

Warping 조건하에서 박판 폐단면 보의 동적 모드 해석

Dynamic Mode Analysis of Thin Walled Closed Section Beams under Warping Conditions

유환신*, 천동준**

Hwan-Shin Yu*, Dong-Joon Chun**

요 약

박판 폐단면을 갖는 프레임과 같은 보가 warping 조건하에서 동적 거동 특성이 어떠한가를 시뮬레이션과 시험을 통하여 제시하였다. 비틀림 모멘트를 받는 보의 단면은 비틀림 뿐만 아니라 warping에 의해서 변형을 일으키게 된다. 어떤 박판 단면을 갖는 보에서는 warping이 매우 커서 축방향과 전단방향 응력을 발생하고 보의 비틀림에 저항이 크게 된다. 이 논문에서는 유한요소에서 warping restraint factor가 보의 변형 거동과 동적 모드에 미치는 영향을 살펴본다. 유한 보 요소와 박판 요소 모델을 사용하여 정적변형과 고유주파수 및 모드해석을 시뮬레이션하며 이 결과를 시험 결과와 비교한다.

Abstract

A dynamic simulation and test of frame with thin walled closed section beams considering warping conditions have been performed. When a beam is subjected under torsional moment, the cross section will deform an warping as well as twist. For some thin-walled sections warping will be large, and accompanying warping restraint will induce axial and shear stresses and reduce the twist of beam which stiffens the beam in torsion. This paper presents that an warping restraint factor in finite element model effects the behavior of beam deformation and dynamic mode shape. The computer modelling of frame is discussed in linear beam element model and linear thin shell element model, also presents a correlation between computer predicted and actual experimental results for static deflection, natural frequencies and mode shapes of frame.

Key words : mode analysis(해석방법), warping restraint factor(와핑구속변수), beam element(보요소), thin shell element(박판요소)

I. Introduction

A structure welded in sub-frames with closed section beams are mostly used in automotive frames demanding high strength and stiffness. So it is great important to

establish the secure frame having high torsional strength as well as bending strength in the viewpoint of safety and cost advantages having thin and light frame materials. Generally in simple beam theory it is assumed that shear center axis and torsional center axis

* 호원대학교 자동차기계공학과(Howon University)

** 벽성대학 자동차과(Byuksung College)

· 제1저자('주저자' First Author): 유환신

· 교신저자(Correspondent Author): 천동준

· 투고일자 : 2012년 3월 12일

· 심사(수정)일자 : 2012년 3월 14일 (수정일자 : 2012년 4월 25일)

· 게재일자 : 2012년 4월 30일

are identical. When a beam is subjected to a torsional moment and the cross section is not symmetrical, the cross section will deform out of plane, which is called warping, as well as twist.[1]-[4]. For the modal and stress analysis of frames which closed section cross members, the compatibility of any warping displacements in the cross member and rate of twist in the side member has to be ensured at the joints. Even when warping free sections are used, any non-zero twisting strain of the beam to which they are joined at a node has to be taken account. The additional displacements arising from the deformation of the cross section can be added to the displacements assumed for open sections and lateral bimoment added to internal loads. The load-displacements for torsion in closed sections are more complex in applying to all closed section.

A beam finite element model with different warping restraint factors are discussed comparing test results.[5]. In this paper through use of structural dynamic behaviour comparing thin shell finite element and beam element model, comprehensive and detailed structural description of technique are described. Also presented is a correlation between computer codes predicted results and actual test results for static results, natural frequencies and mode shapes of frame.

II. Dynamic Mode Analysis

2-1 Mode Analysis

We makes use of a mixed mode of solution for the analysis of structural systems. A direct stiffness matrix approach is employed for the solution of the system joint displacements. The system matrix equation can be written as,

$$F = [(K) - \omega^2 (M)] X \quad (1)$$

or equivalently

$$F = [D] X \quad (2)$$

where, F : column matrix of external forces applied to the joints of the structure

K : square static stiffness matrix

M : diagonal mass matrix

ω : harmonic frequency of the excitation force

X : column matrix of displacements occurring at the joints of the structures

D : dynamic stiffness matrix

To formulate the dynamic stiffness for each span which is made up of number of span segments and lumped masses, a recurrence matrix or transfer matrix approach is used. The recurrence matrix method is described by the following matrix equation,

$$Z_{i+1} = [U]_i Z_i \quad (3)$$

where, Z_{i+1} , Z_i : state vectors at stations $i+1$ and i along the span beams. Stations are designated at the segment ends.

