Optimal Price Strategy Selection for MVNOs in Spectrum Sharing: An Evolutionary Game Approach

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Abstract

The optimal price strategy selection of two bounded rational cognitive mobile virtual network operators (MVNOs) in a duopoly spectrum sharing market is investigated. The bounded rational operators dynamically compete to sell the leased spectrum to secondary users in order to maximize their profits. Meanwhile, the secondary users' heterogeneous preferences to rate and price are taken into consideration. The evolutionary game theory (EGT) is employed to model the dynamic price strategy selection of the MVNOs taking into account the response of the secondary users. The behavior dynamics and the evolutionary stable strategy (ESS) of the operators are derived via replicated dynamics. Furthermore, a reward and punishment mechanism is developed to optimize the performance of the operators. Numerical results show that the proposed evolutionary algorithm is convergent to the ESS, and the incentive mechanism increases the profits of the operators. It may provide some insight about the optimal price strategy selection for MVNOs in the next generation cognitive wireless networks.

Keywords: evolutionary game theory, cognitive radio, mobile virtual network operator, evolutionary stable strategy, replicator dynamics, reward and punishment

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1. Introduction

Currently, dynamic spectrum sharing among primary and secondary users has been drawn great attention due to its potential to improve spectrum efficiency [1][2][3]. Game theory, as one way to optimize and improve the efficiency of dynamic spectrum sharing, is widely used to model and analyze interactive decision-making processes among multiple users [4][5]. A non-cooperative game is employed to solve the problem of dynamic spectrum sharing with multiple strategic primary and secondary users [4], while a cooperative game was proposed to model the distributed spectrum access for unlicensed spectrum sharing [5].

Most existing works using classical game theory mainly rely on the assumption that: (a) the users are completed rational and always following their optimal strategy, (b) have common knowledge of rationality, and (c) fully aware of the game they are playing. Nevertheless, these assumptions may not feasible for all the users in the real cognitive radio network. Not all the users are rational and have the ability to judge correctly and predict perfectly. In fact, the users belong to the bounded rational population. Additionally, the problem becomes even more complicated if the game involves a dynamic process. Therefore, the employment of evolutionary game theory (EGT) is developed, which provides a good means to address the strategic uncertainty that a player faces, reveal the underlying dynamics and find a robust equilibrium strategy [6]. Moreover, it can give a better answer to the selection of multiple equilibria and better tackle the unpredictable behavior of players.

Recently, EGT has been used to many facts. Such as, EGT is employed to model the distributed cooperative sensing over cognitive radio networks [7] and investigate the dynamic network selection [8]. In addition, EGT is also carried out to investigate the new robust equilibrium concepts and describe the dynamics of competition in networking [9], and analyze the dynamics of access strategy in cognitive radio ad hoc network [10].

In this paper, a dynamic spectrum sharing among two bounded rational MVNOs and multiple secondary users is investigated. The MVNO provides mobile phone services but does not have its own licensed frequency allocation of radio spectrum, nor does it necessarily have all of the infrastructures required to provide mobile telephone service [11]. As intermediaries between the spectrum owner and the secondary users, MVNOs can raise the competition level of the wireless markets through providing competitive pricing plans as well as more flexible services [12]. We consider the case that the two MVNOs lease the temporarily unused wireless spectrum and then dynamically compete via price strategy to serve the potential secondary users. The response of the secondary users is modeled through an acceptance probability which reflects its willingness to buy the offered service with the rate and the asked price. The EGT is proposed to investigate the price strategy selection for MVNOs with a dynamic viewpoint, where the quality and service (QoS) of the secondary users are taken into account. Via replicated dynamics, the strategy equilibria of the operators are derived. Then the stability of the proposed algorithm is analyzed based on the concept of evolutionary stable strategy (ESS). Furthermore, in order to maintain the collusion of the MVNOs, a reward and punishment mechanism is introduced into the proposed evolutionary algorithm. The computational results show that the proposed evolutionary algorithm is convergent to ESS, and evidence the feasibility of the proposed incentive-mechanized EGT algorithm in optimizing the total profits of the MVNOs.

The rest of this paper is organized as follows. The background on EGT is briefly reviewed in Section 2. Section 3 presents a description of the system model. Price based evolutionary game and strategy analysis are shown in Section 4. Collusion based on reward and punishment mechanism is introduced in Section 5. Section 6 presents and discusses the numerical results from simulations. Finally, conclusions are drawn in Section 7.

2. Background on Evolutionary Game Theory

EGT originated as an application of game theory to biological evolving populations of life forms. There are two approaches to EGT. The first approach derives from the work of Maynard Smith and Price, and employs the concept of ESS as the principal tool of analysis. [6][13]. A strategy is called an ESS if, whenever all members of the population adopt it, no dissident strategy could invade the population under the influence of natural selection [14][15]. Let us denote $u(p,q)$ as the payoff of a player using strategy p against another player using strategy q. Then, the formal definition of an ESS can be defined as following [16].

Definition 1: A strategy q is an ESS if and only if, for all $p \neq q$, there exists some $\bar{\varepsilon}_y > 0$, such that

$$U(q, \varepsilon p + (1-\varepsilon)q) > U(p, \varepsilon p + (1-\varepsilon)q) \tag{1}$$

holds for all $\varepsilon \in (0, \bar{\varepsilon}_y)$

The second approach of EGT constructs an explicit model of the process by which the frequency of strategies changes in the population and studies properties of the evolutionary dynamics within that model [13]. Replicator dynamics is one of the most studied dynamics in EGT [17][18]. It describes evolutions of the distribution of strategies in the population itself. Replicated dynamic equation is expressed as following [19]:

$$\dot{x}_i = \varepsilon[u(s_i, x_{-s_i}) - \overline{U}(x)]x_i \tag{2}$$

where ε denotes the factor that affects the evolution speed. x_i is the population share of players that select pure strategy s_i at time t. $u(s_i, x_{-s_i})$ represents the average payoff of the individuals that choose strategy s_i, x_{-s_i} is the set of population share who use pure strategies other than s_i, and $\overline{U}(x)$ is the average payoff of the entire population. The intuition behind (2) is as follows: if strategy s_i results in a higher payoff than the average level, the percentage using s_i will grow, and the growth rate $\dot{x}_i(t)/x_i$ can be viewed as the difference between the fitness $u(s_i, x_{-s_i})$ of s_i and the average payoff $\overline{U}(x)$ of the population.

