \left[\begin{array}{l} \left(-1\right)^i C_M^{j-1} C_{M-j+1}^i \\ \times \exp\left(-\left(i+j-1\right)\left(\lambda_1 + \lambda_2\right) x\right) \end{array}\right]. \tag{11}$$

From (11), we get the probability density function (PDF) $f_{X_N}\left(x\right)$ as

$$f_{X_N}\left(x\right) = \sum_{j=1}^{N} \sum_{i=0}^{M-j+1} \left[\begin{array}{l} \left(-1\right)^{i+1} C_M^{j-1} C_{M-j+1}^i \left(i+j-1\right) \\ \left(\lambda_1 + \lambda_2\right) \exp\left(-\left(i+j-1\right)\left(\lambda_1 + \lambda_2\right) x\right) \end{array}\right]. \tag{12}$$

Ⅲ. Performance Evaluation

3-1 Outage Probability

We assume that each node (including relays and the destination) decodes the data successfully if the instantaneous received SNR at this node is larger than a predetermined threshold, γ_{th}. If the chosen relay, e.g., R_c, decodes the source' signal successfully, it will forward the decoded signal to the destination. Otherwise, it keeps silent. Therefore, the outage probability in this case can be formulated as

$$\begin{aligned} P_{out} &= \Pr\left[\gamma_{1c} < \gamma_{th}\right] \Pr\left[\gamma_{2c} < \gamma_{th}\right] \\ &= \Pr\left[\min\left(\gamma_{1c}, \gamma_{2c}\right) < \gamma_{th}\right] = \Pr\left[\gamma_c < \gamma_{th}\right] \end{aligned}$$

$$= F_{X_N}\left(\gamma_{th}\right) \tag{13}$$

3-2 Symbol Error Rate

In fixed decode-and-forward relaying scheme, the equivalent SNR for two hops of each relay can be approximated tightly as [5]

$$\gamma_{eq,i} \approx \min\left(\gamma_{1i}, \gamma_{2i}\right) = \gamma_i \tag{14}$$

For V-ary PAM, by averaging the SER in AWGN channel [9, Eq. (5.2.45)] over the PDF $f_{X_N}(x)$, the SER over Rayleigh fading channel can be calculated approximately by

$$\overline{\mathrm{SER}}_{\mathrm{PAM}} \approx \int_0^{+\infty} \frac{2(V-1)}{V} Q\left(\sqrt{\frac{6x}{V^2-1}}\right) f_{X_N}(x)dx \tag{15}$$

where $Q(.)$ is Q-function defined in [9].

With the help of Appendix A, the average SER in (15) can be calculated approximately by

$$\overline{\mathrm{SER}}_{\mathrm{PAM}} \approx \sum_{j=1}^{N} \sum_{i=0, i+j\neq 1}^{M-j+1} (-1)^{i+1} \frac{(V-1)}{V} C_M^{j-1} C_{M-j+1}^{i} \times \left(1 - \sqrt{\frac{3}{3+\left(V^2-1\right)(i+j-1)\left(\lambda_1+\lambda_2\right)}}\right) \tag{16}$$

For V-ary PSK (V>4), utilizing [9, Eq. (5.2.61)], the average SER over the distribution of the instantaneous SNR can be approximated by

$$\overline{SER}_{\mathrm{PSK}} \approx \int_0^{+\infty} 2Q\left(\sqrt{2\sin^2\left(\frac{\pi}{V}\right)x}\right) f_{X_N}(x)dx \tag{17}$$

Substituting (12) into (17), similar to (16), we also obtain

$$\overline{\mathrm{SER}}_{\mathrm{PSK}} \approx \sum_{j=1}^{N} \sum_{i=0, i+j\neq 1}^{M-j+1} (-1)^{i+1} C_M^{j-1} C_{M-j+1}^{i} \times \left(1 - \sqrt{\frac{\sin^2(\pi/V)}{\sin^2(\pi/V)+(i+j-1)\left(\lambda_1+\lambda_2\right)}}\right) \tag{18}$$

