Calculation Of Mobile Location Based On TOA/SS Measurements

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Received May 13, 2012; revised July 10, 2012; accepted November 19, 2012; published December 27, 2012

Abstract

Localization of mobile station (MS) has now gained considerable attention. Since the hybrid measurements can help to improve the positioning accuracy, several hybrid localization methods have been proposed in the literature. However, the high performance estimator with the closed-form solution and complete performance analysis for time-of-arrival/signal strength (TOA/SS) localization technique is still an opening issue. Two TOA/SS localization algorithms with the closed-form solutions are proposed for the cases with or without uncertainty in the positions of base stations. Furthermore, performance analysis for the TOA/SS localization technique is presented. Both the theoretical variances and Cramer-Rao lower bounds (CRLBs) are derived and the relationship between the cases with or without uncertainty is given. The paper also proves that the TOA/SS scheme has a lower CRLB than the TOA (or SS) scheme. Theoretical analysis and simulations show that the proposed method can reach its CRLB.

Keywords: Mobile location, CRLB, TOA/SS

This work was sponsored by the Research Fund of ZTE Corporation, and the National Natural Science Foundation of China (61201275).

http://dx.doi.org/10.3837/tiis.2012.12.008

1. Introduction

Location estimation of a mobile station (MS) in wireless communication systems has gained considerable attention since the Federal Communication Commission passed a mandate requiring cellular providers to generate accurate location estimates for Enhanced-911 services [1]. Wireless location as an important public safety feature has created many potential applications to future cellular systems such as: location-sensitive billing, fraud protection, person/asset tracking, fleet management, mobile yellow pages, wireless network design, radio resource management, and intelligent transportation systems [2].

Many methods have been proposed to estimate MS position [3][4][5][6][7][8][9][10][111][12][13][14][15][16][17][18]. The iterative methods [3][4][5][6] used the linearization techniques such as Taylor-series expansion, the steepest descent method, and Newton iteration to solve the nonlinear position problems. The closed-form methods have been developed for real-time implementation [7][8][9][10]. Caffery [7] used the straight lines of position (LOP) rather than the circular LOP to determine the MS position. The divide and conquer method [8] utilized the Fisher information to improve location accuracy. The spherical-interpolation (SI) method [9] transformed the nonlinear equations into a set of linear equations by introducing an extra variable. The SI method may lose the optimum as the relationship between the MS position and the extra variable is ignored. Chan and Ho [10] proposed the two-step weighted least squares (WLS) method based on maximum likelihood estimator to improve the location accuracy of the SI method. This method is an unbiased, and can provide an optimum performance and attain the Cramer-Rao lower bound (CRLB) asymptotically. The location methods based on multidimensional scaling have been addressed in the literature [11][12][13][14]. The authors in [15] replaced the least-squares cost function with a robust cost function in the wireless location system. The minimum mean square error estimator with the prior information on MS position was proposed to provide a better performance [16]. Furthermore, the localization algorithms considering the uncertainties in the positions of base stations (BSs) were presented in [17][18].

Conventional geolocation techniques include the time-of-arrival (TOA), the time-difference-of-arrival (TDOA), the angle-of-arrival (AOA), the signal strength (SS) based methods or hybrid location methods. It should be noted that the above studies in [3][4][5][6][7][7][8][9][10][11][12][13][14][15][16][17][18] are all based on a single kind of measurements such as TOA, TDOA, or SS. Since the hybrid measurements can help to improve the positioning accuracy, several hybrid localization methods have been proposed to estimate the position of MS [19][20]. However, the high performance estimator with the closed-form solution and complete performance analysis for time-of-arrival/signal strength (TOA/SS) localization technique is still an opening issue. TOA/RSS is a hybrid localization technique which uses both of time-of-arrival and signal strength measurements to estimate MS position. In this paper, two TOA/SS localization methods are proposed based on two-step WLS estimator for the cases with or without uncertainty in the positions of BSs. In addition, the performance analysis of the proposed methods is also given. Compared with the previous hybrid localization methods, main contributions of this paper are listed as follows:

(1) The existing hybrid methods for TOA/SS in [19][20] require a search process and cannot give a closed-form solution for mobile location. The inefficiency incurred by these algorithms might not be feasible to be applied in practical systems. Another problem for these methods is that these hybrid methods cannot attain the best achievable positioning accuracy which is

evaluated in terms of the CRLB. The proposed methods can not only provide the closed-form solutions but also attain the CRLBs which is verified by the theoretical analysis and simulations. In addition, this paper first considers the case with uncertainty in the TOA/SS localization technique.

(2) Performance analysis for the TOA/SS localization technique is presented in this paper. Both the theoretical variances and CRLBs for the proposed methods are derived and the relationship between the cases with or without uncertainty is given. The paper also proves that the TOA/SS scheme has a lower CRLB than the TOA (or SS) scheme. These have not been addressed in the literature.

Section 2 briefly introduces system model. Two novel TOA/SS methods for the cases with or without uncertainty and its theoretical variances are proposed in Section 3. Section 4 derives the CRLBs for both the cases. Some characteristics of the proposed methods are given in Section 5. In section 6, the performance of the proposed algorithm is simulated in terms of the root mean square error (RMSE). Conclusions of this paper are given in Section 7.

2. System Model

The basic TOA and SS models are briefly introduced in this section. Assuming that (x, y) is the position of a MS and (x_i, y_i) is the position of the i th BS in a N -BSs system. The MS position is a unknown parameter to be estimated. This paper considers both the cases that the positions of BSs are perfectly or imperfectly known. The paper focuses on the TOA/SS location technique which uses TOA and RSS measurements to locate the MS. The TOA measurement $\hat{t}_i$ is the transmission time between MS and BS i. Denote the measurement with noise of $\{*\}$ as $\{\hat{*}\}$. The range measurement $\hat{r}_i$ from the corresponding TOA measurement $\hat{t}_i$ is modeled as:

$$\hat{r}_i = c\hat{t}_i = r_i + n_{ri} = \sqrt{(x - x_i)^2 + (y - y_i)^2} + n_{ri} \tag{1}$$

where c is the speed of light, r_i is the true distance between the MS and BS i. Assuming that the signal propagations between the MS and all BSs are line-of-sight (LOS) such that n_{ri} is a zero-mean Gaussian random process with variance σ_{ri}^2. This assumption is widely used in the radar and sonar systems. Although Non-line-of-sight (NLOS) propagations may degrade the performance of cellular location system, a lot of methods in [21][22][23] can be used to mitigate the NLOS errors. After suppress the NLOS errors using these methods, the proposed algorithms can be used to locate the MS. The true distance r_i can be modeled as:

$$r_i^2 = (x_i - x)^2 + (y_i - y)^2 = k_i - 2x_i x - 2y_i y + k \tag{2}$$

where $k_i = x_i^2 + y_i^2$, and $k = x^2 + y^2$.