3. System Model

3.1 Network Model

A duopoly wireless system with two cognitive MVNOs, operator 1 and operator 2 is considered. Both operators lease spectrum from the same spectrum owner, and participate in price competition to attract the secondary users which equipped with software defined radios within a specified region as shown in Fig. 1. The MVNOs are non-cooperative and both aim to maximize their own profits. The secondary users are assumed to be able to switch between the two operators freely.

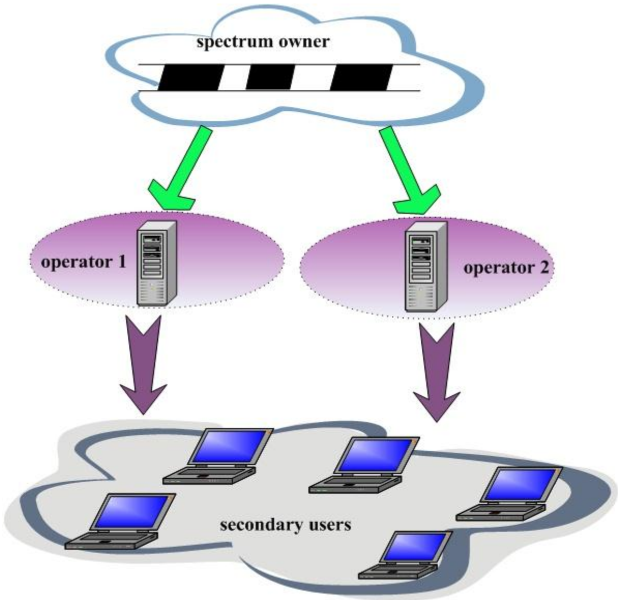

Fig. 1. System model of the MVNOs

3.2 Model for Secondary Users

3.2.1 Utility Function

It is important to take the secondary users heterogeneous preference over the rates R that provided by the operators into account. In general, the utilities of secondary users are assumed to satisfy following properties [20]:

$$\frac{du(R)}{dR} \geq 0, \ \lim_{R \to \infty} u(R) = l \tag{3}$$

The utility function should be an increasing function of R. Meanwhile, once an already high grade of users' satisfaction has been obtained, the utility will not increase. Consequently, the utility function of secondary user is defined as follows [21]:

$$u = \frac{(R/K)^m}{1+(R/K)^m} \tag{4}$$

where R is the rate that the operators offered. K and m (≥ 2) are positive constants that determine the exact shape of the above utility function.

3.2.2 Acceptance Probability

Moreover, an acceptance probability $A(u, p)$ is introduced, where u is the utility function of the secondary user and p is the price that the MVNOs announced. The acceptance probability should have the following properties that it increases as the utility increasing and the price decreasing [20]:

$$\frac{\partial A}{\partial u} \geq 0 \, , \; \frac{\partial A}{\partial p} \leq 0 \tag{5}$$

In more detail, the properties of the acceptance probability can be expressed mathematically:

$$\begin{aligned} &\forall P > 0, \; \lim_{p \to 0} A(u, p) = 1, \; \lim_{p \to \infty} A(u, p) = 0 \\ &\forall u > 0, \; \lim_{u \to 0} A(u, p) = 0, \; \lim_{u \to \infty} A(u, p) = 1 \end{aligned} \tag{6}$$

Therefore, the acceptance probability of the secondary users is defined as [20]:

$$A = e^{-\alpha p^{\beta} u^{-\gamma}} \tag{7}$$

where β is the price sensitivity of the secondary user, γ represents the utility sensitivity of the secondary user, and α is an appropriate constant.

Since u is a function of R, A is a function of R and p. The acceptance probability as a function of offered rate R and price p is shown in Fig. 2. The parameters are assumed to be: $\alpha = 0.05$, $\beta = 2$, $\gamma = 1$, m=2, and $K = 3 \times 10^6$. As expected, the acceptance probability decreases as price increases for fixed R. Moreover, with R increasing to some extend, the acceptance probability no longer increases.

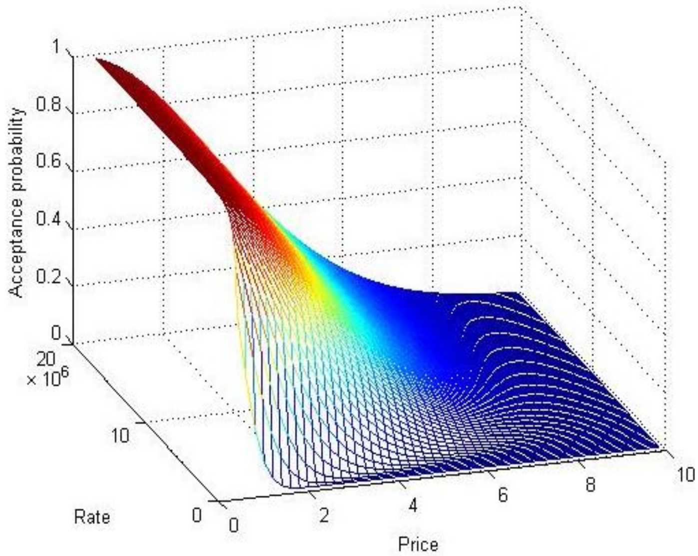

Fig. 2. Acceptance probability of secondary users

4. Price Based Evolutionary Game and Strategy Analysis

4.1 Payoff Matrix for MVNOs

During the process of spectrum sharing, each of the MVNOs has two possible price strategies: either select high price strategy p_{ih} or low price strategy p_{il}, ($i = 1,2$). If both operators select high price strategy p_{ih}, they will gain the profit $\frac{N}{2}(p_{ih}-C_i)A_{ih}$. Otherwise, the operators will obtain $\frac{N}{2}(p_{il}-C_i)A_{il}$ if they both choose low price strategy p_{il}. $N(p_{il}-C_i)A_{il}$ is the profit of the operators given that the operator takes low price strategy, while the other selects high price strategy. Zero is the payoff obtained while the operator takes high price encounters the other chooses low price strategy. The interaction can be expressed by a 2×2 matrix and shown in **Table 1**.

Table 1. Payoff matrix for MVNOs

Operator 2 / Operator 1	p_{2h}	p_{2l}
p_{1h}	$\frac{N}{2}(p_{1h}-C_1)A_{1h}, \frac{N}{2}(p_{2h}-C_2)A_{2h}$	$0,\ N(p_{2l}-C_2)A_{2l}$
p_{1l}	$N(p_{1l}-C_1)A_{1l}$, 0	$\frac{N}{2}(p_{1l}-C_1)A_{1l}, \frac{N}{2}(p_{2l}-C_2)A_{2l}$

where A_{ih} is the acceptance probability with respect to higher price, and A_{il} with respect to lower price. C_i is the fixed cost incurred by the MVNOs, $i=1,2$ represents operator 1 and operator 2, respectively. N is the number of secondary users.

