

Optimal Strategies for Cooperative Spectrum Sensing in Multiple Cross-over Cognitive Radio Networks

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Abstract

To improve the sensing performance, cooperation among secondary users can be utilized to collect space diversity. In this paper, we focus on the optimization of cooperative spectrum sensing in which multiple cognitive users efficiently cooperate to achieve superior detection accuracy with minimum sensing error probability in multiple cross-over cognitive radio networks. The analysis focuses on two fusion strategies: soft information fusion and hard information fusion. Under soft information fusion, the optimal threshold of the energy detector is derived in both noncooperative single-user and cooperative multiuser sensing scenarios. Under hard information fusion, the optimal randomized rule and the optimal decision threshold are derived according to the rule of minimum sensing error (MSE). MSE rule shows better performance on improving the final false alarm and detection probability simultaneously. By simulations, our proposed strategy optimizes the sensing performance for each cognitive user which is randomly distributed in the multiple cross-over cognitive radio networks.

Keywords: Cross-over cognitive radio networks, cooperative spectrum sensing, soft information fusion, hard information fusion

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1. Introduction

Current wireless networks are characterized by a static spectrum allocation policy. However, with the explosive development of wireless services, the policy faces spectrum scarcity in particular spectrum bands, and the limited radio spectrum becomes increasingly crowded. A recent study by Federal Communications Commission (FCC) shows that, in most of time, the actual licensed spectrum is largely under-utilized in vast temporal and geographic dimensions [1].

Cognitive radio enables much higher spectrum efficiency by dynamic spectrum access [2]. Consequently, it is a potential technology to reuse the under-utilized spectrum bands. In cognitive radio systems, the unlicensed wireless users (secondary users or cognitive users) take chances to access the spectrum without causing interference to the licensed users (primary users) so that the spectrum access is dynamic and opportunistic [3]. One of the most fundamental problems of cognitive radio technology is spectrum sensing. By sensing and adapting to the environment, once an idle channel is sensed, the secondary system will access this channel [4]. In order to avoid interference to primary users (PUs), a cognitive radio (CR) needs to efficiently and effectively detect the presence of the primary users. However, many factors make the spectrum sensing problem complicated, such as low signal-to-noise ratio (SNR), little knowledge of primary user and detrimental effects of fading and shadowing. To combat these impacts, cooperative spectrum sensing has been proposed to obtain the space diversity in multiuser CR networks [5], [6]. In cooperative spectrum sensing, each CR user receives the signals from the primary users, independently makes its local decision, and then sends the local observation to the fusion center (FC). Next, FC makes a final decision and immediately responses to CR users once PUs have been detected.

Many studies on cooperative spectrum sensing assume that all CR users share the same occupancy of one primary user, namely, there are two common hypotheses in the spectrum sensing [7]. However, in practical cognitive radio networks, CR users at different locations may experience different spectrum occupancies and opportunistically access different licensed bands since they can be either within or out of the cross-over region of multiple cognitive radio networks. As is shown in Fig. 1, CR1 can only access the licensed band I since it is located within the region of cognitive network I and out of cognitive network II and cognitive network III. However, CR3 can access one of the three licensed bands since it is located within the cross-over region of the three cognitive networks. In this case, the spectrum sensing schemes proposed in previous works may not be suitable.

In the multiple cross-over cognitive radio networks, each CR user needs to carry out a hypothesis test by itself. However, this does not imply that the cooperation among CR users is unnecessary. In this paper, we focus on the optimization of cooperative spectrum sensing in which multiple CR users efficiently cooperate to achieve superior detection accuracy with minimum sensing error probability in multiple cross-over cognitive radio networks. We consider the multiple types of network architecture and investigate the problem of cooperative spectrum sensing for multiple cross-over cognitive radio networks. This is because in practical cognitive radio networks, numbers of CR users are distributed randomly in a certain region, it is possible that some CR users are located within the cross-over region of cognitive networks while others are not. So CR users at different locations may experience different spectrum occupancies.

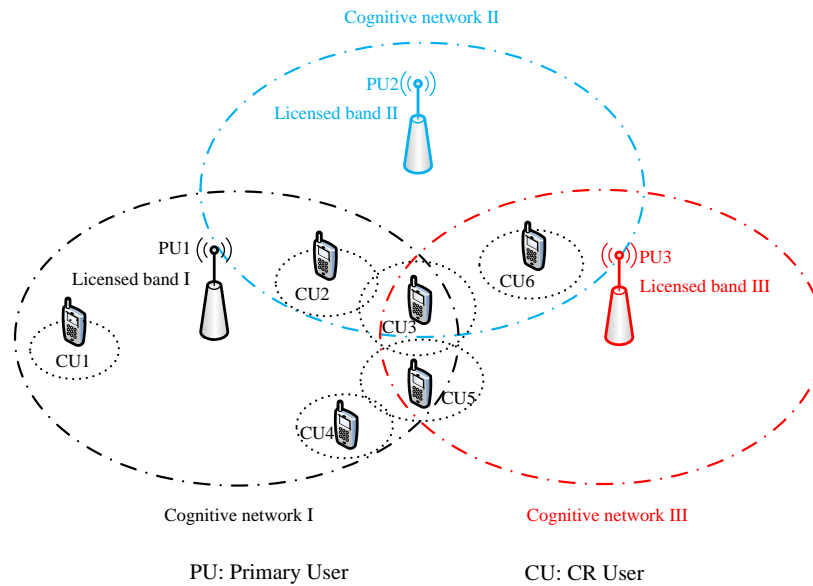


Fig. 1. System model of multiple cross-over cognitive radio networks.