$[U]_i$ is the transfer matrix which relates the state vector at station $i+1$ to that at station i as a function of excitation frequency of the structure.

As many times as the number of segments existing for a given span, equation (3) can be recognized to yield a dynamic stiffness matrix for the entire span.

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \quad (4)$$

where, F_1 , F_2 : force vectors for the fore and after end span joints, K_{11} , K_{12} , K_{21} , K_{22} : elements of the partitioned stiffness matrix, X_1 , X_2 : displacements vectors for the fore and after end span joints

2-2 Warping Restraint

When a beam is subjected to a torsional moment and the cross section is not circular, the cross section will deform out of plane, which is called warp w , as well as twist Θ in Fig. 1. In Saint Venant torsion¹⁾ this warping is free to occur and thus the equation for torsion can be written as,

$$\Theta(x) = \frac{Tx}{GK} \quad (5)$$

or
$$\frac{d\Theta}{dx} = \frac{T}{GK} \quad (6)$$

where Θ : angle of twist
 T : applied torque
 x : distance along the beam
 K : torsional constant
 G : shear modulus

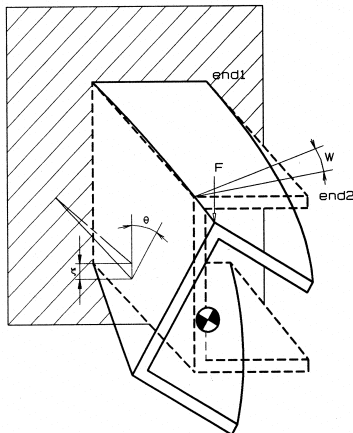


그림 1. 비틀림 하중하의 채널 단면
 Fig. 1. Channel section under torsional load

For the compact solid type cross sections such as rectangles, the amount of warping is small and it has little effect on the torsional behavior. However, for some thin-walled cross sections such as channels and wide flange sections the warping will be quiet large. If the warping is not restrained, then equations (5) and (6) are still valid. Even though in the real world structures, the warping will normally be restrained to some degree since the end of the beam will be connected to other

beams or other structural points. This restraint will induce axial and shear stresses in the beam and reduce the amount of twist of the beam which in effect stiffens the beam in torsion.

When the effects of warping restraint are included, the equation for torsion is

$$EI \frac{d^3\Theta}{dx^3} - GK \frac{d\Theta}{dx} = -T \quad (7)$$

where E : modulus of elasticity
 I_x : warping constant

The general solution of this equations is,

$$\Theta = \frac{Tx}{GK} + A_1 + A_2 \sinh \beta x + A_3 \cosh \beta x \quad (8)$$

where, $\beta^2 = \frac{GK}{EI}$ 또는 $\frac{1}{2(1+\gamma)} \frac{K}{I}$ (9)

Applying these boundary conditions, and assuming that the angle of twist is zero at $x=0$ and that the warping is zero at both end, we get the equation,

$$\Theta(x) = \frac{TL}{G} \left[x - \frac{1}{\beta} \sinh \beta x + \frac{1}{\beta} \tanh \frac{\beta L}{2} (\cos \beta x - 1) \right] \quad (10)$$

Substituting $x=L$ into this equation to get the angle of twist at end we get,

$$\Theta(L) = \frac{TL}{G} \frac{(\beta L - 2 \tanh \frac{\beta L}{2})}{\beta L K} \quad (11)$$

If we try to duplicate this result with equation(5) by replacing the torsional constant K by an effective torsional constant K_e , we get the following expression for the effective torsional constant,

$$K_e = K \frac{\beta L}{(\beta L - 2 \tanh \frac{\beta L}{2})} \quad (12)$$

Looking at equation (12), we see that when $\beta L > 100$, the effective torsional constant is essentially the same as the nominal torsional constant. From this we can say that warping is insignificant when

$$\Gamma < \frac{KL^2}{1 \times 10^5} \tag{13}$$

If we work in mm, the warping constant would have to just be greater than the torsional constant. In most real structures, the torsional warping will not be fully restrained since the end of the beam is connected to a flexible beam not a perfectly rigid wall. The type of joint, welded or bolted, will also effect the degree of warping restraint. By defining a warping restraint factor, we can control the amount of torsional stiffening that occurs as the actual torsional constant used will be an interpolated value between nominal and the effective torsional constant, that is,