4.2. Solution of Evolutionary Stable Equilibrium

For an operator, a pure strategy is either to select high price strategy or to take low price strategy. A mixed strategy is a probabilistic combination of two strategies in which the operator might play one strategy with a probability x and play another with a probability $1-x$. Let the probability of operator i that select high price strategy p_{ih} is $x_i (0 \le x_i \le 1)$, so the proportion of choosing low price strategy p_{il} is $1-x_i$. Using the evolutionary game model, the expected payoff of operator 1 selecting high price strategy is given:

$$u(p_{1h}) = u_{1h} = \frac{N}{2} x_2 (p_{1h}-C_1)A_{1h} \tag{8}$$

and the expected payoff of selecting low price strategy can be expressed as following:

$$\begin{aligned} u(p_{1l}) = u_{1l} &= x_2 N(p_{1l}-C_1)A_{1l} + \frac{N}{2}(1-x_2)(p_{1l}-C_1)A_{1l} \\ &= \frac{N}{2}(1+x_2)(p_{1l}-C_1)A_{1l} \end{aligned} \tag{9}$$

Thus, the average payoff can be written as:

$$\begin{aligned} \overline{U}_1(p_1) &= x_1 u_{1h} + (1-x_1)u_{1l} \\ &= \frac{N}{2} x_1 x_2 (p_{1h}-C_1)A_{1h} + \frac{N}{2}(1-x_1)(1+x_2)(p_{1l}-C_1)A_{1l} \end{aligned} \tag{10}$$

The growth rate dx/dt can be presented as the difference between the payoff of selecting high price strategy and the average expectation utility of the two different strategies. Consequently, using the replicated dynamic equation, the changing rate of bounded rational operator 1 that select high price p_{1h} can be expressed as follows:

$$\begin{aligned}\frac{dx_1}{dt} &= \dot{x}_1 = \mu_1 x_1\left(u_{1h} - \overline{U}_1\right) \\ &= \frac{N}{2}\mu_1 x_1(1-x_1)\left[x_2(p_{1h}-C_1)A_{1h} - (1+x_2)(p_{1l}-C_1)A_{1l}\right]\end{aligned} \tag{11}$$

where μ_i is some positive constant that can be used to tune the rate of convergence.

Analogously, expected payoff and the replicator dynamics equation of operator 2 are written as following respectively:

$$\begin{aligned}\overline{U}_2(p_2) &= x_2 u_{2h} + (1-x_2)u_{2l} \\ &= \frac{N}{2}x_1 x_2(p_{2h}-C_2)A_{2h} + \frac{N}{2}(1+x_1)(1-x_2)(p_{2l}-C_2)A_{2l}\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{dx_2}{dt} &= \dot{x}_2 = \mu_2 x_2\left(u_{2h} - \overline{U}_2\right) \\ &= \frac{N}{2}\mu_2 x_2(1-x_2)\left[x_1(p_{2h}-C_2)A_{2h} - (1+x_1)(p_{2l}-C_2)A_{2l}\right]\end{aligned} \tag{13}$$

According to the replicator dynamics, an equilibrium is a point $(x_1, x_2) \in [0\times1]\times[0\times1]$ such that $\dot{x}_1 = \dot{x}_2 = 0$. Then from (11) and (13) we get five equilibria: (0, 0), (0, 1), (1, 0), (1, 1), and the mixed strategy equilibrium $\left(\frac{(p_{2l}-C_2)A_{2l}}{(p_{2h}-C_2)A_{2h}-(p_{2l}-C_2)A_{2l}}, \frac{(p_{1l}-C_1)A_{1l}}{(p_{1h}-C_1)A_{1h}-(p_{1l}-C_1)A_{1l}}\right)$. The equilibrium (0, 0) and (1, 1) imply that the strategies profiles of operators converge to low price strategy (p_{1l}, p_{2l}) and high price strategy (p_{1h}, p_{2h}), respectively. Similarly, (0, 1) and (1, 0) means the strategy profile will converge to (p_{1l}, p_{2h}) and (p_{1h}, p_{2l}) that one operator selects high price strategy while the other adopts low price strategy. Moreover, $\left(\frac{(p_{2l}-C_2)A_{2l}}{(p_{2h}-C_2)A_{2h}-(p_{2l}-C_2)A_{2l}}, \frac{(p_{1l}-C_1)A_{1l}}{(p_{1h}-C_1)A_{1h}-(p_{1l}-C_1)A_{1l}}\right)$ signifies that the operators would select different strategies corresponding to a certain proportion which leads to a mixed strategy equilibrium.

4.3. Stability of the Algorithm

In terms of [22], when an equilibrium of the replicator dynamics equations that equals to the locally asymptotically stable point is an evolutionary equilibrium in dynamic systems, it is an ESS. Accordingly, whether the five equilibriums are ESSs could be judged via investigating Jacobian Matrix. The Jacobian Matrix of the system can be obtained as follows:

$$J = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \frac{\partial \dot{x}_1}{\partial x_2} \\ \frac{\partial \dot{x}_2}{\partial x_1} & \frac{\partial \dot{x}_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} \tag{14}$$

where

$$\begin{aligned} J_{11} &= \frac{N\mu_1}{2}(1-2x_1)[x_2(p_{1h}-C_1)A_{1h}-(1+x_2)(p_{1l}-C_1)A_{1l}] \\ J_{12} &= \frac{N\mu_1}{2}x_1(1-x_1)[(p_{1h}-C_1)A_{1h}-(p_{1l}-C_1)A_{1l}] \\ J_{21} &= \frac{N\mu_2}{2}x_2(1-x_2)[(p_{2h}-C_2)A_{2h}-(p_{2l}-C_2)A_{2l}] \\ J_{22} &= \frac{N\mu_2}{2}(1-2x_2)[x_1(p_{2h}-C_2)A_{2h}-(1+x_1)(p_{2l}-C_2)A_{2l}] \end{aligned} \tag{15}$$

The local stability of the algorithm is determined by both the determinant and the trace. If the equilibria fit det (J) > 0 and tr (J) < 0, they are asymptotically stable, in other words, they are ESSs of the game [23].