To accurately access the licensed bands for each CR user which is randomly distributed in a certain region, we should optimize the sensing performances of cognitive network I and cognitive network II. Then, FC1 and FC2 immediately response to the CR users which are located within the cross-over region of cognitive network I and cognitive network II so that they can access licensed band I or licensed band II accurately, which is illustrated in **Fig. 2**.

When energy detection is utilized for cooperative spectrum sensing in multiple cross-over cognitive radio networks, CR users report their sensing information to the corresponding fusion centers [8]. The analysis focuses on two fusion strategies: soft information fusion and hard information fusion. The former provides a theoretical bound on the average sensing error probability in an ideal cooperative sensing setup, while the latter leads to practical fusion and decision rules.

With soft information fusion strategy, each CR user simply amplifies the received signal from the primary user and forwards to the fusion center [8]. The framework for two-user and multiple-user cooperative spectrum sensing with soft information fusion was introduced in [5], [6]. However, an analytical study for the false alarm probability and missed detection probability in the cooperative spectrum sensing has not been addressed. In this paper, we provide a rigorous analytical framework for cooperative spectrum sensing with soft information fusion, and derive the optimal threshold of the energy detector by minimizing the average sensing error probability in both noncooperative single-user and cooperative multiuser sensing scenarios. The threshold of the energy detector can be adjusted to improve the efficiency and reliability simultaneously. Then, an efficient spectrum sensing algorithm is proposed for large cognitive networks which requires only a few, not all, CR users in cooperative spectrum sensing to get a target error bound.

With hard information fusion strategy, each CR user makes a “one bit” decision (1 standing for the presence of the primary user, 0 standing for the absence of the primary user) on the primary user activity and then reports the individual decision to the fusion center over a reporting channel. The fusion rule at the fusion center can be OR, AND, or Half-voting rule [9]. In this paper, we employ the randomized rule in the fusion center and analyze the

corresponding performance [10]. In order to minimize the sensing error, the probability in the randomized rule should be adaptive to the threshold of the energy detector. Specifically, the optimal decision threshold is derived for the randomized rule. We further derive the minimum sensing error probability corresponding to different threshold of the local energy detection. Our proposed MSE rule could appropriately improve both the final false alarm and detection probability simultaneously.

In this paper, the optimal cooperative spectrum sensing strategies with soft information fusion and hard information fusion in multiple cross-over cognitive radio networks are proposed. The proposed strategies also minimize the false alarm probability and missed detection probability of the CR users within the cross-over region.

The rest of this paper is organized as follows. The system model and cooperation strategies will be given in Section II. Sections III and IV are devoted to the analysis of cooperative spectrum sensing with soft information fusion and hard information fusion, respectively. The sensing performances of the CR users within the cross-over region will be analyzed in Section V. Simulated verifications will be presented in Section VI, followed by concluding remarks in Section VII.

Notation: Subscripts “ f ”, “ d ”, “ m ”, “ e ” refer to false alarm, detection, missed detection, and average sensing error probability respectively; Superscripts “I”, “II”, “C” refer to cognitive network I, cognitive network II and cross-over region respectively; Subscripts “ s ” and “ h ” refer to soft information fusion and hard information fusion respectively.

