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Discrete-Time Sliding Mode Control with SIIM Fuzzy Adaptive Switching Gain

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Abstract

This paper focuses on discrete-time sliding mode control with SIIM fuzzy adaptive switching gain. The adaptive switching gain is calculated using the simplified indirect inference fuzzy logic. Two fuzzy inputs are the normal distance from the present state trajectory to the switching function and the distance from the present state trajectory to the equilibrium state. The fuzzy output $f_{out}(k)$ is used to adjust the speed the adaptation law depending on the location of the state trajectory. The simulation results showed that the proposed method had no chattering in case of uncertain parameter without disturbance. Moreover the convergent rate of the switching gain was faster and more stable even in case of disturbance.

Keywords: adaptive switching gain, discrete-time, sliding mode control, SIIM fuzzy.

1. INTRODUCTION

Continuous-time sliding mode control (SMC) have been applied to various applications successfully [1]. However, almost all of continuous-time SMC systems are in fact computer-controlled. In these cases, information or measurements about the system are only available at specific time instances and control inputs can only be changed at the time instants. So there is need to convert them into their discrete-time SMC version. It was pointed out that discrete-time SMC system cannot be obtained from their continuous counterpart by means of simple equivalence [2].

Dote and Hoft [3] first designed the discrete-time SMC with a discrete reaching law. Furuta [4] used a Lyapunov-type discrete reaching law which drove the system state to an appropriately determined sector in the state space. Gao et al. [5] defined the concept of the discrete-time quasi-sliding mode control, quasi-sliding mode band, and specified three rules. And they proposed the reaching law which guaranteed the quasi-sliding mode band. Their algorithms can be easily extended to multi-variable systems.

Bartoszewicz [6] redefined the quasi-sliding mode control and suggested new reaching law. His linear reaching law relaxed the second restriction of Gao's and the system showed no longer chattering phenomena. Hui et al. [2] also proposed the linear reaching law, which showed an exact description of the desired trajectories toward the sliding surface.

However, Gao's reaching law and all the linear reaching laws have a quasi-sliding mode band and these lead to excessively large control inputs.

The typical approach to circumvent the excessively large control inputs is to bound the reaching law [7] or to apply the adaptive switching gain [8,9]. Adaptive switching gain methods in the field of continuous-time SMC, which is different from the model adaptive sliding mode control techniques, have been received quite a bit of attention as well. Monsees et al. [10] pointed out that the above methods [8,9] lead to an unstable closed-loop system because of monotonically increasing the switching gain or improper choice of design parameters.

Recently, fuzzy logic algorithm seemed to work well to enhance the performance of the sliding mode control systems [11,12,13].

In this paper, we proposed discrete-time SMC using simplified indirect inference method (SIIM) fuzzy adaptive switching gain. Two fuzzy inputs are the normal distance from the present state trajectory to the switching function and the distance from the present state trajectory to the equilibrium state. The simulation results showed that the proposed method had no chattering in case of uncertain parameter without disturbance. Moreover the convergent rate of the switching gain was faster and more stable than that of Monsees's in case of disturbance.

The paper is organized as follows. In section II, the discrete-time SMC system with SIIM fuzzy adaptive switching gain is derived. Section III describes the SIIM fuzzy logic controller for the adaptive switching gain. The simulated results using the proposed techniques are given in section IV, and finally section V gives the concluding remarks.

2. DISCRETE-TIME SMC USING SIIM FUZZY ADAPTIVE SWITCHING GAIN

Consider the discrete-time system given in regular form

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$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u(k) + \begin{bmatrix} f_u(k) \\ f_m(k) \end{bmatrix} \quad (1)$$

where $x_1(k) \in \Re^{n-m}, x_2(k) \in \Re^m$, and $B_2 \in \Re^{m\times n}$ has full rank. The term $f_u(k)$ is the unmatched uncertainty which is in the null space of $B, f_m(k)$ is the matched uncertainty since it is in the range of B.