For X=(0, 0), the Jacobian Matrix is:

$$J(0,0) = \begin{bmatrix} \frac{N\mu_1}{2}(C_1 - p_{1l})A_{1l} & 0 \\ 0 & \frac{N\mu_2}{2}(C_2 - p_{2l})A_{2l} \end{bmatrix} \tag{16}$$

The determinant of the Jacobian matrix is

$$\det J = J_{11}J_{22} - J_{12}J_{21} = J_{11}J_{22} = \frac{N^2\mu_1\mu_2}{4}(C_1 - p_{1l})(C_2 - p_{2l})A_{1l}A_{2l} > 0 \tag{17}$$

The trace of the Jacobian matrix is

$$trJ = J_{11} + J_{22} = \frac{N\mu_1}{2}(C_1 - p_{1l})A_{1l} + \frac{N\mu_2}{2}(C_2 - p_{2l})A_{2l} < 0 \tag{18}$$

Therefore, the equilibrium $(0,0)$ is ESS.

Similarly, the equilibrium $(1,1)$ is also ESS, while the equilibria $(1,0)$ and $(0,1)$ are unstable points. In addition, the mixed strategy equilibrium point $\left(\frac{(p_{2l}-C_2)A_{2l}}{(p_{2h}-C_2)A_{2h}-(p_{2l}-C_2)A_{2l}}, \frac{(p_{1l}-C_1)A_{1l}}{(p_{1h}-C_1)A_{1h}-(p_{1l}-C_1)A_{1l}}\right)$ is a saddle point.

4.4. Optimal Price Strategy of MVNOs

Based on the evolutionary game algorithm formulation above, the optimal price strategic selection algorithm of the bounded rational MVNOs are described as following:

1: **Initialization:**

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  set the strategy adaptation factor 0 < μ ≤ 1.
  the MVNOs randomly select strategy p_h or strategy p_l .
End Initialization

Evolution Phase: the operators' strategy
loop for each time slot t
   compete to serve the secondary users of the decision period
   compute u_ih , u_il and Ū_i
      if u_ih < Ū_i then
         if (u_ih − Ū_i)/Ū_i < rand() then
            select low price strategy
           else select the original price strategy p_h
         end if
       end if
   updating the strategies of the MVNOs
end loop
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5. Collusion Based on Reward and Punishment Mechanism

As shown in **Table 1**, it can be observed that there are two pure Nash equilibria, given by (p_{1l}, p_{2l}) and (p_{1h}, p_{2h}) in this matrix game.

Proposition 1. If $0 < p_i < 0.5\left(C_i + \sqrt{C_i^2 + 40(R_i/K)^2/\left(1+(R_i/K)^2\right)}\right)$ ($i = 1,2$), Nash equilibrium (p_{1h}, p_{2h}) will achieve a higher profit for the operators.

Proof: Assume that

$$\pi(p) = (p - C)A \tag{19}$$

Substituting (7) into (19), we get

$$\pi(p) = (p - C)\exp\left(-0.05p^2\left(1 + 1/(R/K)^2\right)\right) \tag{20}$$

Differentiating $\pi(p)$ with respect to p:

$$\frac{\partial \pi}{\partial p} = \left(1 - 0.1p\left(1 + 1/(R/K)^2\right)(p - C)\right)\exp\left(-0.05p^2\left(1 + 1/(R/K)^2\right)\right) \tag{21}$$

If $0 < p < 0.5\left(C + \sqrt{C^2 + 40(R/K)^2/\left(1+(R/K)^2\right)}\right)$, $p^2 - pC - 10R^2/\left(1+R^2\right) < 0$, namely, $1 - 0.1p\left(1 + 1/(R/K)^2\right)(p - C) > 0$. Meanwhile, since $\exp\left(-0.05p^2\left(1 + 1/(R/K)^2\right)\right) > 0$, accordingly, $\frac{\partial \pi(p)}{\partial p} > 0$ which indicates that $\pi(p)$ increases when p increases.

As a result, if $p_{ih} > p_{il}$, $\pi_{ih} > \pi_{il}$.

Thus, (p_{1h}, p_{2h}) is the optimal strategy for the operators. ■

Obviously, a better strategy is (p_{1h}, p_{2h}) if $0 < p < 0.5\left(C + \sqrt{C^2 + 40(R/K)^2/\left(1+(R/K)^2\right)}\right)$. That is to say, if one of the two operators chooses high price strategy, it is optimal for the other to choose high price strategy too. However, due to the selfishness, the other operator may select low price strategy in order to maximize its own profit. As a result, introducing a reward and punishment mechanism into this game to maintain the collusion is necessary. The operator chooses high price strategy will obtain a reward. Otherwise the operator will take a punishment for selecting low price strategy. When such mechanism is used, both of the operators are inclined to choose optimal Nash equilibrium from which none of the operators wants to deviate. Accordingly, the payoff matrix of spectrum sharing with a reward and punishment mechanism is shown as following:

Table 2. Payoff matrix for MVNOs with collusion

Operator 1 \ Operator2	p_{2h}	p_{2l}
p_{1h}	$(1+\sigma(t))\frac{N}{2}(p_{1h}-C_1)A_{1h}$, $(1+\sigma(t))\frac{N}{2}(p_{2h}-C_2)A_{2h}$	0, $N(p_{2l}-C_2)A_{2l}-\delta$
p_{1l}	$N(p_{1l}-C_1)A_{1l}-\delta$, 0	$\frac{N}{2}(p_{1l}-C_1)A_{1l}-\delta$, $\frac{N}{2}(p_{2l}-C_2)A_{2l}-\delta$

where $1+\sigma(t)$ is the reward parameter, and the equation $\sigma(t) = 1/(1+e^{-t})$ is assumed. δ is the punishment factor.