2. System Model and Cooperation Strategies

2.1 System Model and Analysis of Cooperative Spectrum Sensing in Multiple Cross-over Cognitive Radio Networks

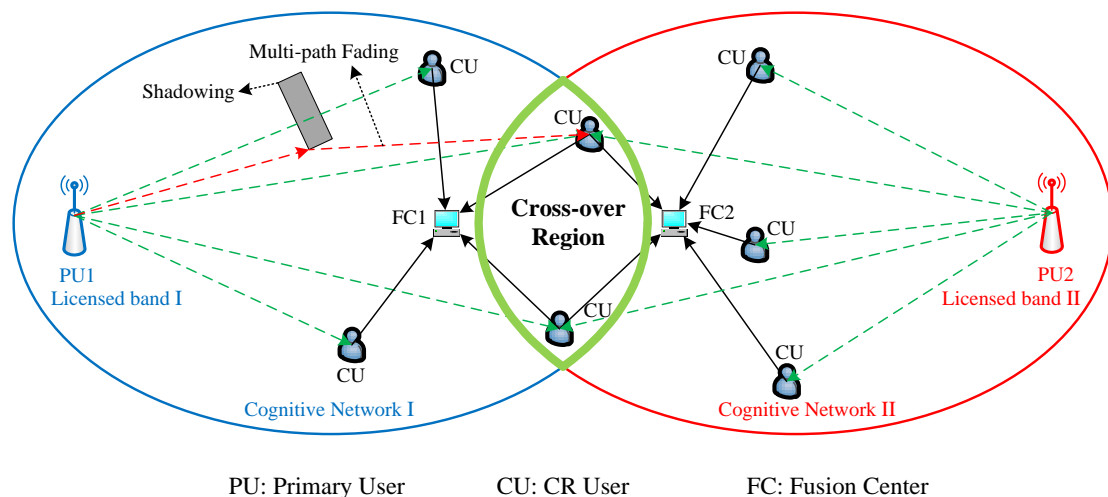


Fig. 2. System model of cooperative spectrum sensing in multiple cross-over cognitive radio networks.

We investigate cooperative spectrum sensing in multiple cross-over cognitive radio networks consisting of two fusion centers and a number of CR users, which is illustrated in Fig. 2. Each primary user communicates using one channel in a scope of area. Assume that the CR users are

distributed randomly within a certain region. These CR users cooperate to sense the activity of the two primary users. It is possible to assume that some CR users are located within the region of only one cognitive network and some are within the cross-over region of both two cognitive networks (the cross-over region is called as region C). Considering that CR users at different locations have different available licensed bands at the same time. We can notice that, if a CR user is located in region C, it can access either the licensed band I or licensed band II.

In multiple cross-over cognitive radio networks, CR users sense the environment and then send their local decision results to FC1 or FC2 depending on whether they are within the region of cognitive network I or cognitive network II. Then, FC1 and FC2 make final decision results to determine which channel to access for the CR users which are located in region C. Assume that N cognitive users locate within the region of cognitive network I and K cognitive users locate within the region of cognitive network II. We focus on the optimization of sensing performance for each CR user which is randomly distributed in the multiple cross-over cognitive radio networks. The first step is to optimize the sensing performances of the cooperated CRs which are located in cognitive network I and cognitive network II respectively. Here, we consider the optimization of cognitive network I. The optimal strategies can also be employed in cognitive network II.

In cognitive network I, we assume that the distance between the CR users is small compared with the distance from PU1 to any CR users. Then the path loss of each CR user is almost identical and the primary signals received at the CR users are considered to be independent and identically distributed (i.i.d.) [11]. For ease of analysis, we assume that the noise power is the same at CR users in the case of an AWGN environment and the same decision rules are employed among the CR users in cognitive network I. Spectrum sensing is to properly determine one of the following two hypotheses H_0 (denote the absence of PU1) and H_1 (denote the presence of PU1) according to the received signal Y_i [11].

$$\begin{aligned} H_0 : y_i(k) &= \varepsilon_i(k), \quad i = 1, \dots, N \\ H_1 : y_i(k) &= h_i s(k) + \varepsilon_i(k), \quad i = 1, \dots, N \end{aligned} \quad (1)$$

where $k = 1, \dots, 2u$, u is the time-bandwidth product of the energy detector, $y_i(k)$ is the received signal at the i th CR, $s(k)$ is the signal of PU1, $\varepsilon_i(k)$ is the additive white Gaussian noise at the i th CR, h_i is the channel gain between PU1 and the i th CR, h_i is i.i.d. among CRs and we do not consider the exact distribution of h_i . The decision statistic of energy detection is [12]

$$\begin{aligned} Y_i &\sim \chi_{2u}^2 \quad H_0 \\ Y_i &\sim \chi_{2u}^2(2\gamma_i) \quad H_1 \end{aligned} \quad (2)$$

where γ_i denotes the instantaneous signal-to-noise ratio (SNR) at the i th CR. For a large u , Y_i approximates the following Gaussian distribution according to the Central Limit Theorem [13]

$$\begin{aligned} Y_i &\sim N(2u, 4u) & H_0 \\ Y_i &\sim N(2u + 2\gamma_i, 4u + 8\gamma_i) & H_1 \end{aligned} \quad (3)$$

Gaussian distribution will be used in the subsequent discussion since it is much easier to deal with in mathematical derivation and provides more insights than chi-square distribution. Since the primary signals received at the CR users are considered to be i.i.d., we can omit the subscript ‘ i ’ of Y_i . Thus, the received signals for different CR users Y_i s are conditionally independent under each hypothesis.