Since the measured received power $\hat{p}_i$ at BS i (in milliwatts) can be modeled as log-normal variable [24], the relation between $\hat{p}_i$ and r_i is:

$$\hat{P}_i = 10\log_{10}\hat{p}_i - P_0 = P_i + n_{si} = -10n_p \log_{10}(r_i / r_0) + n_{si} \tag{3}$$

where n_p is the path loss exponent, n_{si} is a zero-mean Gaussian random process with variance σ_{si}^2 in decibel, $P_i = 10\log_{10} p_i - P_0$, p_i is the received power without the noise, and P_0 is the received power in decibel milliwatts at a reference distance r_0. Typically, $r_0 = 1m$ and P_0 is calculated from the free space path loss formula [24]. From (3), r_i can be obtained:

$$r_i = 10^{-P_i/(10n_p)} \tag{4}$$

With the TOA and SS noises, the error vector derived from (2) and (4) is:

$$\mathbf{e} = \mathbf{Y} - \mathbf{G}\mathbf{Z} \tag{5}$$

where $e = [e_1 \ \cdots \ e_N]^T$, $e_i = \hat{r}_i^2 - r_i^2$,

$$\mathbf{G} = \begin{bmatrix} -2x_1 & -2y_1 & 1 \\ \vdots & \ddots & \vdots \\ -2x_N & -2y_N & 1 \\ -2x_1 & -2y_1 & 1 \\ \vdots & \ddots & \vdots \\ -2x_N & -2y_N & 1 \end{bmatrix}, \ \mathbf{Y} = \begin{bmatrix} \hat{r}_1^2 - k_1 \\ \vdots \\ \hat{r}_N^2 - k_N \\ 10^{-\hat{P}_1/(5n_p)} - k_1 \\ \vdots \\ 10^{-\hat{P}_N/(5n_p)} - k_N \end{bmatrix}, \ \mathbf{Z} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ k \end{bmatrix}.$$

The objective of TOA/SS location method is to estimate the MS position (x, y) from (5).

3. Closed-Form Solutions

3.1 The case without uncertainty

This subsection considers that the positions of BSs are perfectly known.

The first step WLS estimator of $\mathbf{Z}$ can be obtained from (5):

$$\mathbf{Z} = \arg\min\left\{(\mathbf{Y}-\mathbf{G}\mathbf{Z})^T \mathbf{\Psi}^{-1} (\mathbf{Y}-\mathbf{G}\mathbf{Z})\right\} = \left(\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{G}\right)^{-1} \mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{Y} \tag{6}$$

where ψ is the covariance matrix of $\mathbf{e}$:

$$\psi = \mathrm{cov}(\mathbf{e}) = E\left(\mathbf{e}\mathbf{e}^T\right) = \mathbf{BQB} \tag{7}$$

with

$\mathbf{B} = diag\left\{[\mathbf{B_r}, \mathbf{B_r}]\right\}$, $\mathbf{Q} = diag\left\{[\mathbf{Q_r}, \mathbf{Q_s}]\right\}$,

$\mathbf{B_r} = 2diag\left\{[r_1, \cdots, r_N]\right\}$, $\mathbf{Q_r} = diag\left\{\left[\sigma_{r1}^2, \cdots \sigma_{rN}^2\right]\right\}$, $\mathbf{Q_s} = diag\left\{\left[\frac{r_1^2}{b_1}, \cdots, \frac{r_N^2}{b_N}\right]\right\}$, and

$b_i = \left(\frac{10n_p}{\sigma_{si}\ln 10}\right)^2$.

Since the covariance matrix ψ depends on the unknown r_i, the approximate value $\hat{r}_i$ can be used in ψ to make the problem solvable. In fact, (6) provides a initial solution for the MS position since $\mathbf{Z}$ in (6) contains the MS position (x, y). The first step solution of $\mathbf{Z}$ in (6) is based on the assumption of independent x, y, and k. However those parameters are correlated by (2). The estimation accuracy can be further improved using the relationship between x, y, and k. The results can be revised as follows using the relation of (2):

$$\mathbf{e}' = \mathbf{Y}' - \mathbf{G}'\mathbf{Z}' \tag{8}$$

where $\mathbf{e}' = \left[e_1' \ \ e_2' \ \ e_3'\right]^T$ is the error vector,

$$\mathbf{Y}' = \begin{bmatrix} Z_1^2 \\ Z_2^2 \\ Z_3 \end{bmatrix}, \ \mathbf{G}' = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \ \mathbf{Z}' = \begin{bmatrix} x^2 \\ y^2 \end{bmatrix}.$$

It can be seen from (8) that $\mathbf{Z}'=\left[x^2 \quad y^2\right]^T$. To obtain the MS position (x,y), $\mathbf{Z}'$ in (8) should be solved first.

$\mathbf{Z}'$ can be obtained from (8) using the second step WLS solution:

$$\mathbf{Z}'=\left(\mathbf{G}'^T\,\mathbf{\Psi}'^{-1}\,\mathbf{G}'\right)^{-1}\mathbf{G}'^T\,\mathbf{\Psi}'^{-1}\,\mathbf{Y}' \tag{9}$$

where $\mathbf{\Psi}'$ is the covariance matrix of $\mathbf{e}'$. The final estimation of the MS position $\mathbf{Z}''=[x \quad y]^T$ is:

$$\mathbf{Z}''=sign(\mathbf{Z})\sqrt{\mathbf{Z}'} \tag{10}$$

The perturbation approach is used here to calculate $\mathbf{\Psi}'$ for solving (10). Let the estimation errors of x, y, and k be μ_1, μ_2, and μ_3. Then the elements of $\mathbf{Z}$ become:

$$Z_1=x+\mu_1,\ Z_2=y+\mu_2,\ Z_3=k+\mu_3 \tag{11}$$

Substituting (11) into (8) and ignoring the square error term, the entries of $\mathbf{e}'$ can be expressed as:

$$e'_1=2x\mu_1,\ e'_2=2y\mu_2,\ e'_3=\mu_3 \tag{12}$$

Subsequently, the covariance matrix of $\mathbf{e}'$ is:

$$\mathbf{\Psi}'=E\left(\mathbf{e}'\mathbf{e}'^T\right)=\mathbf{B}'\mathrm{cov}(\mathbf{Z})\mathbf{B}' \tag{13}$$

where $\mathbf{B}'=diag\{[2x,2y,1]\}$. In fact, $\mathbf{B}'$ is unknown as $\mathbf{B}'$ contains the true MS position x and y. Like (7), $\mathbf{B}'$ can be approximated as $\mathbf{B}'=diag\{[2Z_1,2Z_2,1]\}$. $\mathrm{cov}(\mathbf{Z})$ is the covariance matrix of $\mathbf{Z}$ and can also be obtained by using the perturbation approach. Δ is denoted as error perturbation. Expressing (6) by Taylor expansion, ignoring the square error term and retaining only the linear perturbation, $\Delta\mathbf{Z}$ can be obtained:

$$\Delta\mathbf{Z}=\left(\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{\Psi}^{-1}\Delta\mathbf{Y}=\left(\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{Bn} \tag{14}$$

where $\mathbf{n}=[n_{r1},\cdots,n_{rN},n_{s1},\cdots,n_{sN}]^T$ is a vector of measurement noise. Substituting (14) into $\mathrm{cov}(\mathbf{Z})$, the covariance matrix of $\mathbf{Z}$ can be obtained:

$$\mathrm{cov}(\mathbf{Z})=E\left[\Delta\mathbf{Z}\Delta\mathbf{Z}^T\right]=\left(\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{Bnn}^T\mathbf{B\Psi}^{-1}\mathbf{G}\left(\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{G}\right)^{-1}=\left(\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{G}\right)^{-1} \tag{15}$$