The switching function is given by

$$s(k) = S_1 x_1(k) + S_2 x_2(k) \quad (2)$$

If we assume that the closed-loop system is in the ideal quasi-sliding mode $s(k) \equiv 0$ when the unknown term $f_u(k)$ is zero, then the system is written with equation (1) and (2)

$$x_1(k+1) = (A_{11} - A_{12} S_2^{-1} S_1) x_1(k) \quad (3)$$

where the matrix $\begin{bmatrix} S_1 & S_2 \end{bmatrix}$ should be chosen such that the matrix $(A_{11} - A_{12} S_2^{-1} S_1)$ is stable, in which the characteristics can be chosen by the pole placement theory or LGQ theory. We choose S_2 such that $S_2B_2=I_m$, where I_m is the identity matrix of size m.

By applying the invertible coordinate transformation $T_s \in \Re^{n\times m}$,

$$\begin{bmatrix} x_1(k) \\ s(k) \end{bmatrix} = T_s \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \quad (4)$$

the system (4) becomes

$$\begin{bmatrix} x_1(k+1) \\ s(k+1) \end{bmatrix} = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ \overline{A}_{21} & \overline{A}_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ s(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 1 & 0 \\ S_1 & S_2 \end{bmatrix} \begin{bmatrix} f_u(k) \\ f_m(k) \end{bmatrix} \quad (5)$$

where $\overline{A}_{11} = (A_{11} - A_{12} S_2^{-1} S_1)$, $\overline{A}_{12} = (A_{12} S_2^{-1})$, $\overline{A}_{21} = (S_1 A_{11} + S_2(A_{21} - A_{22} S_2^{-1} S_1))$, and $\overline{A}_{22} = (S_1 A_{12} S_2^{-1} + S_2 A_{22} S_2^{-1})$.

Now we choose the reaching law

$$s_i(k+1) = \phi_i s_i(k) - K_{s_i} sign(s_i(k)) + f_i(k) \quad (6)$$

where the subscript $i = 1, \cdots, m$ denotes the ith entry of each correspondent vector, $\phi \in \Re^{m\times n}$ is a diagonal design matrix with diagonal entries $0 \le \phi_i < 1$, K_{s_i} is the switching gain, and $f_i(k) = S_2 f_m(k) + S_1 f_u(k)$.

According to Gao et al. [5], the reaching law should hold and have the signum term to guarantee three attributes. The reaching law (6) satisfies their demands, so the trajectory will move monotonically toward the switching hyper-surface and cross it in finite time and remains within the quasi-sliding mode band.

Substituting the reaching law (6) into the lower part of equation (5) leads to the control law as

$$u(k) = -\overline{A}_{21} x_1(k) - (\overline{A}_{22} - \phi) s(k) + u_d(k) \quad (7)$$

where $$u_d(k) = -\begin{bmatrix} K_{s_1} sign(s_1(k)) \\ \vdots \\ K_{s_m} sign(s_m(k)) \end{bmatrix} \quad (8)$$

It is well known [6] that all the attributes which were proposed by Gao et al., are satisfied if

$$K_{s_i} > \frac{1+\phi_i}{1-\phi_i} F_i \quad (9)$$

where we assume that the disturbance $f_i(k)$ is bounded by $\|f_i(k)\| < F_{i.}$

If the condition (9) is met, then the system will converge in finite time to the quasi-sliding mode band Δ given by

$$\Delta_i = K_{s_i} + F_i \quad (10)$$

The equation (10) clearly shows that the quasi-sliding mode band is a function of the switching gain. So to make the switching gain as small as possible unless the condition (9) is not violated, it needed to make the quasi-sliding mode band Δ_i as small as possible.

Therefore, the SIIM fuzzy adaptation law for switching gain should have the following function: The adaptation law increases the switching gain if the state trajectory did not cross the switching hyper-surface in order to reduce the time of the reaching mode, and decreases the switching gain in order to make the quasi-sliding mode band smaller if the state trajectory crossed the switching hyper-space [5].

We choose the following the adaptation law (12) to implement the above condition in place of the Monsees' adaptation law (11) [10].

$$\tilde{K}(k) = |\tilde{K}(k-1) + \gamma sign(s_i(k)) sign(s_i(k-1))| \quad (11)$$

$$\tilde{K}_{s_i}(k) = |\tilde{K}_{s_i}(k-1) + f_{out}(k) sign(s_i(k)) sign(s_i(k-1))| \quad (12)$$

where $f_{out}(k)$ is the output of fuzzy controller.

In (11) and (12), the term $sign(s_i(k))sign(s_i(k-1))$ is +1 if the system did not transpierce the switching hyper-surface, so the adaptation law increases the switching gain. The term $sign(s_i(k))sign(s_i(k-1))$ is -1 if the system transpierce the switching hyper-surface, so the adaptation law decreases the switching gain.

