Honam Mathematical J. **34** (2012), No. 1, pp. 035–044
http://dx.doi.org/10.5831/HMJ.2012.34.1.35

# GENERALIZATIONS OF CERTAIN SUMMATION FORMULA DUE TO RAMANUJAN

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**Abstract.** Motivated by the extension of classical Dixon's summation theorem for the series ${}_3F_2$ given by Lavoie, Grondin, Rathie and Arora, the authors aim at deriving four generalized summation formulas, which, upon specializing their parameters, give many summation identities including, especially, the four very interesting summation formulas due to Ramanujan.

## 1. Introduction

The generalized hypergeometric function with $p$ numeratorial and $q$ denominatorial parameters is defined by the series [8, p. 41]

$$(1.1)\qquad {}_pF_q\left[\begin{array}{c} a_1, a_2, \cdots, a_p \\ b_1, b_2, \cdots, b_q \end{array}; x\right] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \cdots (a_p)_n}{(b_1)_n (b_2)_n \cdots (b_q)_n} \frac{x^n}{n!}.$$

When $q = p$, this series converges for $|x| < \infty$, but when $q = p - 1$, convergence occurs when $|x| < 1$ (unless the series terminates). In (1.1), the Pochhammer symbol (or ascending factorial, since $(1)_n = n!$) is defined for any complex number $\alpha$ by

$$(1.2)\qquad (\alpha)_n = \begin{cases} \alpha(\alpha+1)\cdots(\alpha+n-1) & (n \in \mathbb{N}), \\ 1 & (n = 0). \end{cases}$$

Using the fundamental relation $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$, where $\Gamma$ is the Gamma functions, $(\alpha)_n$ can be written in the form

$$(1.3)\qquad (\alpha)_n = \frac{\Gamma(\alpha+n)}{\Gamma(\alpha)} \quad (n \in \mathbb{N} \cup \{0\}).$$

Received October 31, 2011. Accepted November 16, 2011.
2000 Mathematics Subject Classification. 33C65, 33C60, 33C70, 33C05.
Key words and phrases. generalized hypergeometric series, generalized Dixon's summation theorem, Ramanujan summation formula.
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In 1994, Lavoie *et al.* [5] have obtained a large number of interesting summation formulas contiguous to Dixon's theorem (see [9, p. 250, Eq. (4)]) of the form

(1.4)

$$
{}_3F_2\left[\begin{matrix} a, & b, & c \\ 1+a-b+i, & 1+a-c+i+j \end{matrix}; 1\right]
$$

$$
= \frac{2^{-2c+i+j}\Gamma(1+a-b+i)\Gamma(1+a-c+i+j)\Gamma(b-i)\Gamma(c-i-j)}{\Gamma(a-2c+i+j+1)\Gamma(a-b-c+i+j+1)\Gamma(b)\Gamma(c)}
$$

$$
\times\left\{ A_{ij}\ \frac{\Gamma(\frac{1}{2}a-c+\frac{1}{2}+\left[\frac{i+j+1}{2}\right])\Gamma(\frac{1}{2}a-b-c+1+i+\left[\frac{j+1}{2}\right])}{\Gamma(\frac{1}{2}a+\frac{1}{2})\Gamma(\frac{1}{2}a-b+1+\left[\frac{i}{2}\right])}\right.
$$

$$
\left. +\ B_{ij}\ \frac{\Gamma(\frac{1}{2}a-c+1+\left[\frac{i+j}{2}\right])\Gamma(\frac{1}{2}a-b-c+\frac{3}{2}+i+\left[\frac{j}{2}\right])}{\Gamma(\frac{1}{2}a)\Gamma(\frac{1}{2}a-b+\frac{1}{2}+\left[\frac{i+1}{2}\right])}\right\}
$$

$$
(i,j=0,1,2,3;\quad \Re e(a-2b-2c)+2+2i+j>0)\,.
$$

Here, as usual $[x]$ denotes the greatest integer less than or equal to $x$ and the coefficients $A_{i,j}$, $B_{i,j}$ are given in the following tables.