Substituting (9), (13), and (15) into (10), the MS position is given by:

$$\mathbf{Z}''=sign(\mathbf{Z})\sqrt{\left(\mathbf{G}'^T\,\mathbf{B}'^{-1}\,\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{GB}'^{-1}\,\mathbf{G}'\right)^{-1}\mathbf{G}'^T\,\mathbf{B}'^{-1}\,\mathbf{G}^T\mathbf{\Psi}^{-1}\mathbf{GB}'^{-1}\,\mathbf{Y}'} \tag{16}$$

Covariance matrix of a estimation describes the practical performance of the corresponding estimator. To analyze the real performance of the proposed method, the covariance matrix of the final estimation of MS position $\mathbf{Z}''$ has to be derived.

Since $\mathbf{Z}''$ contains $\mathbf{Z}'$ in (10), the covariance matrix of $\mathbf{Z}'$ should be calculated first. Using the perturbation approach, it can be derived from (9):

$$\mathrm{cov}(\mathbf{Z}')=\left(\mathbf{G}'^T\,\mathbf{\Psi}'^{-1}\,\mathbf{G}'\right)^{-1} \tag{17}$$

From the definition of $\mathbf{Z}'$ in (9) and ignoring the square error term, $\mathbf{Z}'$ can be rewritten as:

$$Z'_1-x^2=2xe_x,\ Z'_2-y^2=2ye_y \tag{18}$$

where e_x and e_y are the estimation errors of x, y respectively. The covariance matrix of $\mathbf{Z}''$ can be obtained from (18):

$$\mathrm{cov}(\mathbf{Z}'')=\mathbf{B}''^{-1}\,\mathrm{cov}(\mathbf{Z}')\mathbf{B}''^{-1} \tag{19}$$

where $\mathbf{B}''=diag\{[2x \quad 2y]\}$.

From (13), (15), (17), and (19), the covariance matrix of $\mathbf{Z}''$ can be finally obtained:

$$\begin{aligned}\operatorname{cov}(\mathbf{Z}'') &= \left(\mathbf{B}''\operatorname{cov}(\mathbf{Z}')^{-1}\mathbf{B}''\right)^{-1} \\ &= \left(\mathbf{B}''\mathbf{G}'^{T}\mathbf{B}'^{-1}\mathbf{G}^{T}\mathbf{B}^{-1}\mathbf{Q}^{-1}\mathbf{B}^{-1}\mathbf{G}\mathbf{B}'^{-1}\mathbf{G}'\mathbf{B}''\right)^{-1} \\ &= \left(\mathbf{B}''\mathbf{G}'^{T}\mathbf{B}'^{-1}\mathbf{G}_{\mathbf{s}}^{T}\mathbf{B}_{\mathbf{r}}^{-1}\left(\mathbf{Q}_{\mathbf{r}}^{-1}+\mathbf{Q}_{\mathbf{s}}^{-1}\right)\mathbf{B}_{\mathbf{r}}^{-1}\mathbf{G}_{\mathbf{s}}\mathbf{B}'^{-1}\mathbf{G}'\mathbf{B}''\right)^{-1}\end{aligned} \tag{20}$$

where $\mathbf{G} = [\mathbf{G}_{\mathbf{s}} \quad \mathbf{G}_{\mathbf{s}}]^T$.

3.2 The case with uncertainty

The positions of BSs in a practical system such as radar, sonar, and wireless sensor networks system are usually provided by GPS receivers. The GPS receivers may not provide exact location information due to cost and complexity constraints applied on devices. The hybrid TOA/RSS localization algorithm considering uncertainty of BS position is needed. In the presence of BS position errors, the BS position can be modeled as [17-18]:

$$\tilde{x}_i = x_i + n_{xi},\ \ \tilde{y}_i = y_i + n_{yi} \tag{21}$$

where the disturbances n_{xi} and n_{yi} are assumed to be independent zero-mean Gaussian distribution with variance σ_{xi}^2 and σ_{yi}^2, respectively.

Using the Taylor expansion and ignoring the square error term, (5) can be rewritten as:

$$\begin{aligned} e_i &= 2r_i\Delta r_i + (2x-2x_i)\Delta x_i + (2y-2y_i)\Delta y_i \\ e_{N+i} &= -\frac{\ln 10}{5n_p}10^{-P_i/(5n_p)}\Delta P_i + (2x-2x_i)\Delta x_i + (2y-2y_i)\Delta y_i \end{aligned} \tag{22}$$

where $i = 1,\cdots,N$. The covariance matrix of $\mathbf{e}$ can be obtained from (22):

$$\boldsymbol{\psi} = \operatorname{cov}(\mathbf{e}) = E\left(\mathbf{e}\mathbf{e}^T\right) = \mathbf{BQB} + \mathbf{B}_1\mathbf{Q}_1\mathbf{B}_1 + \mathbf{B}_2\mathbf{Q}_2\mathbf{B}_2 \tag{23}$$

where $\mathbf{B}_1 = diag\left\{\left[\mathbf{B}_{\mathbf{x}},\mathbf{B}_{\mathbf{x}}\right]\right\}$, $\mathbf{B}_2 = diag\left\{\left[\mathbf{B}_{\mathbf{y}},\mathbf{B}_{\mathbf{y}}\right]\right\}$, $\mathbf{Q}_1 = diag\left\{\left[\mathbf{Q}_{\mathbf{x}},\mathbf{Q}_{\mathbf{x}}\right]\right\}$, $\mathbf{Q}_2 = diag\left\{\left[\mathbf{Q}_{\mathbf{y}},\mathbf{Q}_{\mathbf{y}}\right]\right\}$, $\mathbf{B}_{\mathbf{x}} = 2diag\left\{\left[x-x_1,\cdots,x-x_N\right]\right\}$, $\mathbf{Q}_{\mathbf{x}} = diag\left\{\left[\sigma_{x1}^2,\cdots,\sigma_{xN}^2\right]\right\}$, $\mathbf{B}_{\mathbf{y}} = 2diag\left\{\left[y-y_1,\cdots,y-y_N\right]\right\}$, $\mathbf{Q}_{\mathbf{y}} = diag\left\{\left[\sigma_{y1}^2,\cdots,\sigma_{yN}^2\right]\right\}$.

Like (7), the MS position (x, y) in $\mathbf{B}_{\mathbf{x}}$ and $\mathbf{B}_{\mathbf{y}}$ can be approximated as (Z_1, Z_2).