The probability density function (PDF) of Y can then be written as

$$f_Y(y) = \frac{1}{2\sqrt{2\pi u}} e^{-\frac{(y-2u)^2}{8u}} \quad H_0 \quad (4)$$

$$f_Y(y) = \frac{1}{2\sqrt{2\pi(u+2\gamma)}} e^{-\frac{[y-(2u+2\gamma)]^2}{2(4u+8\gamma)}} \quad H_1 \quad (5)$$

2.2 Cooperative Strategies

In cognitive network I, cooperative spectrum sensing requires the cooperation among multiple CR users. CR users sense the environment and then send their local sensing information to FC1 periodically via the common control channels [14]. The cooperation benefit can be maximized if all the sensing information reaches FC1 without any loss. However, this condition cannot always be satisfied due to limited wireless resource. In this paper, we focus on two fusion strategies: soft information fusion and hard information fusion.

1) Soft Information Fusion: Without complex signal processing at CR users, each CR user simply amplifies the received signal from PU1 and forwards to FC1. Thus, FC1 can obtain the information from the distributed CRs perfectly. Although this is not achievable in practical cognitive radio networks, it provides a theoretical bound on sensing error probability performance in an ideal cooperative sensing setup.

2) Hard Information Fusion: Instead of sending the received signal to FC1 directly, each CR user makes its own “one bit” hard decision on PU1 and then sends the individual decision to FC1 over a reporting channel (which can be with a narrow bandwidth). We employ the randomized rule in FC1 and analyze the corresponding performance.

3. Cooperative Spectrum Sensing with Soft Information Fusion

3.1 Single-user Sensing

First we consider a single-user sensing scheme. In this case, the CR user continuously monitors the signal received from PU1. The distribution of the decision statistic is discussed in section II. Two quantities are usually used to assess spectrum sensing performance, namely, the probability of false alarm p_f and the probability of detection p_d [15]. For a non-fading environment, the probability of false alarm and the probability of detection can be generally computed by

$$p_f = P_{rob}\{Y > \lambda | H_0\} = \int_{\lambda}^{\infty} f_{Y|H_0}(y)dy = Q\left(\frac{\lambda - 2u}{2\sqrt{u}}\right) \quad (6)$$

$$p_d = P_{rob}\{Y > \lambda | H_1\} = \int_{\lambda}^{\infty} f_{Y|H_1}(y)dy = Q\left(\frac{\lambda - 2u - 2\gamma}{2\sqrt{u + 2\gamma}}\right) \quad (7)$$

$$p_m = 1 - p_d \quad (8)$$

where λ is the energy detection threshold, $Q(\cdot)$ is the Q-function defined as

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt.$$

Assumed that $p(H_0)$ and $p(H_1)$ are the prior probabilities of H_0 and H_1 respectively, then the average sensing error is $p_e = p(H_0)p_f + p(H_1)(1 - p_d)$. Since the false alarm and missed detection probability respectively captures the efficiency and reliability of the cognitive system, we will optimize the threshold λ by minimizing the average sensing error probability p_e to balance the system efficiency and reliability. We have

$$\frac{\partial p_e}{\partial \lambda} = p(H_0) \left[-\frac{1}{2\sqrt{2\pi}u} e^{-\frac{(\lambda-2u)^2}{8u}} \right] + p(H_1) \left\{ \frac{1}{2\sqrt{2\pi}(u+2\gamma)} e^{-\frac{[\lambda-(2u+2\gamma)]^2}{2(4u+8\gamma)}} \right\} \quad (9)$$

Setting $\frac{\partial p_e}{\partial \lambda} = 0$, we can obtain the optimal threshold as

$$\lambda_{opt} = u + \frac{1}{\gamma} \sqrt{u^2 \gamma^2 + 2\gamma \left[u\gamma^2 + u(u+2\gamma) \left(2\ln \frac{p(H_0)}{p(H_1)} + \ln \frac{u+2\gamma}{u} \right) \right]} \quad (10)$$

Then, the minimum sensing error probability is

$$p_{e,\min} = p(H_0)Q\left(\frac{\lambda_{opt} - 2u}{2\sqrt{u}}\right) + p(H_1) \left[1 - Q\left(\frac{\lambda_{opt} - 2u - 2\gamma}{2\sqrt{u + 2\gamma}}\right) \right] \quad (11)$$

3.2 Multiuser Sensing with Soft Information Fusion

With soft information fusion strategy, FC1 receives Y_1, Y_2, \dots, Y_N from the distributed CR users, where N is the number of CR users which are located within the region of cognitive network I and Y_i s are i.i.d. under both H_0 and H_1 . Assume that the amplification factor is 1. So, in FC1, $Y_s = \sum_{i=1}^N Y_i$, where 's' refers to soft information fusion. Since Y_i s are i.i.d., according to (3), we have

$$f_{Y_s}(y) = \frac{1}{2\sqrt{2\pi Nu}} e^{-\frac{(y-2Nu)^2}{8Nu}} \quad H_0 \quad (12)$$

$$f_{Y_s}(y) = \frac{1}{2\sqrt{2\pi N(u+2\gamma)}} e^{-\frac{[y-2N(u+\gamma)]^2}{2N(4u+8\gamma)}} \quad H_1 \quad (13)$$