| $i \backslash j$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 1 | -1 | $\frac{1}{2}\left[(a-b-c+1)^2 +(c-1)(c-3) -b^2+a\right]$ | $c(a-b-c+4) -(a+1)(a+2) -(a-1)(b-1) +3ab$ |
| 1 | 1 | $c-a-1$ | $a(a-1) +(b+c-3) \times(c-2a-1)$ | — |
| 2 | $\frac{1}{2}(a-1)(a-4) -(b^2-5a+1) -(a-b+1) \times(b+c)$ | $(b-1)(b-2) -(a-b+1) \times(a-b-c+3)$ | $\frac{1}{2}(a-c+2) \times(a-2b-c+5) \times\left[(a-c+2)(a-2b+2) -a(c-3)\right] -(b-1)(b-2) \times(c-2)(c-3)$ | — |
| 3 | $5a-b^2 +(a+1)^2 -(2a-b+1) \times(b+c)$ | — | — | — |

Table 1. Values of $A_{ij}$.

| $i \backslash j$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 0 | 1 | -2 | $(a+2)(a+4)$ $-b(2a+5)$ $-3c(a-b-c+4)$ $+3$ |
| 1 | -1 | $a-2b-c+3$ | $(b-1)(b-c+1)$ $-(a-b-c+2)$ $\times(a-b-c+3)$ | — |
| 2 | -2 | $(a-b-2c+5)$ $\times(a-b-c+3)$ $-(b-1)(b-2)$ | $-2(a-c+2)$ $\times(a-2b-c+5)$ | — |
| 3 | $-a+3b^2$ $-(a+3)^2$ $+(2a-3b+5)$ $\times(b+c)$ | — | — | — |

Table 2. Values of $B_{ij}$.

The aim of this paper is to give four general summation formulas and thereafter their some special cases.

## 2. Main summation formulas

The 64 summation formulas established in this paper can be presented in the following four unified forms.

$$\begin{aligned}
&\sum_{n=0}^{\infty} \frac{(\frac{1}{2})_n (1-x)_n}{(\frac{3}{2}+i)_n (1+x+i+j)_n} \\
&= 1 - \frac{1}{(3+2i)} \cdot \frac{(x-1)}{(1+x+i+j)} \\
&\quad + \frac{1\cdot 3}{(3+2i)(5+2i)} \cdot \frac{(x-1)(x-2)}{(1+x+i+j)(2+x+i+j)} - \cdots \\
&= \frac{2^{2x-2+i+j}\ \Gamma(\frac{3}{2}+i)\Gamma(1+x+i+j)\Gamma(\frac{1}{2}-i)\Gamma(1-x-i-j)}{\Gamma(2x+i+j)\Gamma(x+\frac{1}{2}+i+j)\Gamma(\frac{1}{2})\Gamma(1-x)} \\
&\quad \times \left\{ C_{ij}\ \frac{\Gamma(x+[\frac{i+j+1}{2}])\Gamma(x+i+[\frac{j+1}{2}])}{\Gamma(1+[\frac{i}{2}])} \right. \\
&\quad \left. + D_{ij}\ \frac{\Gamma(x+\frac{1}{2}+[\frac{i+j}{2}])\ \Gamma(x+\frac{1}{2}+i+[\frac{j}{2}])}{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}+[\frac{i+1}{2}])} \right\}
\end{aligned} \tag{2.1}$$

for $i, j = 0, 1, 2, 3$ and (2.1) holds true when $\Delta_1 := \Re e(x) + 2i + j > 0$ for convergence.

The coefficients $C_{ij}$ and $D_{ij}$ can be obtained by simply putting $a = 1$, $b = \frac{1}{2}$ and $c = 1 - x$ in $A_{ij}$ and $B_{ij}$ given at the tables.