For square V-ary QAM, similar as above, the average SER over Rayleigh fading channel is calculated as

$$\overline{SER}_{\mathrm{QAM}} \approx \int_0^{+\infty} \left[\begin{array}{l} \frac{4\left(\sqrt{V}-1\right)}{\sqrt{V}} Q\left(\sqrt{\frac{3x}{V-1}}\right) \\ -\frac{4\left(\sqrt{V}-1\right)^2}{V} Q^2\left(\sqrt{\frac{3x}{V-1}}\right) \end{array}\right] f_{X_N}(x)dx \tag{19}$$

Substituting (12) into (19) and with the help of Appendices A and B, the high transmit SNR approximation of SER is calculated by

$$\overline{SER}_{\mathrm{QAM}} \approx \sum_{j=1}^{N} \sum_{i=0}^{M-j+1} (-1)^{i+1} C_M^{j-1} C_{M-j+1}^{i} \times \left[\begin{array}{l} \frac{2\left(\sqrt{V}-1\right)}{\sqrt{V}}\left(1-\sqrt{\phi}\right) - \\ \frac{\left(\sqrt{V}-1\right)^2}{V}\left(1-\frac{4}{\pi}\right)\sqrt{\phi}\tan^{-1}\left(\frac{1}{\phi}\right) \end{array}\right] \tag{20}$$

where $\phi = \dfrac{3}{3+2(M-1)\left(\lambda_1+\lambda_2\right)}$.

Ⅳ. Simulation Results

In this section, we provide some numerical results that have been developed in Section III and we verify these results with Monte-Carlo simulations. In all simulations, we assume that the transmit power of the source and relays are same and the expected values Ω_1 and Ω_2 equal to 1.

Fig. 2 shows the outage probability of the system as a function of the transmit SNR $\bar{\gamma}$ in dB. In this simulation, we assign the value of M and γ_{th} as 3 and 2, respectively, while the value of N varies from 1 to 3. As shown in Fig. 2, the outage performance is smallest when the best relay (N=1) is chosen. This is due to the fact that the obtained diversity order decreases with the increasing of N.

In Fig. 3, we investigate the effect of threshold γ_{th}

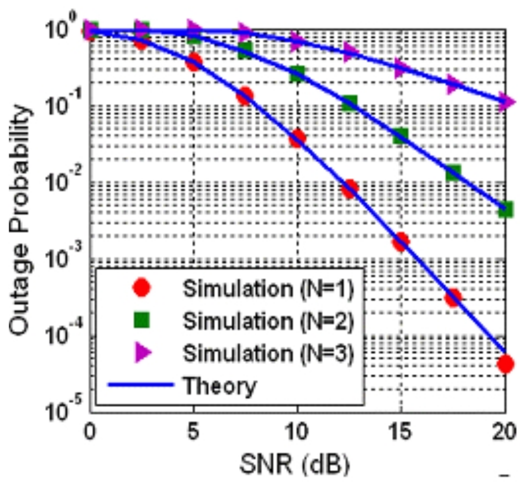

Fig. 2. Outage probability as a function of the transmit SNR $(\bar{\gamma})$ in dB when M=3 and $\gamma_{th} = 2$.

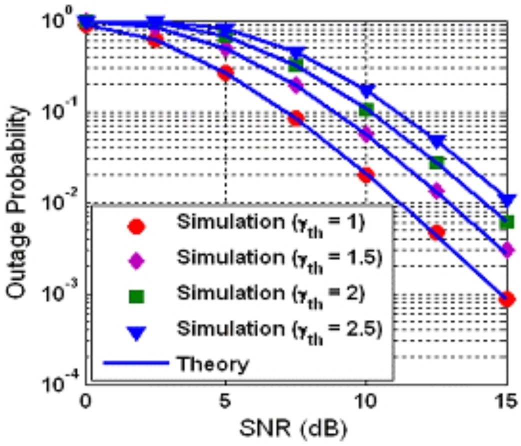

Fig. 3. Outage probability as a function of the transmit SNR $(\bar{\gamma})$ in dB when M=4 and N=2.

on the outage performance. In this figure, we assume that M=4 and N=2. As expected, the outage performance decreases with decreasing γ_{th}. Figs. 2~3 show that the theoretical and simulation results are in good agreement, which verifies the accuracy of our analyses.