Following the same deriving processes as the case without uncertainty, the MS position under the case with uncertainty can be calculated by substituting (23) into (16), and the covariance matrix of $\mathbf{Z}''$ can be derived as:

$$\operatorname{cov}(\mathbf{Z}'') = \left(\mathbf{B}''\mathbf{G}'^{T}\mathbf{B}'^{-1}\mathbf{G}^{T}\left(\mathbf{BQB}+\mathbf{B}_1\mathbf{Q}_1\mathbf{B}_1+\mathbf{B}_2\mathbf{Q}_2\mathbf{B}_2\right)^{-1}\mathbf{G}\mathbf{B}'^{-1}\mathbf{G}'\mathbf{B}''\right)^{-1} \tag{24}$$

4. Cramer–Rao Lower Bound

It is well known that the CRLB sets a lower limit for the covariance matrix of any unbiased estimate of parameters and determines the physical impossibility of the variance of an unbiased estimator being less than the bound [25]. Compared with the covariance matrix determining the practical performance of the proposed method, CRLB provides a benchmark to evaluate the performance of any unbiased estimator. This section derives the CRLBs of the hybrid TOA/RSS location approach for the cases with or without uncertainty.

Let $\hat{\mathbf{m}} = \left[\hat{\mathbf{r}}^T \quad \hat{\mathbf{P}}^T\right]^T$ be a measurement vector and a parameter vector $\boldsymbol{\theta}$ to be estimated, where $\boldsymbol{\theta}$ is $[x, y]^T$ for the case without uncertainty and $[x, y, x_1, \cdots, x_N, y_1, \cdots, y_N]^T$ for the case with uncertainty. The vector of range measurements $\hat{\mathbf{r}}$ is $[\hat{r}_1, \cdots, \hat{r}_N]^T$ and the vector of received power measurements $\hat{\mathbf{P}}$ is $\left[\hat{P}_1, \cdots, \hat{P}_N\right]^T$. The CRLB matrix is defined as the inverse of the Fisher information matrix (FIM) $\mathbf{J}_{\boldsymbol{\theta}}$:

$$E\left(\left(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\right)\left(\hat{\boldsymbol{\theta}}-\boldsymbol{\theta}\right)^T\right) \geq \mathbf{J}_{\boldsymbol{\theta}}^{-1} \tag{25}$$

where $\hat{\boldsymbol{\theta}}$ is an estimate of $\boldsymbol{\theta}$.

The FIM is determined by [25]:

$$\mathbf{J}_{\boldsymbol{\theta}} = E\left[\frac{\partial \ln f\left(\hat{\mathbf{m}};\boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\left(\frac{\partial \ln f\left(\hat{\mathbf{m}};\boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\right)^T\right] \tag{26}$$

where $f\left(\hat{\mathbf{m}};\boldsymbol{\theta}\right)$ is probability density function (PDF).

Using the Bayes' theorem,

$$f\left(\hat{\mathbf{m}};\boldsymbol{\theta}\right) = f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right) f\left(\boldsymbol{\theta}\right) \tag{27}$$

Substituting (27) into (26), gives

$$\mathbf{J}_{\boldsymbol{\theta}} = E\left[\frac{\partial \ln f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\left(\frac{\partial \ln f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\right)^T\right] + E\left[\frac{\partial \ln f\left(\boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\left(\frac{\partial \ln f\left(\boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\right)^T\right] \tag{28}$$

From (1) and (3), the joint conditional PDF $f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right)$ can be written as:

$$f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right) = \prod_{i=1}^{N} f\left(\hat{r}_i \mid \boldsymbol{\theta}\right) f\left(\hat{P}_i \mid \boldsymbol{\theta}\right) \tag{29}$$

where

$$f\left(\hat{r}_i \mid \boldsymbol{\theta}\right) = \frac{1}{\sqrt{2\pi}\sigma_{ri}} \exp\left(-\frac{\left(\hat{r}_i - r_i\right)^2}{2\sigma_{ri}^2}\right), f\left(\hat{P}_i \mid \boldsymbol{\theta}\right) = \frac{1}{\sqrt{2\pi}\sigma_{si}} \exp\left(-\frac{\left(\hat{P}_i - P_i\right)^2}{2\sigma_{si}^2}\right)$$

The log of $f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right)$ is:

$$\ln f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right) = \sum_{i=1}^{N}\left(\ln f\left(\hat{r}_i \mid \boldsymbol{\theta}\right) + \ln f\left(\hat{P}_i \mid \boldsymbol{\theta}\right)\right) \tag{30}$$

Substituting (30) into $\partial \ln f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right) / \partial \boldsymbol{\theta}_i$, gives:

$$\frac{\partial \ln f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}_i} = \sum_{i=1}^{N}\left(\frac{\left(\hat{r}_i - r_i\right)}{\sigma_{ri}^2} - \frac{10 n_p \left(\hat{P}_i - P_i\right)}{r_i \ln 10 \sigma_{si}^2}\right)\frac{\partial r_i}{\partial \boldsymbol{\theta}_i} \tag{31}$$

where

$\frac{\partial r_i}{\partial x} = \frac{x - x_i}{r_i}$, $\frac{\partial r_i}{\partial y} = \frac{y - y_i}{r_i}$, $\frac{\partial r_i}{\partial x_i} = \frac{x_i - x}{r_i}$, and $\frac{\partial r_i}{\partial y_i} = \frac{y_i - y}{r_i}$.

4.1 CRLB for the case without uncertainty

Since the BS position is perfectly known in this case, the FIM $\mathbf{J}_{\boldsymbol{\theta}}$ in (28) becomes:

$$\mathbf{J}_1 = E\left[\frac{\partial \ln f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\left(\frac{\partial \ln f\left(\hat{\mathbf{m}} \mid \boldsymbol{\theta}\right)}{\partial \boldsymbol{\theta}}\right)^T\right] \tag{32}$$

Substituting (31) into FIM $\mathbf{J}_1$, gives:

$$\mathbf{J}_1 = \mathbf{H}_1\mathbf{Q}_\mathbf{r}^{-1}\mathbf{H}_1^T + \mathbf{H}_1\mathbf{Q}_\mathbf{s}^{-1}\mathbf{H}_1^T \tag{33}$$

where $\mathbf{H}_1 = \begin{bmatrix} \partial r_1/\partial x & \cdots & \partial r_N/\partial x \\ \partial r_1/\partial y & \cdots & \partial r_N/\partial y \end{bmatrix}$, $\frac{\partial r_i}{\partial x} = \frac{x - x_i}{r_i}$, and $\frac{\partial r_i}{\partial y} = \frac{y - y_i}{r_i}$. Since $\mathbf{H}_1\mathbf{Q}_\mathbf{r}^{-1}\mathbf{H}_1^T$ and $\mathbf{H}_1\mathbf{Q}_\mathbf{s}^{-1}\mathbf{H}_1^T$ are the FIMs for TOA and RSS techniques respectively, the FIM for TOA/RSS hybrid scheme can be rewritten as:

$$\mathbf{J}_1 = \mathbf{J}_{TOA} + \mathbf{J}_{SS} = \mathbf{H}_1\mathbf{Q}_\mathbf{r}^{-1}\mathbf{H}_1^T + \mathbf{H}_1\mathbf{Q}_\mathbf{s}^{-1}\mathbf{H}_1^T \tag{34}$$

Substituting (34) into (25), the CRLB for the case without uncertainty can be obtained. (34) shows that the CRLB of the TOA/SS location technique for the case without uncertainty depends on the geometrical structure of MS and BSs $\mathbf{H}_1$, covariance matrix of TOA measurements $\mathbf{Q}_\mathbf{r}$, and covariance matrix of SS measurements $\mathbf{Q}_\mathbf{s}$. Therefore the performance of TOA/SS location technique can be improved throough adjusting the geometrical structure of MS and BSs and reducing the measurement noise of TOA and SS measurements.