(2.2)

$$\begin{aligned}
&\sum_{n=0}^{\infty} \frac{(1-x)_n(1-x)_n}{(1+x+i)_n(1+x+i+j)_n} \\
&= 1 + \frac{(x-1)^2}{(x+1+i)_n(x+1+i+j)} \\
&\quad + \frac{(x-1)^2(x-2)^2}{(x+1+i)(x+2+i)(x+1+i+j)(x+2+i+j)} + \cdots \\
&= \frac{2^{-2(1-x)+i+j}\Gamma(1+x+i)\Gamma(1+x+i+j)\Gamma(1-x-i)\Gamma(1-x-i-j)}{\Gamma(x+i+j)\Gamma(2x+i+j)\Gamma^2(1-x)} \\
&\quad \times \left\{ E_{ij} \frac{\Gamma(x+[\frac{i+j-1}{2}])\Gamma(2x-\frac{1}{2}+i+[\frac{j+1}{2}])}{\Gamma(\frac{1}{2}+x+[\frac{i}{2}])} \right. \\
&\qquad \left. + F_{ij} \frac{\Gamma(\frac{1}{2}+x+[\frac{i+j}{2}])\Gamma(2x+i+[\frac{j}{2}])}{\Gamma(\frac{1}{2})\Gamma(x+[\frac{i+1}{2}])} \right\}
\end{aligned}$$

for $i, j = 0, 1, 2, 3$ and (2.2) holds true when $\Delta_2 := 3\Re e(x) + 2i + j - 1 > 0$ for convergence. The coefficients $E_{ij}$ and $F_{ij}$ can be obtained from the table of $A_{ij}$ and $B_{ij}$ by putting $a = 1$ and $b = c = 1 - x$.

$$\begin{aligned}
&\sum_{n=0}^{\infty} \frac{(\frac{3}{2})_n(1-x)n}{(\frac{1}{2}+i)_n(1+x+i+j)_n} \\
&= 1 - \frac{3(x-1)}{(1+2i)(1+x+i+j)} + \frac{3\cdot 5}{(1+2i)(3+2i)} \\
&\qquad \times \frac{(x-1)(x-2)}{(1+x+i+j)(2+x+i+j)} - \cdots \\
&= \frac{2^{2x-1+i+j}\Gamma(\frac{1}{2}+i)\Gamma(1+x+i+j)\Gamma(\frac{3}{2}-i)\Gamma(1-x-i-j)}{\Gamma(2x+i+j)\Gamma(x-\frac{1}{2}+i+j)\Gamma(\frac{1}{2})\Gamma(1-x)} \\
&\quad \times \left\{ G_{ij} \frac{\Gamma(x+[\frac{i+j+1}{2}])\Gamma(x-1+i+[\frac{j+1}{2}])}{\Gamma([\frac{i}{2}])} \right. \\
&\qquad \left. + H_{ij} \frac{\Gamma(\frac{1}{2}+[\frac{i+j}{2}]) + \Gamma(x-\frac{1}{2}+i+[\frac{j}{2}])}{\Gamma(\frac{1}{2})\Gamma([\frac{i+1}{2}]-\frac{1}{2})} \right\}
\end{aligned} \tag{2.3}$$

for $i, j = 0, 1, 2, 3$ and (2.3) holds true when $\Delta_3 := 2\Re e(x)+2i+j-2 > 0$ for convergence. The coefficients $G_{ij}$ and $H_{ij}$ can be obtained from the tables of $A_{ij}$ and $B_{ij}$ by putting $a = 1$, $b = \frac{3}{2}$ and $c = 1 - x$.

(2.4)

$$\begin{aligned}
&\sum_{n=0}^{\infty} \frac{(x)_n^3}{(1+i)_n(1+i+j)_n\, n!} \\
&= 1 + \frac{x^3}{(1+i)(1+i+j)} + \frac{x^3(1-x)^3}{(1+i)(2+i)(1+i+j)(2+i+j)\, 2!} + \cdots \\
&= \frac{2^{-2x+i+j}\ \Gamma(1+i)\Gamma(1+i+j)\Gamma(x-i)\Gamma(x-i-j)}{\Gamma(1-x+i+j)\Gamma(1+x+i+j)\Gamma^2(x)} \\
&\quad \times \left\{ I_{ij}\ \frac{\Gamma(\frac{1}{2}-\frac{1}{2}x+[\frac{i+j+1}{2}])\Gamma(1-\frac{3}{2}x+i+[\frac{j+1}{2}])}{\Gamma(\frac{1}{2}x+\frac{1}{2})\Gamma(1-\frac{1}{2}x+[\frac{i}{2}])} \right. \\
&\qquad \left. +J_{ij}\ \frac{\Gamma(1-\frac{1}{2}x+[\frac{i+j}{2}])\Gamma(\frac{3}{2}-\frac{3}{2}x+i+[\frac{i}{2}])}{\Gamma(\frac{1}{2}x)\Gamma(\frac{1}{2}-\frac{1}{2}x+[\frac{i+1}{2}])} \right\}
\end{aligned}$$