$$K'_e = fK_e + (1 - f)K \tag{14}$$

$$0.0 < f < 1.0 \tag{15}$$

The problem occurs when a beam is broken up into several segments for reasons due to intersecting beams, for example, when there are slight bends in the beam. The warping restraint is only at the far ends of the beam, and the restraint at the middle points will be less. Thus it is not valid to model these beam segments with the same warping restraint.

2-3 Modal Test

To determine the dynamic characteristics of frame, frequency response was measured under free-free supporting condition. Each natural frequency represents specific patterns and deformation patterns of frame, which is called mode shape, is determined by adjusting oscillator frequency with natural frequency of frame. This deformed data is read by A-D converter and mode

shapes are plotted. The overall test procedure was processed according to general modal test procedure as follows in Fig. 2.

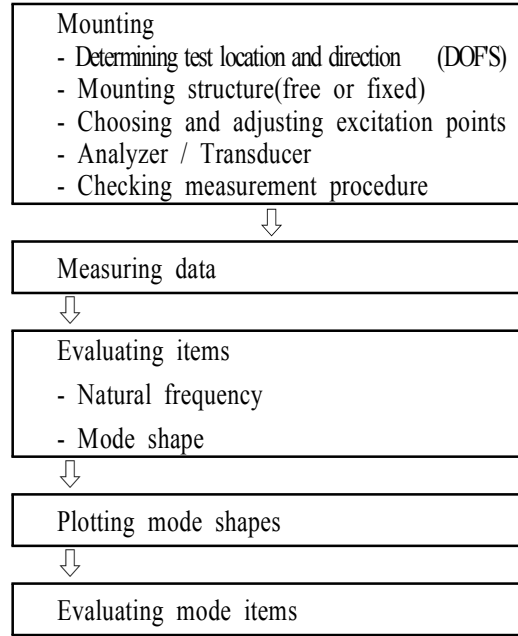


그림 2. 모드 시험 순서

Fig. 2. The set up for modal test procedure

The body mounting points were chosen as excitation points and impacted in vertical and horizontal directions. The frame were free-free condition and measured at the fifty-one frame points. The modal test measurement System set-up is represented in Fig. 3.

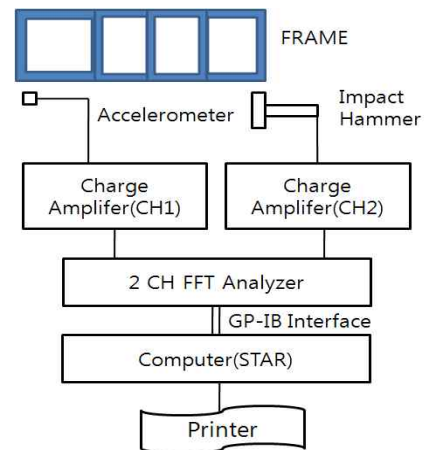
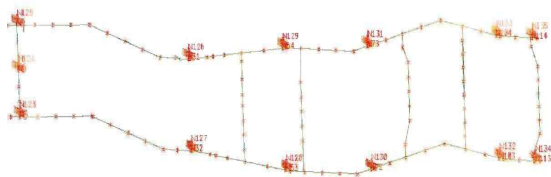


그림 3. 모드 시험 장치

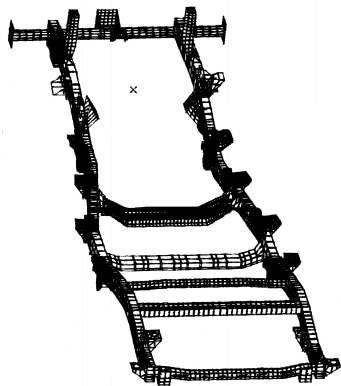
Fig. 3. Modal Test Measurement System Set-Up

2-4 Analysis Model and Displacement test

For comparing warping restraint effect, automotive frame was modelled using finite element method. In beam element model, cross member and side rail were connected node to node linear beam elements in Fig. 4(a), and cross member and side rail were connected rigid element model in Fig. 4(b). Also entire frame was modelled with linear thin shell element model. The results were compared with test results using UPM-60 displacements equipment.