The false alarm and detection probabilities of cooperative spectrum sensing with soft information fusion are as follows

$$Q_{f,s} = P_{rob}\{Y_s > \lambda_s | H_0\} = \int_{\lambda_s}^{\infty} f_{Y_s|H_0}(y) dy = Q\left(\frac{\lambda_s - 2Nu}{2\sqrt{Nu}}\right) \quad (14)$$

$$Q_{d,s} = P_{rob}\{Y_s > \lambda_s | H_1\} = \int_{\lambda_s}^{\infty} f_{Y_s|H_1}(y) dy = Q\left(\frac{\lambda_s - 2Nu - 2N\gamma}{2\sqrt{N(u+2\gamma)}}\right) \quad (15)$$

Accordingly, the average sensing error probability is $Q_{e,s} = p(H_0)Q_{f,s} + p(H_1)(1 - Q_{d,s})$.

Similarly, setting $\frac{\partial Q_{e,s}}{\partial \lambda_s} = 0$, we can obtain the optimal threshold as

$$\lambda_{s,opt} = Nu + \frac{1}{\gamma} \sqrt{N^2 u^2 \gamma^2 + 2\gamma \left[N^2 u \gamma^2 + Nu(u+2\gamma) \left(2 \ln \frac{p(H_0)}{p(H_1)} + \ln \frac{u+2\gamma}{u} \right) \right]} \quad (16)$$

The minimum sensing error probability of cognitive network I with soft information fusion can be derived as

$$Q_{e,s_min} = p(H_0)Q\left(\frac{\lambda_{s,opt} - 2Nu}{2\sqrt{Nu}}\right) + p(H_1) \left[1 - Q\left(\frac{\lambda_{s,opt} - 2Nu - 2N\gamma}{2\sqrt{N(u+2\gamma)}}\right) \right] \quad (17)$$

4. Cooperative Spectrum Sensing with Hard Information Fusion

With the hard information fusion strategy, each CR user makes individual decision and then sends the “one bit” decision to FC1. After collecting all decisions, FC1 utilizes all “one bit” decisions to make a final decision according to the proposed fusion rules. Let Λ denotes the number of CR users reporting existence of PU1. In FC1, the final decision strategy may be expressed in the randomized rule

if $\Lambda > n$ decide H_1

if $\Lambda = n$ decide H_1 with probability ϕ ($0 \leq \phi \leq 1$, a coin with $P_r(\text{head}) = \phi$ is tossed, and the decision is taken to be H_1 if the result is heads, and H_0 otherwise)

if $\Lambda < n$ decide H_0

where n is an integer, and $n = 1, 2, \dots, N$ is the decision threshold at FC1, ϕ indicates that FC1 employs the randomized rule. The final false alarm probability and final detection probability are given by

$$Q_{f,h} = \sum_{i=n+1}^N \binom{N}{i} (p_f)^i (1-p_f)^{(N-i)} + \phi \binom{N}{n} (p_f)^n (1-p_f)^{(N-n)} \quad (18)$$

$$Q_{d,h} = \sum_{i=n+1}^N \binom{N}{i} (p_d)^i (1-p_d)^{(N-i)} + \phi \binom{N}{n} (p_d)^n (1-p_d)^{(N-n)} \quad (19)$$

$$Q_{m,h} = 1 - Q_{d,h} \quad (20)$$

$n=1,2,\dots,N$ and it is defined that when $n=N$ the first item of $Q_{f,h}$ and $Q_{d,h}$ is equal to 0.

Under hard information fusion, there are two levels of decision making, each level having its own decision performance. Hence, to minimize the average sensing error probability $Q_{e,h} = p(H_0)Q_{f,h} + p(H_1)(1-Q_{d,h})$, we need to jointly choose both the optimal local decision threshold λ_{opt} at the energy detector and the optimal final decision threshold n_{opt} at FC1. λ_{opt} is derived according to section III. Our objective is to find the optimal n to minimize the sensing error probability and do not introduce any extra overhead for second system.

According to (18) and (19), we have

$$\begin{aligned} Q_{e,h} &= p(H_0)Q_{f,h} + p(H_1)(1-Q_{d,h}) \\ &= A + \phi \cdot B + p(H_1) \end{aligned} \quad (21)$$

where

$$A = \sum_{i=n+1}^N \binom{N}{i} \left[p(H_0)(p_f)^i (1-p_f)^{(N-i)} - p(H_1)(p_d)^i (1-p_d)^{(N-i)} \right] \quad (22)$$

$$B = \binom{N}{n} \left[p(H_0)(p_f)^n (1-p_f)^{(N-n)} - p(H_1)(p_d)^n (1-p_d)^{(N-n)} \right] \quad (23)$$