In Fig. 4, we investigate the average SER performance when different modulation techniques are used. In this figure, the number of relays is fixed at 3 and the second best relay is chosen for the cooperation. The performance is best when 16-QAM is used and the worst performance is obtained when 16-PAM is used. In addition, the theoretical and simulation results match well at high SNR region. The small gap between them comes from the approximate method which is made by (14).

Ⅴ. Conclusion

In this paper, we evaluate the performance of decode-and-forward relaying scheme with N^{th} best relay selection by deriving expressions of outage probability and symbol error rate over Rayleigh fading channels. Monte Carlo simulations are also presented to verify our derivations. The simulation results show that the simulation and theoretical results match very well at high signal to noise ratio (SNR).

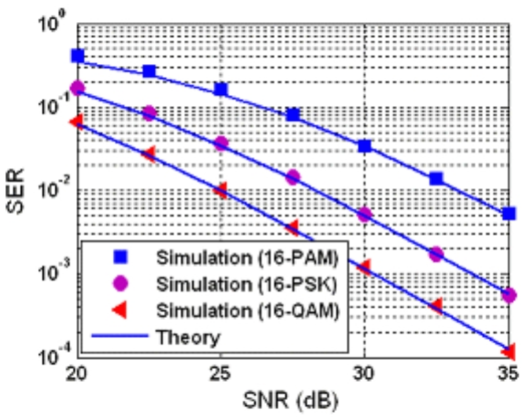

Fig. 4. Average SER as a function of the transmit SNR $(\bar{\gamma})$ in dB when M=3 and N=2.

Appendix A

Considering the following integral:

$$A(a,b,c)=\int_0^{+\infty} x^a \exp(-bx)Q\left(\sqrt{cx}\right)dx \tag{A.1}$$

Substituting the Q function [10, Eq. (4.2)], $Q(x)=\frac{1}{\pi}\int_0^{\pi/2}\exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta$ into (A.1), and after some manipulation, we have

$$A(a,b,c)=\frac{\Gamma(a+1)}{\pi b^{1+a}}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta+c/2b}\right)^{1+a}d\theta \tag{A.2}$$

In case that a=0, after some manipulation, $A(0,b,c)$ can be calculated as

$$A(0,b,c)=\frac{1}{2b}\left(1-\sqrt{\frac{c/2b}{1+c/2b}}\right) \tag{A.3}$$

Appendix B

Considering the following integral:

$$B(a,b,c)=\int_0^{+\infty} x^a \exp(-bx)Q^2\left(\sqrt{cx}\right)dx \tag{B.1}$$

Substituting the Q^2 function [10, Eq. (4.9)], $Q^2(x)=\frac{1}{\pi}\int_0^{\pi/4}\exp\left(-\frac{x^2}{2\sin^2\theta}\right)d\theta$ into (B.1), and after some manipulation, we have

$$B(a,b,c)=\frac{\Gamma(a+1)}{\pi b^{1+a}}\int_0^{\pi/4}\left(\frac{\sin^2\theta}{\sin^2\theta+c/2b}\right)^{1+a}d\theta \tag{B.2}$$

For case of a=0, substituting [10, Eq. (5A.13)] into (B.2) yields

$$B(0,b,c)=\frac{1}{4b}\left[1-\frac{c/2b}{1+c/2b}\left(\frac{4}{\pi}\tan^{-1}\sqrt{\frac{1+c/2b}{c/2b}}\right)\right] \tag{B.3}$$

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