(a) 선형 보 요소 모델
(a) Linear Beam Element Model



(b) 선형 박판 요소 모델
(b) Linear Thin Shell Element Model
그림 4. 자동차 프레임의 유한 요소 모델

Fig. 4. Finite Element Model of Automotive Frame



그림 5. 프레임의 변형 시험
Fig. 5. Displacement Test of Frame

III. Results and Discussion

A static test was performed in which the frame was clamped at all four mounting bracket points, and loads were imposed to cause the frame twist and bend in both the vertical and horizontal planes. By predicting with computer analysis and test result, warping restraint factor(w.r.f.) effect was examined to torsion and bending deformation of frame in Fig. 6. In a short term, it can be said that linear beam element model is more effective than linear thin shell element model to measure the deformation of frame. In Table 1 the result of beam element model with w.r.f. is 0.35 is more closed to the test result than that of beam element model with w.r.f. is 1 in bending test. The close correlation in bending results than in torsion results explains axial stress due to torsion is of a greater magnitude than that due to bending. For further investigation we may compare the correlation with modal test.

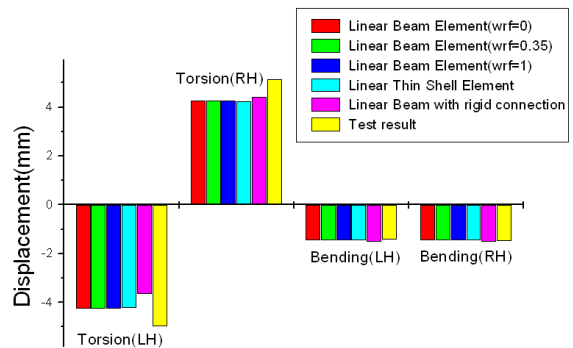


그림 6. 프레임의 변형 결과 비교
Fig. 6. Comparison of displacement results of frame

표 1. 프레임 시험과 해석의 변형 결과

Table 1. Displacement results of frame test and analysis.

Test Condition	Model	Linear Beam Element Model			Linear Thin Shell Element Model	UPM60 Test
		wrf:0	wrf:0.35	wrf:1		
Torsion Test (mm)	LH	-4.28	-4.27	-4.27	-3.68	-4.99
	RH	4.28	4.27	4.27	4.41	5.14
Bending Test (mm)	LH	-1.47	-1.47	-1.47	-1.54	-1.46
	RH	-1.47	-1.47	-1.47	-1.54	-1.53

Fig. 7 ~ Fig. 15 shows the computer analysis results of dynamic modal analysis. The peak deflections predicted by the analysis model compares within the standard engineering accuracy. A comparison of the analysis predicted to the actual (experimentally obtained) natural frequencies of the frame is given in Fig. 16 and Fig. 17. Fig. 16 and Fig. 17 indicated natural frequencies correlation for modes are well accuracy of engineering standard error. However higher the modes, higher the correlation error. The third torsion bending shows correlation of approximately 21%. A possible explanation for the lesser degree of correlation is found in studying the internal loading of this mode.

It is most easily visualized as the bending of a beam with the frame side-rails acting as the flanges and the cross-member acting as the web. For bending of a beam the web acts as a shear panel in plane loading in transmitting the bending load from one flange to the other. For the frame, this in plane loading is taken axially through the cross-members at six discrete locations rather than across a continuous plate. High localized loading occurs in cross-member to side-rail connection, and for the thin walled cross sections this loading can result in significant local distortion in the members. A beam representation of the member does not account for these added deformation, and model predicts this natural frequency higher than actual found in the test, as seen in Fig. 16. This frame joint deformation, joint flexibilities (or joint stiffness) and joint slippage can significantly affect mode dynamic behaviour of frame mode shape.

A number of structural analysis computer codes are available to model frame elements, and the results are similar tendencies as shown in Fig. 18.