Assume that N is a fixed value, given n (n is an integer, and $n=1,2,\dots,N$), then A and B are fixed. In order to minimize the average sensing error probability, the problem is equivalent to

$$\phi = \begin{cases} 0 & B \geq 0 \\ 1 & B < 0 \end{cases} \quad (24)$$

Therefore, if

$$B = \binom{N}{n} \left[p(H_0)(p_f)^n (1-p_f)^{(N-n)} - p(H_1)(p_d)^n (1-p_d)^{(N-n)} \right] \geq 0 \quad (25)$$

We have

$$\left(\frac{1-p_f}{1-p_d}\right)^{N-n} \geq \frac{p(H_1)}{p(H_0)} \left(\frac{p_d}{p_f}\right)^n \tag{26}$$

$$(N-n) \cdot [\ln(1-p_f) - \ln(1-p_d)] \geq \ln \frac{p(H_1)}{p(H_0)} + n \cdot [\ln(p_d) - \ln(p_f)] \tag{27}$$

$$n[\ln(p_d) - \ln(p_f) + \ln(1-p_f) - \ln(1-p_d)] \leq N \cdot [\ln(1-p_f) - \ln(1-p_d)] - \ln \frac{p(H_1)}{p(H_0)} \tag{28}$$

Theorem 1: For p_f and p_d defined in (6) and (7), respectively, given a sampling size u , $\ln(p_d) - \ln(p_f) + \ln(1-p_f) - \ln(1-p_d) > 0$.

Proof: According to (6) and (7), $p_f = Q\left(\frac{\lambda-2u}{2\sqrt{u}}\right)$, $p_d = Q\left(\frac{\lambda-2u-2\gamma}{2\sqrt{u+2\gamma}}\right)$.

As

$$\begin{aligned} \frac{\lambda-2u}{2\sqrt{u}} - \frac{\lambda-2u-2\gamma}{2\sqrt{u+2\gamma}} &= \frac{\sqrt{u+2\gamma}(\lambda-2u) - \sqrt{u}(\lambda-2u-2\gamma)}{2\sqrt{u(u+2\gamma)}} \\ &= \frac{\lambda(\sqrt{u+2\gamma} - \sqrt{u}) + \sqrt{u} \cdot (2u+2\gamma) - \sqrt{u+2\gamma} \cdot 2u}{2\sqrt{u(u+2\gamma)}} \end{aligned} \tag{29}$$

and

$$\sqrt{u+2\gamma} - \sqrt{u} > 0, \left[\sqrt{u} \cdot (2u+2\gamma)\right]^2 - \left[\sqrt{u+2\gamma} \cdot 2u\right]^2 = 4u\gamma^2 > 0 \tag{30}$$

Then

$$\sqrt{u} \cdot (2u+2\gamma) - \sqrt{u+2\gamma} \cdot 2u > 0, \frac{\lambda-2u}{2\sqrt{u}} - \frac{\lambda-2u-2\gamma}{2\sqrt{u+2\gamma}} > 0 \tag{31}$$

Thus

$$\frac{\lambda-2u}{2\sqrt{u}} > \frac{\lambda-2u-2\gamma}{2\sqrt{u+2\gamma}} \tag{32}$$

Since $Q(x)$ is a decreasing function of x , according to (32), we have $p_d > p_f$, then $1-p_f > 1-p_d$. Therefore, $\ln(p_d) > \ln(p_f)$, $\ln(1-p_f) > \ln(1-p_d)$.

Then, $\ln(p_d) - \ln(p_f) + \ln(1-p_f) - \ln(1-p_d) > 0$, Theorem 1 is proved.

Therefore, according to (28), we have $n \leq \rho$, where

$$\rho = \frac{N \cdot [\ln(1 - p_f) - \ln(1 - p_d)] \cdot \ln \frac{p(H_1)}{p(H_0)}}{\ln(p_d) \cdot \ln(p_f) + \ln(1 - p_f) - \ln(1 - p_d)} \quad (33)$$

Similarly, if $B < 0$, we can obtain $n > \rho$. Thus, the probability in the randomized rule should be set as

$$\phi = \begin{cases} 0, & \text{if } n \leq \rho \\ 1, & \text{if } n > \rho \end{cases} \quad (34)$$

Furthermore, the optimal final decision threshold n_{opt} can be derived as

$$\begin{aligned} \mathbf{n}_{opt} &= \lfloor \rho \rfloor, & \mathbf{j} &= \mathbf{0} \\ \mathbf{n}_{opt} &= \lceil \rho \rceil, & \mathbf{j} &= \mathbf{1} \end{aligned} \quad (35)$$

Therefore, in our proposed minimum sensing error (MSE) rule, the minimum average sensing error probability can be calculated by

$$Q_{e,h_min} = \sum_{i=\lceil \rho \rceil}^N \binom{N}{i} \left[p(H_0) (p_f)^i (1 - p_f)^{(N-i)} - p(H_1) (p_d)^i (1 - p_d)^{(N-i)} \right] + p(H_1) \quad (36)$$