Calculation method is the same as the previous section that without reward and punishment mechanism. Consequently, the expectation profit of selecting high price strategy and low price strategy for operator 1 can be calculated respectively as follows:

$$u(p_{1h}) = u_{1h} = \frac{N}{2}x_2(1+\sigma(t))(p_{1h}-C_1)A_{1h} \tag{22}$$

$$\begin{aligned} u(p_{1l}) = u_{1l} &= x_2[N(p_{1l}-C_1)A_{1l}-\delta] + (1-x_2)\left[\frac{N}{2}(p_{1l}-C_1)A_{1l}-\delta\right] \\ &= \frac{N}{2}(1+x_2)(p_{1l}-C_1)A_{1l}-\delta \end{aligned} \tag{23}$$

The average profit can be written as:

$$\begin{aligned} \overline{U}_1(p_1) &= x_1u_{1h} + (1-x_1)u_{1l} \\ &= \frac{N}{2}x_1x_2(1+\sigma(t))(p_{1h}-C_1)A_{1h} + (1-x_1)\left[\frac{N}{2}(1+x_2)(p_{1l}-C_1)A_{1l}-\delta\right] \end{aligned} \tag{24}$$

The replicated dynamic equation of operator 1 can be represented as follows:

$$\begin{aligned}\frac{dx_1}{dt} &= \dot{x}_1 = x_1\left(u_{1h} - \overline{U}_1\right) \\ &= \tfrac{1}{2}\mu_1 x_1(1-x_1)\left[Nx_2(1+\sigma(t))(p_{1h}-C_1)A_{1h} - N(1+x_2)(p_{1l}-C_1)A_{1l} + 2\delta\right]\end{aligned} \tag{25}$$

Similarly the replicator dynamics equation of operator 2 is written as:

$$\begin{aligned}\frac{dx_2}{dt} &= \dot{x}_2 = \mu_2 x_2\left(u_{2h} - \overline{U}_2\right) \\ &= \frac{N}{2}\mu_2 x_2(1-x_2)\left[x_1(p_{2h}-C_2)A_{2h} - (1+x_1)(p_{2l}-C_2)A_{2l}\right]\end{aligned} \tag{26}$$

Let $\dot{x}_1 = \dot{x}_2 = 0$. The five equilibria: $(0,0), (0,1), (1,0), (1,0)$ and the mixed strategy equilibrium $\left(\frac{(p_{2l}-C_2)A_{2l} - 2\delta/N}{(1+\sigma(t))(p_{2h}-C_2)A_{2h} - (p_{2l}-C_2)A_{2l}}, \frac{(p_{1l}-C_1)A_{1l} - 2\delta/N}{(1+\sigma(t))(p_{1h}-C_1)A_{1h} - (p_{1l}-C_1)A_{1l}}\right)$ can be deduced.

6. Simulation Results

In this section, a dynamic spectrum sharing cognitive radio system among two bounded rational operators and multiple secondary users are considered. The parameters used in the simulation are as follows. The constants of the acceptance probability are $\alpha = 0.05$, $\beta = 2$, $\gamma = 1$. And the parameter of the utility function is $m = 2$, $K = 3\times10^6$. The fixed costs for operator 1 and operator 2 are $C_1 = 0.1$, $C_2 = 0.05$ respectively. The rate of operator 1 is 15Mbps, and the rate of operator 2 is 5Mbps. The positive constant μ_1 and μ_2 are both assumed to be 0.3. Without loss of generality, the high price and the low price for operator 1 are assumed to be $p_{1h} = 3$ and $p_{1l} = 1$ respectively. For operator 2, the high price and the low price are assumed to be $p_{2h} = 2.4$ and $p_{2l} = 0.8$ respectively. The number of the secondary users is $N = 20$.

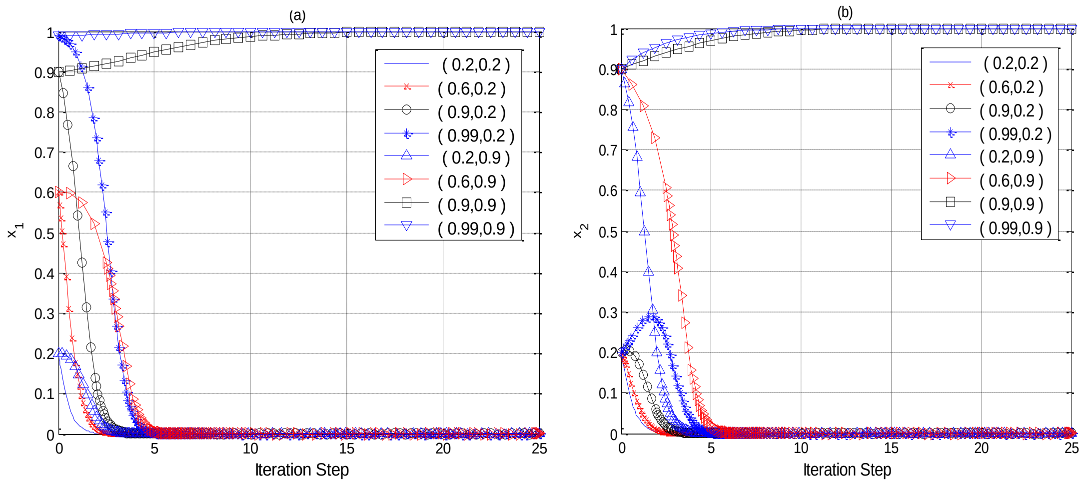

Fig. 3 (a) x_1 **Fig. 3 (b)** x_2

Fig. 3. Strategy dynamics of operators

Fig. 3 shows the convergence of the proposed evolutionary game with different initial strategy selection probabilities $(x_1(0), x_2(0))$ for operator 1 and operator 2, respectively. It is observed that the recommended evolutionary algorithm is convergent, and the initial strategy selection probability of the operators is an important factor that influences the game results. If $x_1(0) > 0.8250$, $x_2(0) > 0.8884$, the strategy selection probability (x_1, x_2) will evolve to the equilibrium (1, 1) that is the ESS. It means that with iteration both of the operators prefer to select higher price strategy finally. Otherwise, the initial state will evolve to the equilibrium (0, 0), which indicates that the low price strategy is the ESS. As the ESS is achieved, no operator will deviate from their current strategies.