for $i, j = 0, 1, 2, 3$ and (2.4) holds true when $\Delta_4 := 3\Re e(x) - 2i - j < 2$ for convergence. The coefficients $I_{ij}$ and $J_{ij}$ can be obtained from the tables of $A_{ij}$ and $B_{ij}$ by putting $a = b = c = x$.

Note that the special case of (1.4) when $i = j = 0$ reduces immediately to the classical Dixon's theorem. Some interesting summation formulas due to Ramanujan (See [2, p. 22, Ex.11; p. 20, Ex.3; p. 21, Ex.8; p. 25, Ex.20]) are easily derivable as some special cases of our main results.

## 3. Proofs of (2.1) to (2.4)

The series (2.1) corresponds to

$${}_3F_2\left(\begin{matrix} 1, & \frac{1}{2}, & 1-x \\ \frac{3}{2}+i, & 1+x+i+j & \end{matrix}\ ;1\right)$$

which is a special case of extended Dixon's summation theorem (1.4) for $a = 1$, $b = \frac{1}{2}$ and $c = 1 - x$. It can easily be evaluated by (1.4) and we get the right-hand side of the result (2.1).

The series (2.2) corresponds to

$$ {}_3F_2\left(\begin{matrix} 1, & 1-x, & 1-x \\ 1+x+i, & 1+x+i+j & \end{matrix}\ ;1\right) $$

which is a special case of extended Dixon's summation theorem (1.4) for $a = 1$ and $b = c = 1 - x$.

The series (2.3) corresponds to

$$ {}_3F_2\left(\begin{matrix} 1, & \frac{3}{2}, & 1-x \\ \frac{1}{2}+i, & 1+x+i+j & \end{matrix}\ ;1\right) $$

which is a special case of extended Dixon's summation theorem (1.4) for $a = 1$, $b = \frac{3}{2}$ and $c = 1 - x$.

The series (2.4) corresponds to

$$ {}_3F_2\left(\begin{matrix} x, & x, & x \\ 1+i, & 1+i+j & \end{matrix}\ ;1\right) $$

which is a special case of extended Dixon's summation theorem (1.4) for $a = b = c = x$.

## 4. Special cases

In (2.1), if we take $i = 0$, $j = 0, 1, 2, 3$; $i = 1$, $j = 0, 1, 2$; $i = 2$, $j = 0, 1, 2$ and $i = 3$, $j = 0$, we get the following interesting summations.

For $i = j = 0$, we have

$$ 1 - \frac{1}{3}\frac{(x-1)}{(x+1)} + \frac{1}{5}\frac{(x-1)(x-2)}{(x+1)(x+2)} - \cdots = \frac{2^{4x}\ \Gamma^4(x+1)}{4x\ \Gamma^2(2x+1)}, \tag{4.1} $$

provided $\Re e(x) > 0$. This can be found in [2, p. 22, Ex.11].

For $i = 0$ and $j = 1$, we have

$$ \begin{aligned} &1 - \frac{1}{3}\frac{(x-1)}{(x+2)} + \frac{1}{5}\frac{(x-1)(x-2)}{(x+2)(x+3)} - \cdots \\ &= \frac{(x+1)}{2x\ \Gamma^2(2x+1)}\left\{\frac{2^{4x}\ \Gamma^4(x+1)}{(2x+1)} - \frac{1}{2}\right\}, \end{aligned} \tag{4.2} $$

provided $\Re e(x) > -1$ and $x$ is not an integer.