The difference is as follows. In (11), the adaptation gain would be increased unceasingly when the term $sign(s_i(k))sign(s_i(k-1))$ is not zero. In order to circumvent the problem, Monsees et al. proposed the time-dependent adaptation constant using the exponential function of the sampling time. But it had the drawback that it is possible that the adaptation constant is decreased too fast. They proposed another method, so called goal-dependent adaptation constant. But it needed to monitor the gain through the whole simulation period. We choose the fuzzy input $d_i(k)$ as the normal distance from the present state to the sliding hyper-surface and the distance from the present state to the equilibrium point. The fuzzy output $f_{out}(k)$ has the ability to adjust the speed the adaptation law depending on the location of the state trajectory. Each fuzzy input variable is normalized by the initial distance individually.

3. SIIM FUZZY LOGIC CONTROL

The simplified indirect fuzzy inference method proposed by Sugeno is a simplification of the indirect fuzzy inference method proposed by Tsukamoto. It has the advantages it is simple and easy to apply to.

We use monotone membership functions. The linear function given as Fig. 1, is used in the antecedent part, and the nonlinear function given as Fig. 2, is used in the consequent part. In Fig. 1 and Fig. 2, L is the discourse universe of the fuzzy set, where we can usually choose it as the unit because we normalize the input and output variables.

Our antecedent and consequent parts of fuzzy set are different with Hayashi et al. [14], because arctan function chosen by Hayashi et al., does not describe the discourse universe properly. Our method has better performance than their method [15].

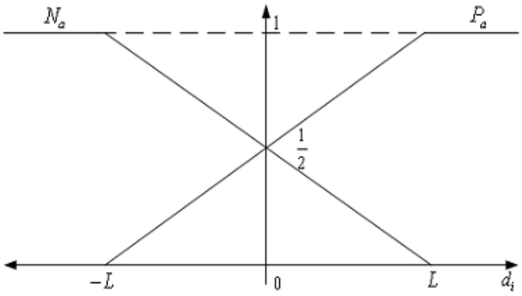


Fig. 1 The antecedent part of the fuzzy set

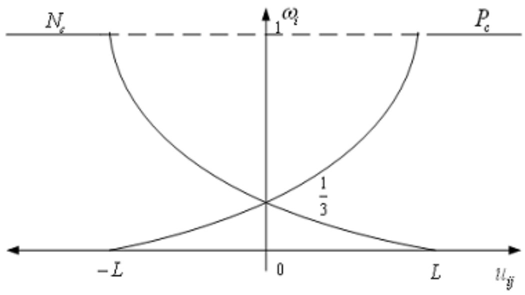


Fig. 2 The consequent part of the fuzzy set

We consider the fuzzy control rules consisting of the following two simple contrasting rules for each fuzzy input $d_i(k)$.

Rule 1: If $d_i(k)$ is P_{a_i}, then $u_{i_1}(k)$ is $P_{c_i}^{-1}$.

Rule 2: If $d_i(k)$ is N_{a_i}, then $u_{i_2}(k)$ is $N_{c_i}^{-1}$.

When a crisp value $d_i(k)$ is the input of fuzzy controller, the matching degrees in the antecedent part of each control rule are $\omega_{i_1}(k)$ and $\omega_{i_2}(k)$.

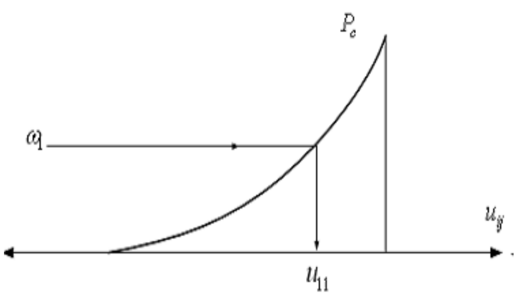
$$\begin{aligned} \omega_{i_1}(k) &= P_{a_i}(d_i(k)) = (d_i(k)+L)/L \\ \omega_{i_2}(k) &= N_{a_i}(d_i(k)) = (-d_i(k)+L)/L \end{aligned} \tag{13}$$

where $\omega_{i_1}(k)+\omega_{i_2}(k)=1$.