4.2 CRLB for the case with uncertainty

For this case, the PDF of the BS positions $\tilde{x}_i$ and $\tilde{y}_i$ can be obtained from (21):

$$f(\boldsymbol{\theta}) = \prod_{i=1}^{N} f(x_i)\, f(y_i) \tag{35}$$

where

$$f(x_i) = \frac{1}{\sqrt{2\pi}\sigma_{xi}}\exp\left(-\frac{(\tilde{x}_i - x_i)^2}{2\sigma_{xi}^2}\right),\ f(y_i) = \frac{1}{\sqrt{2\pi}\sigma_{yi}}\exp\left(-\frac{(\tilde{y}_i - y_i)^2}{2\sigma_{yi}^2}\right) \tag{36}$$

Substituting (35) into $\partial \ln f(\boldsymbol{\theta})/\partial \boldsymbol{\theta}_i$, gives:

$$\frac{\partial \ln f(\boldsymbol{\theta})}{\partial x} = \frac{\partial \ln f(\boldsymbol{\theta})}{\partial y} = 0 \tag{37}$$

$$\frac{\partial \ln f(\boldsymbol{\theta})}{\partial x_i} = -\frac{\tilde{x}_i - x_i}{\sigma_{xi}^2}$$

$$\frac{\partial \ln f(\boldsymbol{\theta})}{\partial y_i} = -\frac{\tilde{y}_i - y_i}{\sigma_{yi}^2}$$

$$E\left[\frac{\partial \ln f(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\left(\frac{\partial \ln f(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^T\right] = \mathbf{Q}_4 = \begin{bmatrix} \mathbf{0}_{2\times 2} & \mathbf{0}_{2\times 2N} \\ \mathbf{0}_{2N\times 2} & \mathbf{Q}_3^{-1} \end{bmatrix}$$

where $\mathbf{Q}_3 = diag\{[\mathbf{Q}_\mathbf{x}, \mathbf{Q}_\mathbf{y}]\}$, $\mathbf{Q}_\mathbf{x} = diag\{[\sigma_{x1}^2, \cdots, \sigma_{xN}^2]\}$, and $\mathbf{Q}_\mathbf{y} = diag\{[\sigma_{y1}^2, \cdots, \sigma_{yN}^2]\}$.

From (31),

$$E\left[\frac{\partial \ln f(\hat{\mathbf{m}}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\left(\frac{\partial \ln f(\hat{\mathbf{m}}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^T\right] = \mathbf{H}_2\mathbf{Q}_\mathbf{r}^{-1}\mathbf{H}_2^T + \mathbf{H}_2\mathbf{Q}_\mathbf{s}^{-1}\mathbf{H}_2^T \tag{38}$$

where

$$\mathbf{H}_2=\begin{bmatrix}\partial r_1/\partial x & \cdots & \partial r_N/\partial x\\ \partial r_1/\partial y & \cdots & \partial r_N/\partial y\\ \partial r_1/\partial x_1 & \cdots & \partial r_N/\partial x_1\\ \vdots & \cdots & \vdots\\ \partial r_1/\partial x_N & \cdots & \partial r_N/\partial x_N\\ \partial r_1/\partial y_1 & \cdots & \partial r_N/\partial y_1\\ \vdots & \cdots & \vdots\\ \partial r_1/\partial y_N & \cdots & \partial r_N/\partial y_N\end{bmatrix},\ \frac{\partial r_i}{\partial x}=\frac{x-x_i}{r_i},\ \frac{\partial r_i}{\partial y}=\frac{y-y_i}{r_i},\ \frac{\partial r_i}{\partial x_j}=\begin{cases}-\dfrac{x-x_i}{r_i} & i=j\\ 0 & i\neq j\end{cases},$$

and $\frac{\partial r_i}{\partial y_j}=\begin{cases}-\dfrac{y-y_i}{r_i} & i=j\\ 0 & i\neq j\end{cases}$.

Substituting (37) and (38) into (28), gives:

$$\mathbf{J}_2=\mathbf{H}_2\mathbf{Q}_{\mathbf{r}}^{-1}\mathbf{H}_2^T+\mathbf{H}_2\mathbf{Q}_{\mathbf{s}}^{-1}\mathbf{H}_2^T+\mathbf{Q}_4 \tag{39}$$

Compared (34) with (39), geometrical structure matrix $\mathbf{H}_2$ and BS position errors $\mathbf{Q}_4$ result in the performance difference between the CRLBs with or without uncertainty. Relationship between the two CRLBs will be discussed in the section 5.

5. Performance Analysis

The characteristics of TOA/RSS localization technique are provided in the following propositions.

Proposition 1: *Under the Gaussian noise environments, the proposed method for the case without uncertainty can attain its CRLB:*

$$\mathrm{cov}(\mathbf{Z}'')=\left(\mathbf{B}''\mathbf{G}'^T\mathbf{B}'^{-1}\mathbf{G}_{\mathbf{s}}^T\mathbf{B}_{\mathbf{r}}^{-1}\left(\mathbf{Q}_{\mathbf{r}}^{-1}+\mathbf{Q}_{\mathbf{s}}^{-1}\right)\mathbf{B}_{\mathbf{r}}^{-1}\mathbf{G}_{\mathbf{s}}\mathbf{B}'^{-1}\mathbf{G}'\mathbf{B}''\right)^{-1}=\left(\mathbf{H}_1\mathbf{Q}_{\mathbf{r}}^{-1}\mathbf{H}_1^T+\mathbf{H}_1\mathbf{Q}_{\mathbf{s}}^{-1}\mathbf{H}_1^T\right)^{-1}=\mathbf{J}_1^{-1} \tag{40}$$

Proof: Substituting $\mathbf{B}''$, $\mathbf{G}'$, and $\mathbf{B}'$ into $\mathbf{B}''\mathbf{G}'^T\mathbf{B}'^{-1}$, gives:

$$\mathbf{B}''\mathbf{G}'^T\mathbf{B}'^{-1}=\begin{bmatrix}2x & 0\\ 0 & 2y\end{bmatrix}\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{2x} & 0 & 0\\ 0 & \frac{1}{2y} & 0\\ 0 & 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 0 & 2x\\ 0 & 1 & 2y\end{bmatrix} \tag{41}$$