For $i = 0$ and $j = 2$, we have

$$\begin{aligned}
&1 - \frac{1}{3}\frac{(x-1)}{(x+3)} + \frac{1}{5}\frac{(x-1)(x-2)}{(x+3)(x+4)} - \cdots \\
&= \frac{(x+2)}{x(2x+3)}\left\{\frac{2^{4x}\ (x+1)\ \Gamma^4(x+1)}{(2x+1)\ \Gamma^2(2x+1)} - 1\right\},
\end{aligned} \tag{4.3}$$

provided $\Re e(x) > -2$ and $x$ is not an integer.

For $i = 0$ and $j = 3$, we have

$$\begin{aligned}
&1 - \frac{1}{3}\frac{(x-1)}{(x+4)} + \frac{1}{5}\frac{(x-1)(x-2)}{(x+4)(x+5)} - \cdots \\
&= \frac{(x+3)}{x(x+1)(2x+3)(2x+5)} \\
&\quad \times \left\{\frac{2^{4x+1}\ (x+1)(x+2)\ \Gamma^3(x+1)}{(2x+1)\ \Gamma^2(2x+1)} - \frac{(6x^2+15x+18)}{2}\right\},
\end{aligned} \tag{4.4}$$

provided $\Re e(x) > -3$ and $x$ is not an integer.

For $i = 1$ and $j = 0$, we have

$$\begin{aligned}
&1 - \frac{1}{5}\frac{(x-1)}{(x+2)} + \frac{1\cdot 3}{5\cdot 7}\frac{(x-1)(x-2)}{(x+2)(x+3)} - \cdots \\
&= \frac{3(x+1)}{x}\left\{\frac{2^{4x-1}\ \Gamma^4(x+1)}{(2x+1)\ \Gamma^2(2x+1)} - \frac{(2x+1)}{2}\right\},
\end{aligned} \tag{4.5}$$

provided $\Re e(x) > -2$ and $x$ is not an integer.

For $i = 1$ and $j = 1$, we have

$$\begin{aligned}
&1 - \frac{1}{5}\frac{(x-1)}{(x+3)} + \frac{1\cdot 3}{5\cdot 7}\frac{(x-1)(x-2)}{(x+3)(x+4)} - \cdots \\
&= \frac{3(x+2)}{\sqrt{\pi}\ x(2x+1)^2(2x+3)} \\
&\quad \times \left\{\frac{2^{2x+1}(x+1)^2\Gamma^4(x+1)}{\Gamma(2x+1)} - \frac{2^{4-2x}(x+2)\Gamma(2x+1)}{(2x+1)}\right\},
\end{aligned} \tag{4.6}$$

provided $\Re e(x) > -3$ and $x$ is not an integer.

For $i=1$ and $j=2$, we have

(4.7)
$$\begin{aligned}
&1-\frac{1}{5}\frac{(x-1)}{(x+4)}+\frac{1\cdot 3}{5\cdot 7}\frac{(x-1)(x-2)}{(x+4)(x+5)}-\cdots\\
&=\frac{3\cdot 2^{4x+2}(x+3)}{x(x+1)(2x+1)^2(2x+5)}\\
&\qquad\times\left\{\frac{(4x^2+14x+13)(2x+1)^2}{2^{4x+4}}-\frac{(x+1)^2(x+2)\Gamma^4(x+1)}{2\Gamma^2(2x+1)}\right\},
\end{aligned}$$

provided $\Re e(x)>-4$ and $x$ is not an integer.

For $i=2$ and $j=0$, we have

$$\begin{aligned}
&1-\frac{1}{7}\frac{(x-1)}{(x+3)}+\frac{1}{3\cdot 7}\frac{(x-1)(x-2)}{(x+3)(x+4)}-\cdots\\
&=\frac{5\cdot 2^{4x+1}(x+2)}{x(2x+1)^2(2x+3)}\\
&\qquad\times\left\{\frac{(x+1)(6x+5)\Gamma^4(x+1)}{4\Gamma^2(2x+1)}-\frac{(2x+1)^2(2x+3)}{2^{4x+1}}\right\},
\end{aligned}\tag{4.8}$$

provided $\Re e(x)>-4$ and $x$ is not an integer.