Because P_c and N_c chosen by (14) are one-to-one functions, the inference results $u_{i_1}(k)$ and $u_{i_2}(k)$ are computed by using (15) as shown Fig. 3, in the consequent part of each control rule.

$$\begin{aligned} P_c &= 4.5-\sqrt{21.25-(d_i(k)/L+2)^2} \\ N_c &= 4.5-\sqrt{21.25-(d_i(k)/L-2)^2} \end{aligned} \tag{14}$$

$$\begin{aligned} u_{i_1}(k) &= P_{c_i}(\omega_{i_1}(k)) \\ &= L\times\left[-2+\sqrt{21.25-(\omega_{i_1}(k)-4.5)^2}\right] \\ u_{i_2}(k) &= N_{c_i}(\omega_{i_2}(k)) \\ &= L\times\left[2-\sqrt{21.25-(\omega_{i_2}(k)-4.5)^2}\right] \end{aligned} \tag{15}$$



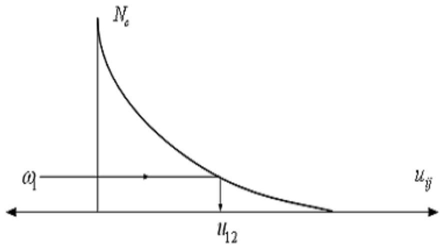


Fig. 3 The inference results $u_{i_1}(k)$ and $u_{i_2}(k)$

The overall inference results $O(k)$ and $f_{out}(k)$ are obtained by using the weighted sum mean as in (16) and (17).

$$O(k) = \frac{\sum_{i=1}^{n}(\omega_{i_1}(k)u_{i_1}(k) + \omega_{i_2}(k)u_{i_2}(k))}{\sum_{i=1}^{n}(\omega_{i_1}(k) + \omega_{i_2}(k))} \tag{16}$$

$$f_{out}(k) = \eta O(k) \tag{17}$$

where η is the fuzzy output scaling factor.

4. SIMULATION EXAMPLES

Consider a second order system [6] given as in regular form

$$\begin{aligned} x_1(k+1) &= 1.2x_1(k) + 0.1x_2(k) \\ x_2(k+1) &= \sigma x_1(k) + 0.6x_2(k) + u(k) + f(k) \end{aligned} \tag{18}$$

where σ is an uncertain parameter.

Let the switching function be

$$s(k) = c_1 x_1(k) + c_2 x_2(k) \tag{19}$$

where $c_1 = 5.0$ and $c_2 = 1.0$.

We first consider the system with uncertain parameter σ, which σ is changed from -1 to 1, or vice versa every 5 sampling period (in case of $f(k) = 0$).

Setting $x_1(0) = 2.0,\ s(0) = 4.0,\ qT = 0.25,$ and $\varepsilon T = 0.5$ for the Gao's [5] discrete-time SMC, the response of $x_1(k)$ and $s(k)$ are shown as Fig. 4.

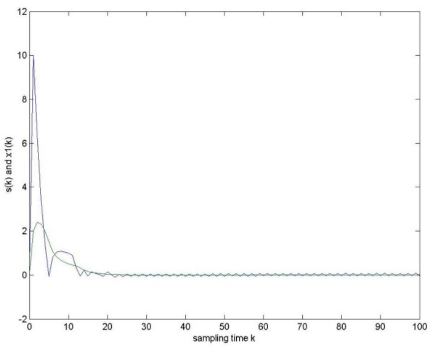


Fig. 4 Gao's results of $x_1(k)$ and $s(k)$

The simulation results using our design technique with the initial switching gain and the adaptation gain are shown in Fig. 5. In Fig.4, there is a chattering in the quasi-sliding mode, but there is no chattering in the proposed design technique.