5. Sensing Performances of CR Users within the Cross-over Region

In the cross-over cognitive radio networks, FC1 and FC2 make final decision results to determine which channel to access for the CR users which are located in region C. These CR users can choose one licensed band to access either PU1 or PU2 is absent. When PU1 and PU2 are both absent, they can access licensed band I or licensed band II with probability of $\frac{1}{2}$. In FCi ($i=1,2$), we assume the final decision remark D_i (D_1 denotes the final decision remark of cognitive network I, and D_2 denotes the final decision remark of cognitive network II) is

$$D_i = \begin{cases} 0 & \text{if FCi decides } H_0 \\ 1 & \text{if FCi decides } H_1 \end{cases} \quad (37)$$

Assume that S_i denotes the activity of PUi ($S_i = 0$: PUi is absent; $S_i = 1$: PUi is present; $i = 1, 2$). Now consider the sensing performances of the CR users which are located in region C.

1) PU1 is present and PU2 is present, i.e., $S_1 = 1, S_2 = 1$. According to the decision remarks D_1 and D_2 , in this case, the probability of false alarm is

$$Q_{f,1}^C = 0 \quad (38)$$

The probability of missed detection is

$$\begin{aligned}
 Q_{m,1}^C &= p\{D_1 = 1, D_2 = 0 | S_1 = 1, S_2 = 1\} + p\{D_1 = 0, D_2 = 1 | S_1 = 1, S_2 = 1\} \\
 &\quad + p\{D_1 = 0, D_2 = 0 | S_1 = 1, S_2 = 1\} \\
 &= p(H_1^I)Q_d^I \cdot p(H_1^{II})Q_m^{II} + p(H_1^I)Q_m^I \cdot p(H_1^{II})Q_d^{II} + p(H_1^I)Q_m^I \cdot p(H_1^{II})Q_m^{II} \\
 &= p(H_1^I)p(H_1^{II})(Q_d^I \cdot Q_m^{II} + Q_m^I)
 \end{aligned} \tag{39}$$

2) PU1 is present and PU2 is absent, i.e., $S_1 = 1, S_2 = 0$. According to the decision remarks D_1 and D_2 , in this case, the probability of false alarm is

$$\begin{aligned}
 Q_{f,2}^C &= p\{D_1 = 1, D_2 = 1 | S_1 = 1, S_2 = 0\} + p\{D_1 = 0, D_2 = 1 | S_1 = 1, S_2 = 0\} \\
 &= p(H_1^I)Q_d^I \cdot p(H_0^{II})Q_f^{II} + p(H_1^I)Q_m^I \cdot p(H_0^{II})Q_f^{II} \\
 &= p(H_1^I)p(H_0^{II})Q_f^{II}
 \end{aligned} \tag{40}$$

The probability of missed detection is

$$\begin{aligned}
 Q_{m,2}^C &= p\{D_1 = 0, D_2 = 1 | S_1 = 1, S_2 = 0\} + p\{D_1 = 0, D_2 = 0 | S_1 = 1, S_2 = 0\} \\
 &= p(H_1^I)Q_m^I \cdot p(H_0^{II})Q_f^{II} + p(H_1^I)Q_m^I \cdot \frac{1}{2} p(H_0^{II})(1 - Q_f^{II}) \\
 &= \frac{1}{2} p(H_1^I)p(H_0^{II})Q_m^I(1 + Q_f^{II})
 \end{aligned} \tag{41}$$

3) PU1 is absent and PU2 is present, i.e., $S_1 = 0, S_2 = 1$. According to the decision remarks D_1 and D_2 , in this case, the probability of false alarm is

$$\begin{aligned}
 Q_{f,3}^C &= p\{D_1 = 1, D_2 = 1 | S_1 = 0, S_2 = 1\} + p\{D_1 = 1, D_2 = 0 | S_1 = 0, S_2 = 1\} \\
 &= p(H_0^I)Q_f^I \cdot p(H_1^{II})Q_d^{II} + p(H_0^I)Q_f^I \cdot p(H_1^{II})Q_m^{II} \\
 &= p(H_0^I)p(H_1^{II})Q_f^I
 \end{aligned} \tag{42}$$

The probability of missed detection is

$$\begin{aligned}
 Q_{m,3}^C &= p\{D_1 = 1, D_2 = 0 | S_1 = 0, S_2 = 1\} + p\{D_1 = 0, D_2 = 0 | S_1 = 0, S_2 = 1\} \\
 &= p(H_0^I)Q_f^I \cdot p(H_1^{II})Q_m^{II} + \frac{1}{2} p(H_0^I)(1 - Q_f^I) \cdot p(H_1^{II})Q_m^{II} \\
 &= \frac{1}{2} p(H_0^I)p(H_1^{II})Q_m^{II}(1 + Q_f^I)
 \end{aligned} \tag{43}$$