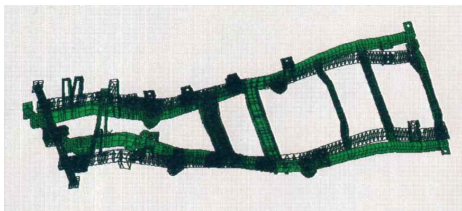


그림 7. 수직 비틀림 모드
Fig. 7. 1st torsion vertical mode

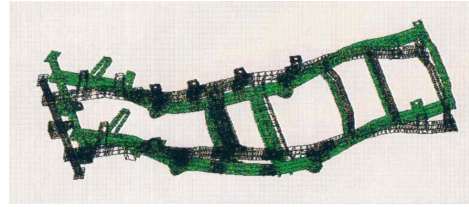


그림 8. 1 수직 굽힘 모드
Fig. 8. 1st bending vertical mode

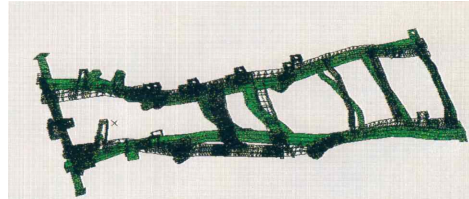


그림 9. 2 수직 비틀림 모드
Fig. 9. 2nd torsion vertical mode

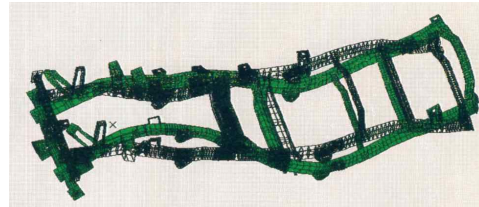


그림 10. 2 수직 굽힘 모드
Fig. 10 2nd bending vertical mode

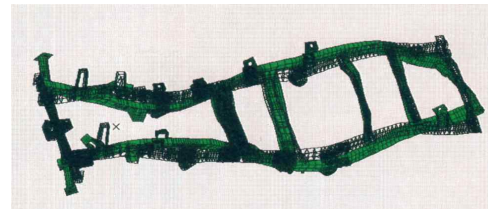


그림 11. 3 수직 비틀림 모드
Fig. 11 3rd torsion vertical mode

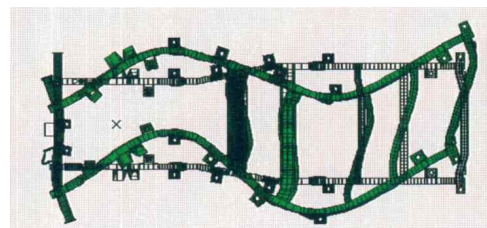


그림 12. 1 수평 굽힘 모드
Fig. 12 1st lateral bending mode

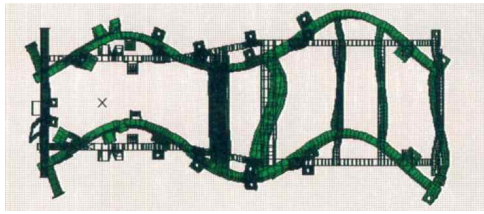


그림 13. 2 수평 굽힘 모드
Fig. 13 2nd lateral bending mode

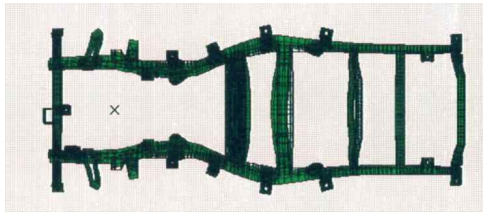


그림 14. 6 수평 굽힘 모드
Fig. 14 6th lateral bending mode

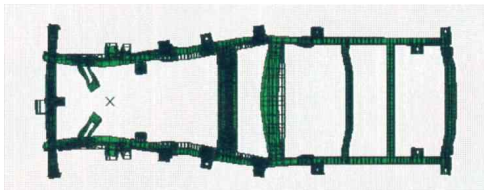


그림 15. 7 수평 굽힘 모드
Fig. 15 7th lateral bending mode

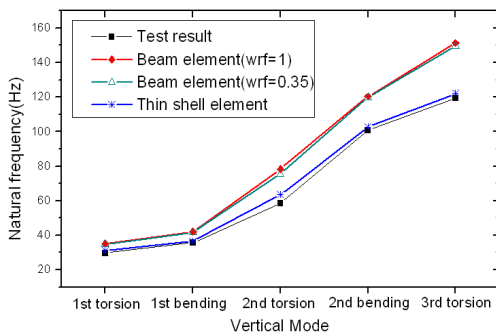


그림 16. 세가지 모델과 시험의 고유주파수에 대한 수직모드 비교
Fig. 16 Comparison of vertical mode natural frequencies with three different models and test