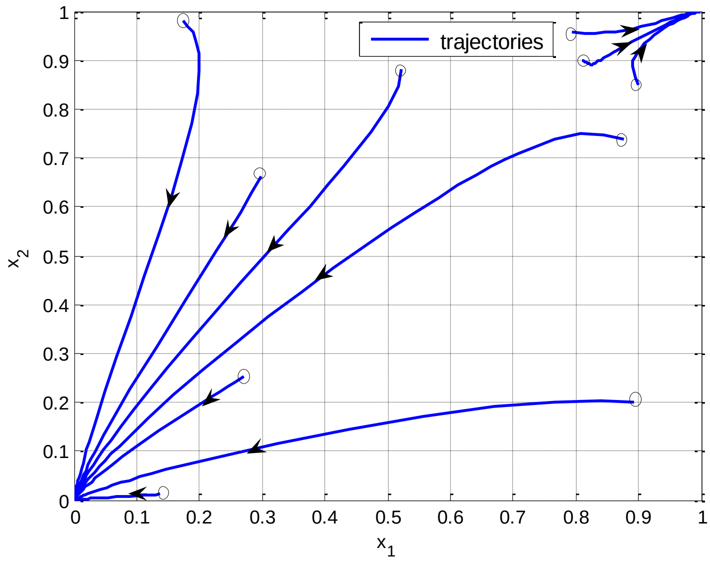

Fig. 4. Strategy selection probability trajectories for the operators

The evolution of strategy adaptation for the operators via using the phase plane of replicator dynamics is displayed in **Fig. 4**. The initial states $(x_1(0), x_2(0))$ of the strategy selection probability are randomly selected. With average profit information, the operator can adapt its price strategy selection to reach the evolutionary equilibrium. In particular, if the current profit of the operator is lower than the average profit of different strategies, the operator may deviate from the current strategy to obtain a better performance. It is observed that the direction of adaptation in the strategy selection for the operators is similar to the convergence of the proposed game with different initial strategy selection probabilities shown in **Fig. 3**. Most of the strategy selection probabilities evolve to the evolutionary equilibrium $(x_1^*, x_2^*) = (0, 0)$, and a few reach to the evolutionary equilibrium $(x_1^*, x_2^*) = (1, 1)$.

Profits of different strategies for operator 1 and operator 2 are shown in **Fig. 5** and **Fig. 6** respectively. In **Fig. 5** it is observed that with the strategy selection probability (x_1, x_2) evolving to the evolutionary equilibrium (0, 0), the profit of selecting low price strategy p_{1l} is 8.544 which equals to the average profit of the two different strategies for operator 1, and the profit of selecting high price strategy p_{1h} evolves to zero ultimately. On the reverse, if both of the operators adopt high price strategy ultimately, the profit of choosing high price strategy p_{1h} is 18.161 which equals to the average profit, and the profit of choosing low price strategy

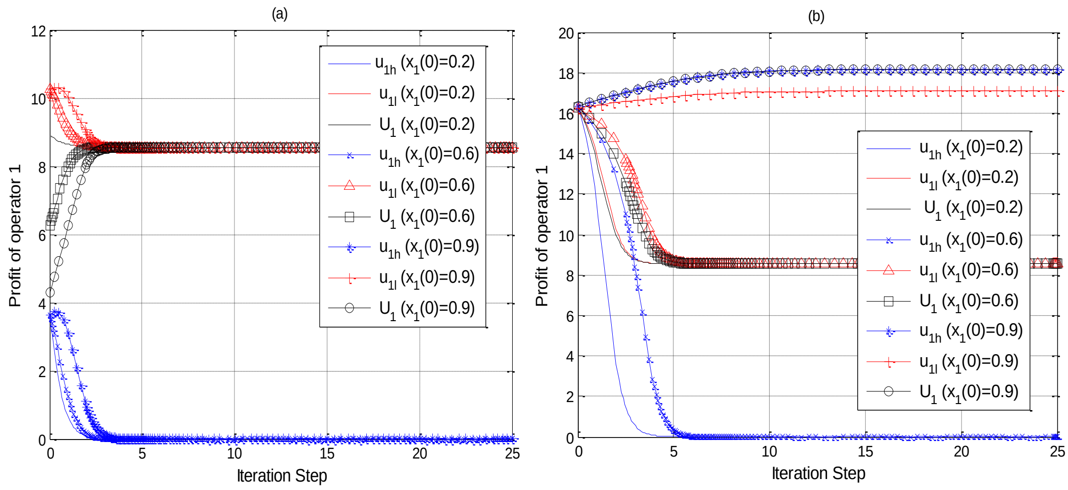

Fig. 5 (a) $x_2(0)=0.2$ **Fig. 5 (b)** $x_2(0)=0.9$

Fig. 5. Profit of operator 1

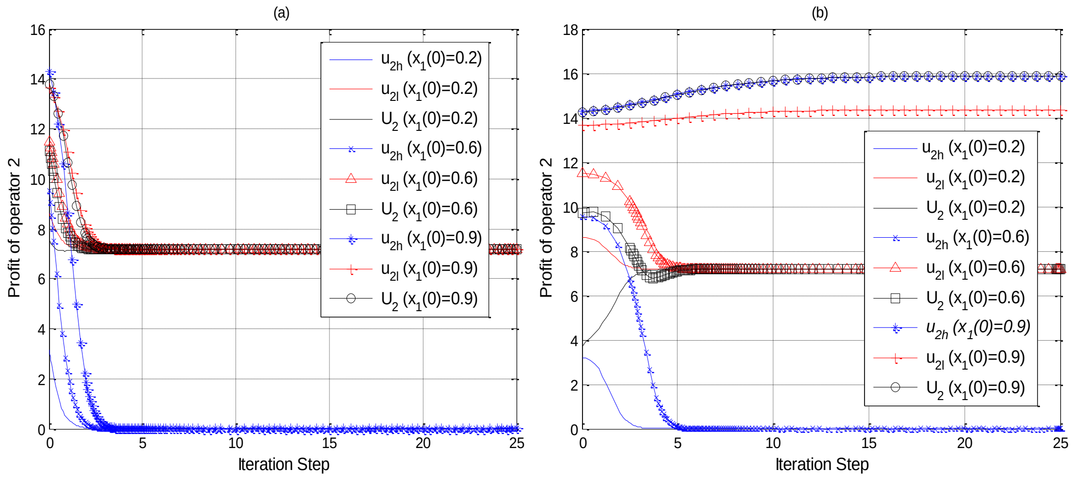

Fig. 6. (a) $x_2(0)=0.2$ **Fig. 6. (b)** $x_2(0)=0.9$

Fig. 6. Profit of operator 2

is 17.088 that is higher than the former case. These results indicate that the evolutionary equilibrium (1, 1) is superior to the evolutionary equilibrium (0, 0). In other words, with iterations if both of the operators adopt high price strategy ultimately, they would obtain the highest profits. Similar phenomenon can be found in profit evolution of operator 2, as shown in **Fig. 6**.

Fig. 7 shows the impact of the parameter μ on the dynamics of strategy adaption with initial strategy state X (0.5, 0.8). We observe that the parameter μ doesn't change the convergence trend but have an influence on the rate of the strategy adaption. With μ decreasing, the iterative convergence speed will become slower.