Substituting $\mathbf{G}_{\mathbf{s}}$, and $\mathbf{B}_{\mathbf{r}}$ into $\mathbf{G}_{\mathbf{s}}^T\mathbf{B}_{\mathbf{r}}^{-1}$, gives:

$$\mathbf{G}_{\mathbf{s}}^T\mathbf{B}_{\mathbf{r}}^{-1}=\begin{bmatrix}-\frac{x_1}{r_1} & \cdots & -\frac{x_N}{r_N}\\ -\frac{y_1}{r_1} & \cdots & -\frac{y_N}{r_N}\\ \frac{1}{2r_1} & \cdots & \frac{1}{2r_N}\end{bmatrix} \tag{42}$$

Substituting (41) and (42) into $\mathbf{B}''\mathbf{G}'^T\mathbf{B}'^{-1}\mathbf{G}_{\mathbf{s}}^T\mathbf{B}_{\mathbf{r}}^{-1}$, gives:

$$\mathbf{B}''\mathbf{G}'^T\mathbf{B}'^{-1}\mathbf{G}_{\mathbf{s}}^T\mathbf{B}_{\mathbf{r}}^{-1}=\mathbf{H}_1=\begin{bmatrix}\dfrac{x-x_1}{r_1} & \cdots & \dfrac{x-x_N}{r_N}\\ \dfrac{y-y_1}{r_1} & \cdots & \dfrac{y-y_N}{r_N}\end{bmatrix} \tag{43}$$

Substituting (43) into (19), the covariance matrix of the proposed method for the case with uncertainty becomes:

$$\mathrm{cov}(\mathbf{Z}'')=\left(\mathbf{H}_1\mathbf{Q}_{\mathbf{r}}^{-1}\mathbf{H}_1^T+\mathbf{H}_1\mathbf{Q}_{\mathbf{s}}^{-1}\mathbf{H}_1^T\right)^{-1} \tag{44}$$

It can be seen from (44) and (34) that the proposed method for the case without uncertainty can attain the corresponding CRLB.

Proposition 2: *The TOA/SS scheme has a lower CRLB than the TOA (or SS) scheme:*

$$tr\left\{\mathbf{J}_1^{-1}\right\} = tr\left\{\left(\mathbf{J}_{TOA} + \mathbf{J}_{SS}\right)^{-1}\right\} \le \min\left\{tr\left\{\mathbf{J}_{TOA}^{-1}\right\}, tr\left\{\mathbf{J}_{SS}^{-1}\right\}\right\} \quad (45)$$

where $\mathbf{J}_{TOA} = \mathbf{H}_1\mathbf{Q}_\mathbf{r}^{-1}\mathbf{H}_1^T$ *and* $\mathbf{J}_{SS} = \mathbf{H}_1\mathbf{Q}_\mathbf{s}^{-1}\mathbf{H}_1^T$ *are the FIMs for TOA and SS localization techniques respectively.*

Proof: It can be seen from Appendix A that $\mathbf{J}_{TOA}$ and $\mathbf{J}_{SS}$ are positive definite matrices. Therefore proposition 2 can be proved from Appendix B.

Proposition 3: *In the TOA/SS localization technique, the CRLB of the case with uncertainty is higher than that of the case without uncertainty.*

Proof: From (39), The FIM for the case with uncertainty $\mathbf{J}_2$ can be rewritten as:

$$\mathbf{J}_2 = \begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_{ur} \\ \mathbf{J}_{ur}^T & \mathbf{J}_{lr} \end{bmatrix} \quad (46)$$

where $\mathbf{J}_1$ is the FIM for the case without uncertainty, $\mathbf{J}_{ur}$ is an $2\times 2N$ matrix, and $\mathbf{J}_{lr}$ is an $2N\times 2N$ symmetric matrix.

Let $\left[\mathbf{J}_2^{-1}\right]_{ul}$ be the upper left 2×2 block of $\mathbf{J}_2^{-1}$. From the matrix inversion lemma [26],

$$\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \left(\mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21}\right)^{-1} & \mathbf{A}_{11}^{-1}\mathbf{A}_{12}\left(\mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12} - \mathbf{A}_{22}\right)^{-1} \\ \left(\mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12} - \mathbf{A}_{22}\right)^{-1}\mathbf{A}_{21}\mathbf{A}_{11}^{-1} & \left(\mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-1}\mathbf{A}_{12}\right)^{-1} \end{bmatrix} \quad (47)$$

Using the above equation, $\left[\mathbf{J}_2^{-1}\right]_{ul}$ becomes:

$$\left[\mathbf{J}_2^{-1}\right]_{ul} = \left(\mathbf{J}_1 - \mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T\right)^{-1} \quad (48)$$

It can be seen from appendix A.1 and A.2 that $\mathbf{J}_{lr}^{-1}$ and $\mathbf{J}_1 - \mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T$ are positive definite matrices.

Since $\mathbf{J}_{ur}$ is a full row rank matrix and $\mathbf{J}_{lr}^{-1}$ is a positive definite matrix, It can be seen from appendix A.2(4) that $\mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T$ is also a positive definite matrix. The FIM of case without uncertainty can be rewritten as:

$$\mathbf{J}_1^{-1} = \left(\mathbf{J}_1 - \mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T + \mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T\right)^{-1} \quad (49)$$

Note that $\mathbf{J}_1 - \mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T$ and $\mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T$ are positive definite matrices. Thus, it can be seen from appendix B that:

$$tr\left\{\left(\mathbf{J}_1 - \mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T + \mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T\right)^{-1}\right\} \le tr\left\{\left(\mathbf{J}_1 - \mathbf{J}_{ur}\mathbf{J}_{lr}^{-1}\mathbf{J}_{ur}^T\right)^{-1}\right\} \quad (50)$$

From (48), (49), and (50),

$$tr\left\{\mathbf{J}_1^{-1}\right\} \le tr\left\{\left[\mathbf{J}_2^{-1}\right]_{ul}\right\} \quad (51)$$

Thus, the case without uncertainty has the lower CRLB than the case with uncertainty.

6. Simulation Results

The simulations are based on the cells of 2000m of diameter. A hexagonal test cell is surrounded by 6 neighboring cells as shown in **Fig. 1**. The coordinates of BSs are BS_1 (0,0)m, BS_2 (1732,1000)m, BS_3 (0,2000)m, BS_4 (-1732,1000) m, BS_5 (-1732,-1000)m, BS_6 (0,-2000)m, and BS_7 (-1730,-1000)m. The position of MS is uniformly distributed in the square space $-1000 \le x, y \le 1000$ m.