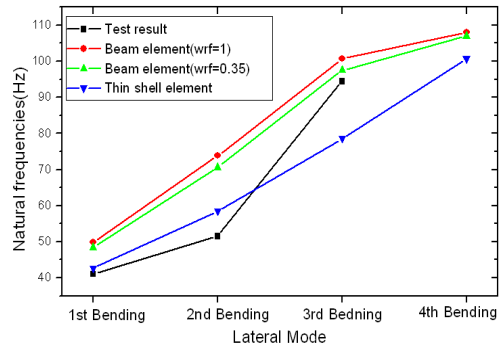


그림 17. 세가지 모델과 시험의 고유주파수에 대한 수평모드 비교
Fig. 17 Comparison of lateral mode natural frequencies with three different models and test

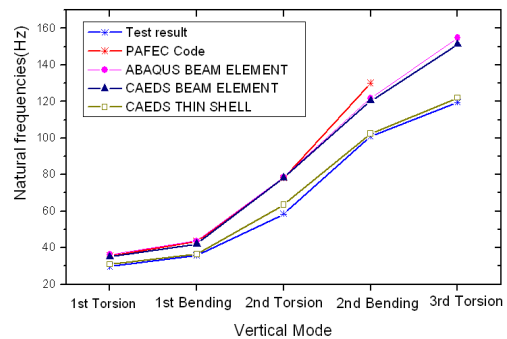


그림 18. 컴퓨터 코드 및 시험의 고유주파수 비교
Fig. 18 Comparison of natural frequencies in different computer codes with test

V. Conclusion

In this study a method for efficient and accurate analysis of frame has been established. The warping conditions of beam elements was discussed. The beam elements method analysis was well matched with experimental results in the frame displacement test. In the displacement test results, the linear beam element results were well matched than thin shell element model with experimental result. However it is not valid to model beam segments with the same warping restraint. In a long beam, the warping restraint in the middle segment might actually be zero.

On the contrary natural frequency of the thin shell element model is well matched with the test result. It can be the influence of joint flexibilities of the frame,

so it is more desirable to use thin shell element model in dynamic modal test. In dynamic modal analysis the analysis predicted results well matched with test results. The frame was excited vertically and the resulting vertical motion measured at a point near the front of the frame on the left side-rail. The close correlation obtained over the entire frequency range.

Further study is indicated optimizing mounting engine bracket location with open and closed sectioned beam applied torsional loading using the result of dynamic mode of frame considering warping restraint factor of beam properties. The computer program appears to be promising but results need test results guide.

Acknowledgment

The Authors are Grateful for Supports from Howon University in 2012

References

- [1] J. K. Hay and J. M. Blew, "Dynamic Testing and Computer Analysis of Automotive Frames", *SAE*, 720046
- [2] R. E. Alliston and R. E. Sherrer, "Axial Stresses Due to Torsion in Truck Frame Rails", *SAE*, 770396
- [3] W.J. Stronge and T.G. Zhang, "Warping of Prismatic Bars in Torsion", *Int. J. Solids Structures*, Vol.30, No.5, pp.601-606, 1993
- [4] S. Krenk and L. Dankilde, "Warping of Joints in I-Beam Assemblages", *J. of Engineering Mechanics*, Vol.117, No.15, pp. 2457-2474, 1991
- [5] 천동준 등, "보의 warping restraint factor 해석", *한국기계기술학회지*, 제7권 제4호, pp13-17, 2005

유 환 신 (劉桓臣)



1993년 2월: 동국대학교 전자공학(공학사)

2001년 2월: 국민대학교 자동차공학
(공학석사)

2006년 2월: 국민대학교 자동차공학
(공학박사)

2006~현재:호원대학교 자동차기계

공학과 교수

관심분야 : car electronics

천 동 준 (千東濬)



1987년 2월 : 전남대학교 기계공학과
(공학사)

1989년 9월 : 단국대학교 기계공학과
(공학석사)

1999년 3월 : 경희대학교 기계공학과
(공학박사)

1997년~현재:벽성대학 자동차과교수

관심분야: 열유체 및 플랜트