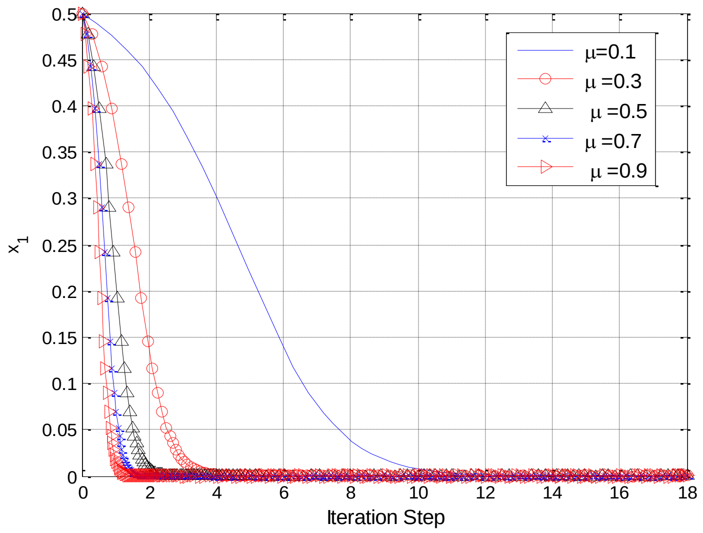

Fig. 7. Impact of μ on the dynamics of strategy adaption

In the following simulation, the reward and punishment mechanism is introduced into the proposed evolutionary algorithm of the dynamic spectrum sharing. For the sake of analysis, the punishment factor $\delta = 1$ is assumed. **Fig. 8 (a)** and **(b)** show the convergence of the proposed game with different initial selection probabilities for operator 1 and operator 2 with collusion respectively. Comparing the results shown in **Fig. 3** and **Fig. 8**, **Fig. 4** and **Fig. 9**, it can be found that more strategy selection probabilities evolve to the evolutionary equilibrium (1, 1) with iteration for both operators in **Fig. 8** and **Fig. 9**. It suggests that the operators are more willing to choose high price strategy finally with different initial strategy selection probabilities. This is because that if the operator chooses high price strategy, it gets a reward, on the contrary the operator obtains a punishment. As a result, both of the operators are inclined to adopt high price strategy, which is the optimal choice to maximize their profits. In addition, the convergence process becomes quicker with collusion.

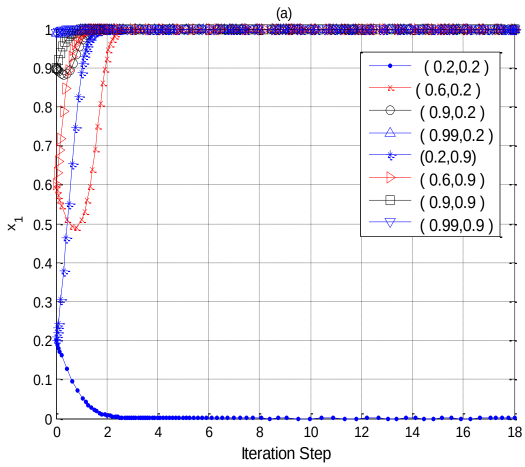

Fig. 8. (a) x_1

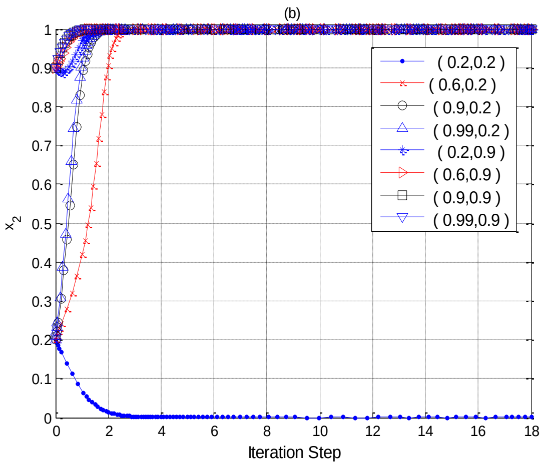

Fig. 8. (b) x_2

Fig. 8. Strategy dynamics of operators with collusion

The evolution of strategy adaptation with collusion for the operators by using the phase plane of replicator dynamics is shown in **Fig. 9.** The initial states $(x_1(0), x_2(0))$ of the strategy selection probability are also randomly selected. Comparing the results displayed in **Fig. 4** and **Fig. 9**, it is clearly that in **Fig. 9** the algorithm with collusion changes the convergence trend.

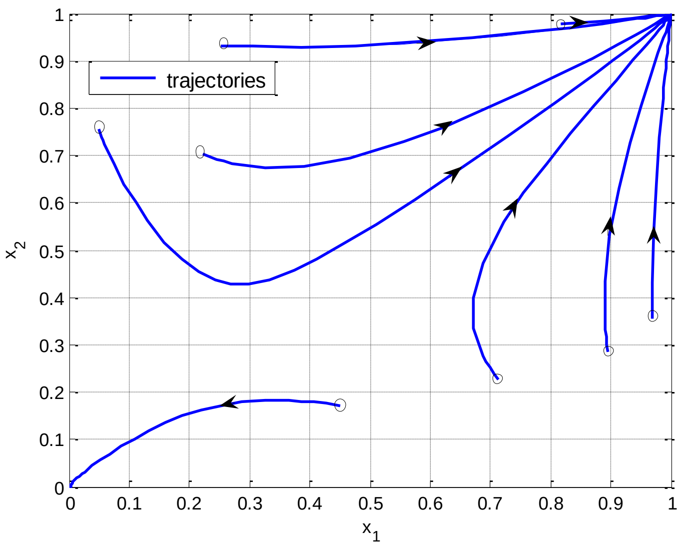

Fig. 9. Strategy selection probability trajectories for the operators with collusion