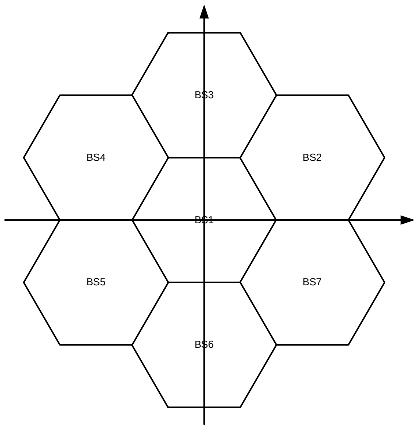

Fig. 1. Cell layout

The RMSEs are defined as $\sqrt{E\left[(x-\hat{x})^2+(y-\hat{y})^2\right]}$ in the units of m, and are obtained from the average of 5000 independent runs. To compare with the proposed methods, the least square localization method is selected here due to its wide application in the wireless location system [27]. Using the least square method, the MS position can be estimated as:

$$\mathbf{Z}=\left(\mathbf{G}^T\mathbf{G}\right)^{-1}\mathbf{G}^T\mathbf{Y}$$

where $\mathbf{Z}$ contains the MS position, $\mathbf{G}$ and $\mathbf{Y}$ can be obtained from (5).

Fig. 2 shows the RMSEs versus standard deviations (stds) of range measurements for the case without uncertainty when the std of the measured received power is 2dB. It can be seen from figure that the proposed method provides much better performance than the least square method. As the range noise becomes small, the positioning accuracy of the proposed method increases. However, the least square method has almost the same performance under different range noises. This is because the least square method gives a unified weight to each measurement. Although the range noise becomes small, the noise of SS is still large. In this case, the least square method using the unified weight will lead to large poisoning error whereas the position accuracy can be improved using different weights in the proposed method. **Fig. 3** shows the RMSEs versus stds of the measured received power when the std of the range noise is 10m. It can be also observed that the proposed method outperforms the least square method.

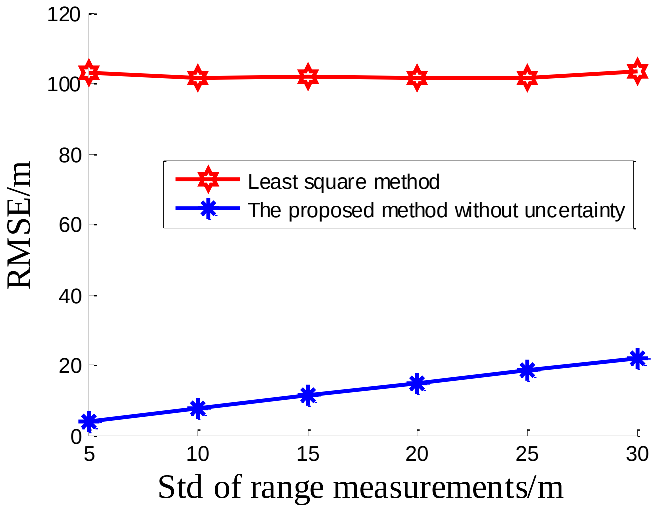

Fig. 2. Comparison between the least square method and the proposed method under different range noises

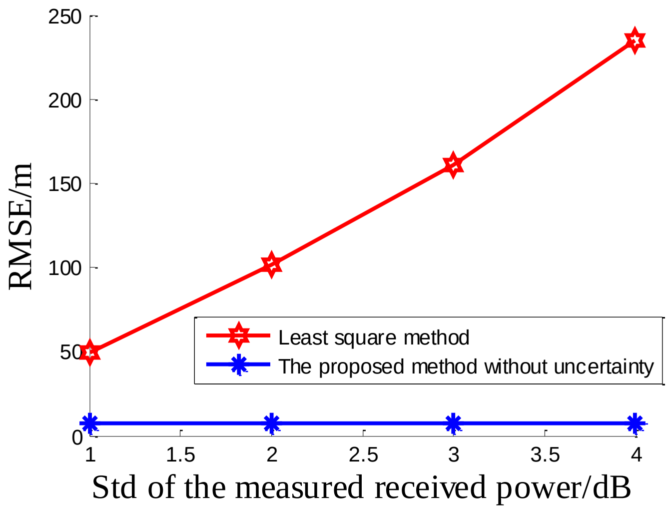

Fig. 3. Comparison between the least square method and the proposed method under different power noises

Comparisons among the proposed methods, its theory variances and CRLBs are recorded in the **Fig. 4**. The std of range measurements is varied from 5m to 30m and the std of received measured power is 2dB. For the case with uncertainty, σ_{xi} and σ_{yi} are set to be 10m. **Fig. 4** shows that the proposed methods can attain the corresponding CRLBs and the CRLB of the case with uncertainty is higher than that of the case without uncertainty, which matches the proposition 1 and 3.

Fig. 5 is performed to study the differences among the TOA, SS, and hybrid TOA/SS localization techniques. In this simulation, the std of range measurements is varied from 5m to 30m and the std of received measured power is 0.5dB. The results show that the TOA/SS scheme has a lower CRLB than the TOA (or SS) scheme, which verifies the proposition 2.

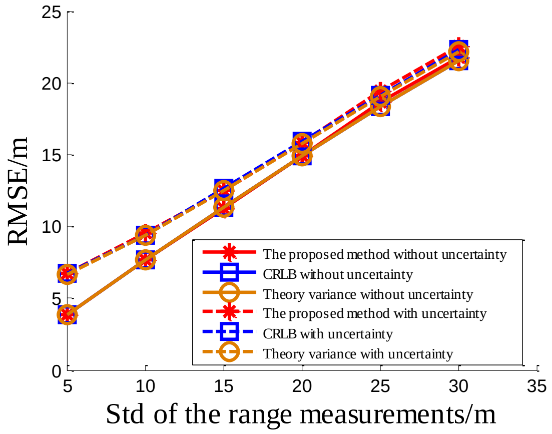

Fig. 4. Comparisons among the proposed methods, theory variances, and CRLBs for both the cases with or without uncertainty

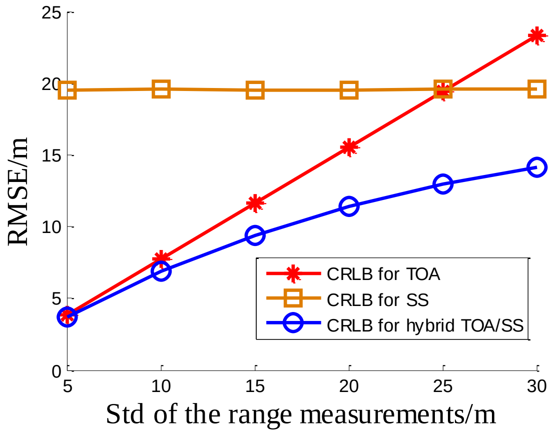

Fig. 5. CRLB comparisons among the TOA, SS, and hybrid TOA/RSS methods

7. Conclusion

Two TOA/SS localization methods are proposed based on two-step WLS estimator for the cases with or without uncertainty in the positions of BSs. Compared with other hybrid localization methods, the proposed methods can not only provide the closed-form solutions but also attain the CRLB which is verified by the theoretical analysis and simulations. In addition, this paper first considers the case with uncertainty in the TOA/SS localization technique. Both the theoretical variances and CRLBs for the proposed methods are derived and the relationship between the cases with or without uncertainty is given. The paper also proves that the TOA/SS scheme has a lower CRLB than the TOA (or SS) scheme.