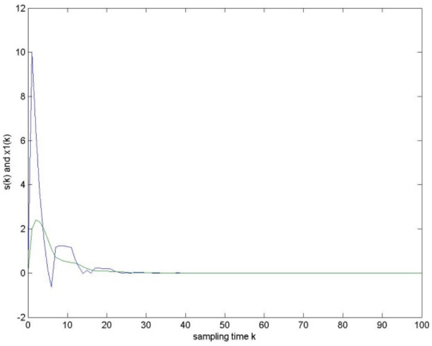


Fig. 5 The proposed results of $x_1(k)$ and $s(k)$

Now we consider the system with disturbance $f(k)$ (in case of $\sigma = 0$).

$$f(k) = \begin{bmatrix} 0 \\ 0.2\sin(k/4\pi) \end{bmatrix} \tag{20}$$

Setting the initial switching gain $K_s(0) = 0$ and the fuzzy output scaling factor $\eta = 0.001$, the response of $x_1(k)$ and $s(k)$ are shown as Fig. 6. The simulation result of the adaptive switching gain for Monsees's is shown in Fig. 7, and the proposed results with the same parameter as the above are shown in Fig. 8 and Fig. 9.

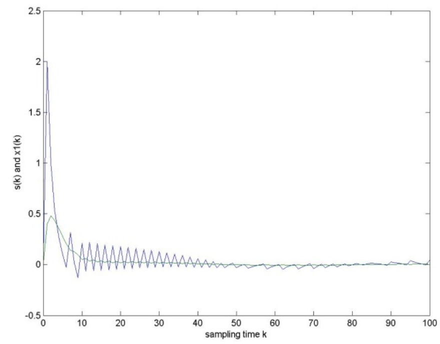


Fig. 6 Monsees's results of $x_1(k)$ and $s(k)$

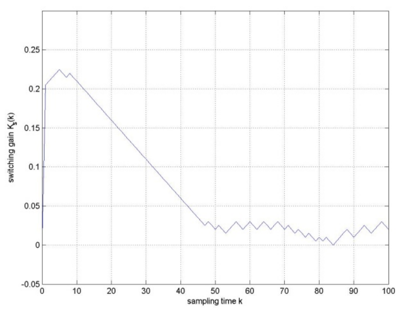


Fig. 7 Monsees's adaptive switching gain

In Fig.6, chattering is apparent at the first stage of the quasi-sliding mode, but it is decreasing as the switching gain approaches to the equilibrium state. In Fig.7, the adaptive switching gain shows somewhat to follow the period of the disturbance.

In Fig.8, the proposed design technique shows that the system enters the quasi-sliding mode earlier owing to a larger switching gain than that of Monsees's. Chattering is apparent at the first stage of the quasi-sliding mode, but it is decreasing exponentially as the switching gain approaches to the equilibrium state. In Fig.9, the switching gain shows somewhat to follow the period of the disturbance, but it is more moderate than the Monsees's result.

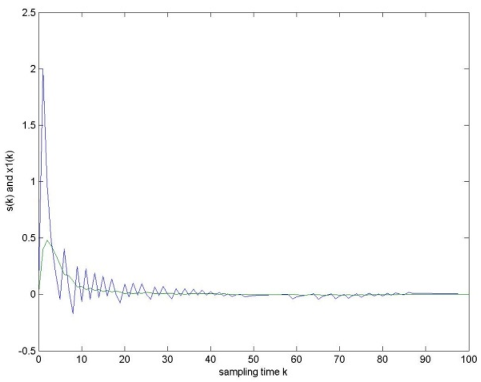


Fig. 8 The proposed results of $x_1(k)$ and $s(k)$

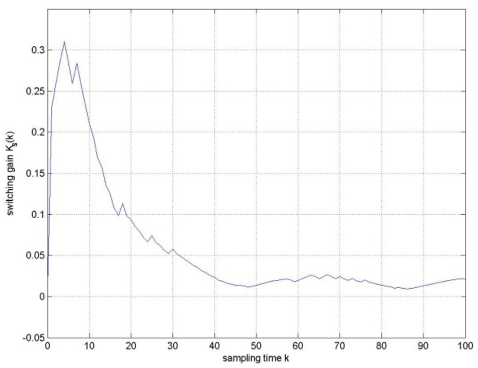


Fig. 9 The proposed adaptive switching gain

5. CONCLUSIONS

This paper focuses on discrete-time sliding mode control with SIIM fuzzy adaptive switching gain. We used the simplified indirect inference fuzzy logic to calculate the adaptive switching gain using two fuzzy inputs, one is the normal distance from the present state trajectory to the switching function and another is the distance from the present state trajectory to the equilibrium state. Simulation results for the nonlinear systems with uncertainty or disturbances showed that the propose method had no chattering in case of uncertain parameter without disturbance. Moreover the convergent rate of the switching gain was faster and more stable than that of Monsees's.

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