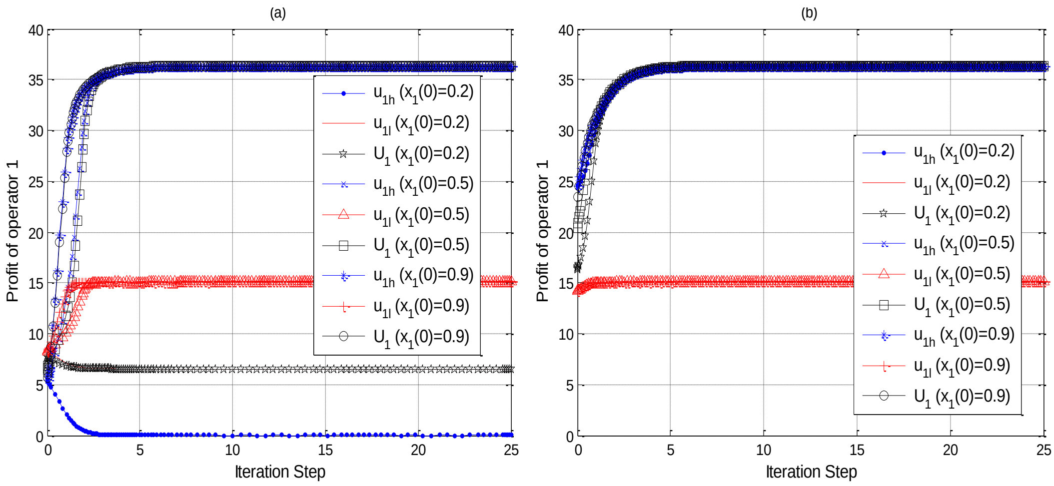

Fig. 10. (a) $x_2(0) = 0.2$ **Fig. 10. (b)** $x_2(0) = 0.9$

Fig. 10. Profit of operator 1 with collusion

Fig. 10 shows the profit of the algorithm with reward and punishment mechanism for operator 1. It can be found that the variation is similar as the algorithm that without collusion. However, as expected the evolutionary game algorithm with incentive mechanism achieves higher profits for the operator no matter the strategy selection probability evolves to the evolutionary equilibrium (0, 0) or (1, 1). This is attributed to that when the incentive mechanism is used, both of the operators are aware of the reward and the punishment. Collusion between the operators can be maintained to select the optimal price strategy so that the highest profit will be obtained in the long run. As a result, the reward and punishment is an effective method to promote the profits of the operators.

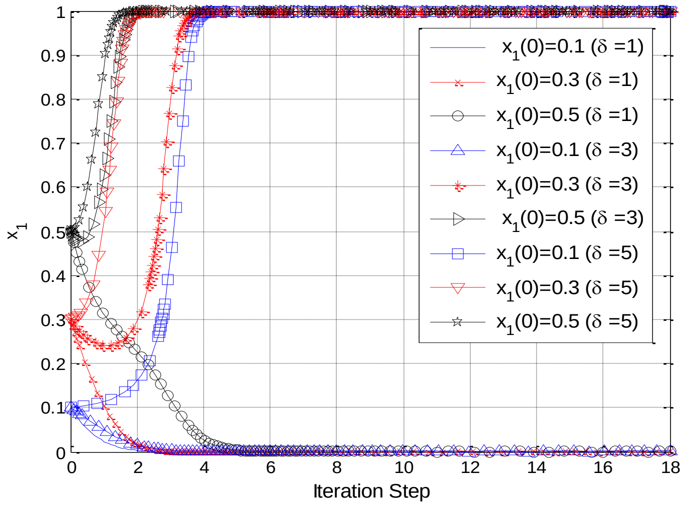

Fig. 11. Effect of δ on price strategy selection probability ($x_2(0) = 0.2$)

The effect of punishment factor δ on the price strategy selection probability x_1 is shown in **Fig. 11**. As the punishment factor increases to some extent for the same initial price strategy selection probability, the operator would choose high price strategy ultimately with iteration. In other words, the higher the punishment factor becomes, the operator is more incline to select high price strategy. In addition, with the same initial strategy selection probability $x_1(0)$, the convergence time is shorter as δ increases.

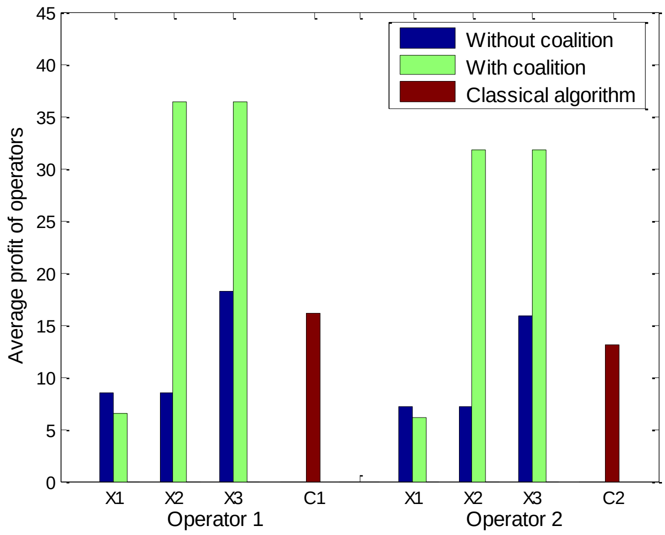

Fig. 12. Average profit of operators for EGT with and without coalition as well as classical game algorithm

As shown in **Fig. 12**, we compare the average profit of the operators based on the proposed EGT algorithm with and without coalition as well as the classical game algorithm. The different initial strategy selection probability of the operators X1=(0.2, 0.2), X2=(0.2, 0.9) and X3=(0.9, 0.9) are selected for the proposed EGT algorithm. C1 and C2 denote the classical game algorithm of operator 1 and 2, respectively. It is clearly that, increasing the initial strategy selection probability, the operators obtain higher average profits with coalition than the other algorithms. It indicates that the proposed reward and punishment mechanism EGT algorithm has a better performance. This is because with coalition, both the operators are inclined to choose high price strategy ultimately, which will leads to higher profits. The computational results evidence the feasibility of the proposed algorithm in optimizing the total profits of operators in spectrum sharing to some extent.

7. Conclusion

In this paper, the demand responsive pricing competition between two bounded rational MVNOs in a duopoly spectrum market is investigated. The optimal price strategy selection that takes into account both the profits of operators and QoS requirement of the secondary users is modeled. EGT is employed to model the dynamic price strategy selection for MVNOs. Using replicator dynamics, the operators are able to try two different price strategies and learn a better strategy via strategic interactions. The ESSs are characterized and studied with the proposed game. In addition, the operators can benefit from collusion with increased average profit when collusion is maintained between them via reward and punishment mechanism. Simulation results reveal that the proposed game could make the operators adjust their strategies to achieve ESS using replicated dynamics. The initial strategy selections of operators play an important role in affecting the ESSs. Moreover, the dynamic spectrum sharing algorithm with reward and punishment mechanism has a better performance than that without such mechanism. Such results may give some insight about the optimal price strategy selection for MVNOs in the next generation cognitive wireless networks.

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