8. Appendix

8.1 Some properties of matrix

The following results are well known and can be easily derived.

***A.1* Proposition 4:** *the FIM* $\mathbf{J}$ *is a positive definite matrix.*

Proof: From the FIM definition equation (26) and note that $\mathbf{J}$ is the real matrix, the quadratic form of the FIM $\mathbf{J}$ is

$$\mathbf{XJX}^T = \mathbf{X}E\left[\frac{\partial \ln f(\mathbf{r}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \cdot \left(\frac{\partial \ln f(\mathbf{r}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^T\right]\mathbf{X}^T = E\left[\left(\mathbf{X}\frac{\partial \ln f(\mathbf{r}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)\left(\mathbf{X}\frac{\partial \ln f(\mathbf{r}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^T\right] \quad \text{(A.1)}$$

Note that $\mathbf{X}$ is a constant vector and $\frac{\partial \ln f(\mathbf{r}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}$ is a random vector. Thus, $\mathbf{X}\frac{\partial \ln f(\mathbf{r}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} \neq 0$. Then

$$\mathbf{XJX}^T = E\left[\left(\mathbf{X}\frac{\partial \ln f(\mathbf{r}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)\left(\mathbf{X}\frac{\partial \ln f(\mathbf{r}|\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right)^T\right] > 0, \forall \mathbf{X} \neq 0 \quad \text{(A.2)}$$

Then we can get that the FIM $\mathbf{J}$ is a positive definite matrix.

A.2*: if the* $n \times n$ *Hermitian matrix* $\mathbf{A}$ *is a positive definite matrix, then*

(1) $\mathbf{A}^{-1}$ *is also a positive definite matrix.*

(2) there is a $n \times n$ *non-singular matrix* $\mathbf{a}$ *, which makes* $\mathbf{A} = \mathbf{a}\mathbf{a}^H$ *.*

(3) the matrix, which is the submatrix deleted the k *th row and the* k *th column from* $\mathbf{A}$ *(*$1 \le k \le n$*), is also a positive definite matrix.*

(4) the matrix $\mathbf{CAC}^H$, where $\mathbf{C}$ is the $m \times n$ matrix with full row rank, is also a positive definite matrix.

8.2 Proposition 5

If $\mathbf{J}_1$ *and* $\mathbf{J}_2$ *are both positive definite matrices, then*

$$tr\left\{(\mathbf{J}_1 + \mathbf{J}_2)^{-1}\right\} \le \min\left\{tr\left\{\mathbf{J}_2^{-1}\right\}, tr\left\{\mathbf{J}_2^{-1}\right\}\right\}$$

Proof: Since $\mathbf{J}_1$ and $\mathbf{J}_2$ are positive definite matrices, we have:

$$\mathbf{J}_1 = bb^T, \ \mathbf{J}_2 = aa^T \quad \text{(B.1)}$$

where a and b are non-singular square matrices.

Substituting (B.1) into $(\mathbf{J}_1 + \mathbf{J}_2)^{-1}$ and using the matrix inversion lemma [26], we have:

$$(\mathbf{J}_1 + \mathbf{J}_2)^{-1} = (\mathbf{J}_1 + aa^T)^{-1} = \mathbf{J}_1^{-1} - \mathbf{J}_1^{-1}a(\mathbf{I} + a^T\mathbf{J}_1^{-1}a)^{-1}a^T\mathbf{J}_1^{-1} = \mathbf{J}_1^{-1} - \mathbf{J}_1^{-1}a\left(\mathbf{I} + a^Tb^{-1}(b^T)^{-1}a\right)^{-1}a^T\mathbf{J}_1^{-1} \quad \text{(B.2)}$$

The quadratic form of $\mathbf{I} + a^Tb^{-1}(b^T)^{-1}a$ is given by:

$$\mathbf{X}\left(\mathbf{I} + a^Tb^{-1}(b^T)^{-1}a\right)\mathbf{X}^T = \mathbf{XX}^T + \mathbf{X}a^Tb^{-1}(b^T)^{-1}a\mathbf{X}^T = \mathbf{XX}^T + (\mathbf{X}a^Tb^{-1})(\mathbf{X}a^Tb^{-1})^T > 0, \ \forall \mathbf{X} \neq 0 \quad \text{(B.3)}$$

It can be seen from (B.3) that $\mathbf{I} + a^Tb^{-1}(b^T)^{-1}a$ is a positive definite matrix. Thus, $\left(\mathbf{I} + a^Tb^{-1}(b^T)^{-1}a\right)^{-1}$ is also a positive definite matrix and can be decomposed as:

$$\left(\mathbf{I} + a^Tb^{-1}(b^T)^{-1}a\right)^{-1} = dd^T \quad \text{(B.4)}$$

where d is a non-singular square matrix.

Substituting (B.4) into (B.2), we have:

$$(\mathbf{J}_1+\mathbf{J}_2)^{-1}=\mathbf{J}_1^{-1}-\mathbf{J}_1^{-1}add^Ta^T\mathbf{J}_1^{-1} \tag{B.5}$$

The quadratic form of $\mathbf{J}_1^{-1}add^Ta^T\mathbf{J}_1^{-1}$ is given by:

$$\mathbf{X}\mathbf{J}_1^{-1}add^Ta^T\mathbf{J}_1^{-1}\mathbf{X}^T=\mathbf{X}\mathbf{J}_1^{-1}ad\left(\mathbf{X}\mathbf{J}_1^{-1}ad\right)^T\geq 0,\ \forall\mathbf{X}\neq 0 \tag{B.6}$$

Therefore, $\mathbf{J}_1^{-1}add^Ta^T\mathbf{J}_1^{-1}$ is a semi-positive definite matrix and:

$$tr\left\{\mathbf{J}_1^{-1}add^Ta^T\mathbf{J}_1^{-1}\right\}\geq 0 \tag{B.7}$$

From (B.5) and (B.7), we have:

$$tr\left\{(\mathbf{J}_1+\mathbf{J}_2)^{-1}\right\}=tr\left\{\mathbf{J}_1^{-1}-\mathbf{J}_1^{-1}add^Ta^T\mathbf{J}_1^{-1}\right\}=tr\left\{\mathbf{J}_1^{-1}\right\}-tr\left\{\mathbf{J}_1^{-1}add^Ta^T\mathbf{J}_1^{-1}\right\}\leq tr\left\{\mathbf{J}_1^{-1}\right\} \tag{B.8}$$

Using the same proof processing, we have:

$$tr\left\{(\mathbf{J}_1+\mathbf{J}_2)^{-1}\right\}\leq tr\left\{\mathbf{J}_2^{-1}\right\} \tag{B.9}$$

From (B.8) and (B.9), we have:

$$tr\left\{(\mathbf{J}_1+\mathbf{J}_2)^{-1}\right\}\leq\min\left\{tr\left\{\mathbf{J}_1^{-1}\right\},tr\left\{\mathbf{J}_2^{-1}\right\}\right\} \tag{B.10}$$

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