For $i=2$ and $j=1$, we have

$$\begin{aligned}
&1-\frac{1}{7}\frac{(x-1)}{(x+4)}+\frac{1}{3\cdot 7}\frac{(x-1)(x-2)}{(x+4)(x+5)}-\cdots\\
&=\frac{5\cdot 2^{4x+2}(x+3)}{x(x+1)(2x+1)^2(2x+3)(2x+5)}\\
&\qquad\times\left\{\frac{3(x+1)^2(x+2)^2\Gamma^4(x+1)}{2\Gamma^2(2x+1)}\right.\\
&\qquad\qquad\left.-\frac{(4x^2+17x+16)(2x+1)^2(2x+3)}{2^{4x+3}}\right\},
\end{aligned}\tag{4.9}$$

provided $\Re e(x)>-5$ and $x$ is not an integer.

For $i=2$ and $j=2$, we have

$$
\begin{aligned}
&1-\frac{1}{7}\frac{(x-1)}{(x+5)}+\frac{1}{3\cdot 7}\frac{(x-1)(x-2)}{(x+5)(x+6)}-\cdots \\
&=\frac{5\cdot 2^{4(x+1)}\ (x+2)(x+4)}{x(x+1)(2x+1)^2(2x+3)^2(2x+5)(2x+7)} \\
&\times\left\{\frac{3(x+3)(x^2+5x+7)(x+1)^2\ \Gamma^4(x+1)}{2\Gamma^2(2x+1)}\right. \\
&\qquad\left.-\ \frac{(2x+1)^2(2x+3)^2(2x+5)}{2^{4x+3}}\right\}
\end{aligned}
\tag{4.10}
$$

provided $\Re e(x)>-6$ and $x$ is not an interger

For $i=3$ and $j=0$, we have

$$
\begin{aligned}
&1-\frac{1}{9}\frac{(x-1)}{(x+4)}+\frac{1}{3\cdot 11}\frac{(x-1)(x-2)}{(x+4)(x+5)}-\cdots \\
&=\frac{7\cdot 2^{4x+2}(x+3)}{x(x+1)(2x+1)^2(2x+3)(2x+5)} \\
&\left\{\frac{5(5-2x)(x+1)^2(x+2)\Gamma^4(x+1)}{4\Gamma^2(2x+1)}\right. \\
&\qquad\left.-\ \frac{(11x+16)(2x+1)^2(2x+3)(2x+5)}{3\cdot 2^{4x+3}}\right\}
\end{aligned}
\tag{4.11}
$$

provided $\Re e(x)>-6$ and $x$ is not an integer.

Setting $(i,j)=(0,0)$ in (2.2), (2.3) and (2.4) respectively, we obtain the resulting identities,

$$
1+\frac{(x-1)^2}{(x+1)^2}+\frac{(x-1)^2(x-2)^2}{(x+1)^2(x+2)^2}+\cdots=\frac{2x\Gamma^4(x+1)\Gamma(4x+1)}{(4x-1)\Gamma^4(2x+1)}
\tag{4.12}
$$

provided $\Re e(x)>\frac{1}{4}$. This can be found in [2, p. 20, Ex.3].

$$
1-3\ \frac{(x-2)}{(x+1)}+5\ \frac{(x-1)(x-2)}{(x+1)(x+2)}-\cdots=0
\tag{4.13}
$$

provided $\Re e(x)>1$ This can be found in [2, p. 21, Ex.8]

$$
1+\left(\frac{x}{1!}\right)^3+\left(\frac{x(x+1)}{2!}\right)^3+\cdots=\frac{6\ \sin(\frac{1}{2}\pi x)\ \sin\pi x\ \Gamma^3(\frac{1}{2}x+1)}{\pi^2\ x^2\ (1+2\cos\pi x)\ \Gamma(\frac{3}{2}x+1)}
\tag{4.14}
$$

provided $\Re e(x)<\frac{2}{3}$. This can be founded in [2, p. 25, Ex.20].

Similarly, some other results in [2] can be obtained.

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