4) PU1 is absent and PU2 is absent, i.e., $S_1 = 0, S_2 = 0$. According to the decision remarks D_1 and D_2 , in this case, the probability of false alarm is

$$\begin{aligned}
Q_{f,4}^C &= p\{D_1 = 1, D_2 = 1 | S_1 = 0, S_2 = 0\} \\
&= p(H_0^I)Q_f^I \cdot p(H_0^{II})Q_f^{II} \\
&= p(H_0^I)p(H_0^{II})Q_f^I Q_f^{II}
\end{aligned} \tag{44}$$

The probability of missed detection is

$$Q_{m,4}^C = 0 \tag{45}$$

Therefore, the equivalent false alarm and missed detection probabilities for the CR users which are located in region C can be evaluated as

$$\begin{aligned}
Q_f^C &= Q_{f,1}^C + Q_{f,2}^C + Q_{f,3}^C + Q_{f,4}^C \\
&= p(H_1^I)p(H_0^{II})Q_f^{II} + p(H_0^I)p(H_1^{II})Q_f^I + p(H_0^I)p(H_0^{II})Q_f^I Q_f^{II} \\
Q_m^C &= Q_{m,1}^C + Q_{m,2}^C + Q_{m,3}^C + Q_{m,4}^C \\
&= p(H_1^I)p(H_1^{II})(Q_m^I \cdot Q_m^{II} + Q_m^I) + \frac{1}{2}p(H_1^I)p(H_0^{II})Q_m^I (1 + Q_f^{II}) + \frac{1}{2}p(H_0^I)p(H_1^{II})Q_m^{II} (1 + Q_f^I)
\end{aligned} \tag{46}$$

$$\tag{47}$$

6. Simulations

6.1 Soft Information Fusion

We first show the optimal performance of the energy detector with soft information fusion strategy, which is an important portion in cooperative spectrum sensing that has been little addressed. For single-user sensing and multiuser sensing with soft information fusion, we use the threshold by (10) and (16) to obtain the minimum average sensing error probability respectively. These probabilities are shown as the solid curves in **Fig. 3**. Then we change the decision threshold to $\lambda = 25$ for single-user sensing and $\lambda_s = N \times 25$ for multiuser sensing with soft information fusion and obtain the dashed curves. By comparing the solid curves with the dashed curves, we can find that p_e with optimal threshold (10) is much lower than that with fixed threshold $\lambda = 25$. Then, for cooperative sensing with N CR users, with soft information fusion strategy, $Q_{e,s}$ with optimal threshold (16) is also lower than that with fixed threshold $\lambda_s = N \times 25$.

In a CR network with a large number of cooperative partners, cooperative spectrum sensing may become impractical because in a time slot only one CR user should send its local decision to FC so as to separate decisions easily at FC. Hence, it may make the whole sensing time intolerably long. When the number of CR users tends to be very large, the bandwidth for reporting their sensing results to the fusion center will be very huge. To address this issue, we propose next an efficient sensing algorithm that guarantees a target error bound by requiring the least number of cooperated CR users in cooperative spectrum sensing instead of all of them. The bandwidth used by reporting channel can be saved. Now let us look an example of the proposed efficient sensing algorithm in a network with a total of 50 CR users illustrated in **Fig. 4**. We can generate **Fig. 4** according to $Q_{e,s}$ with respect to various N when the optimal

threshold $\lambda_{s,opt}$ is applied. Setting $Q_{e,s} \leq 0.01$, we find that the least numbers of CRs to get the target error bound are 8, 18 and 40 for SNR values of 8, 6 and 4 dB, respectively. This indicates that it is sufficient to employ minimal cooperation to obtain a required QoS (Quality of Sensing).

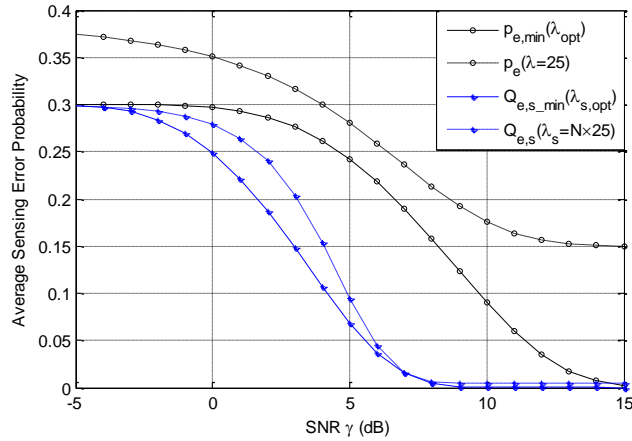


Fig. 3. Single user sensing and multiuser sensing with soft information fusion strategy ($N = 10$) in cognitive network I. Solid curves: optimal threshold; Dashed curves: fixed threshold $\lambda = 25$; $\lambda_s = N \times 25$; $p(H_0^1) = 0.7$.

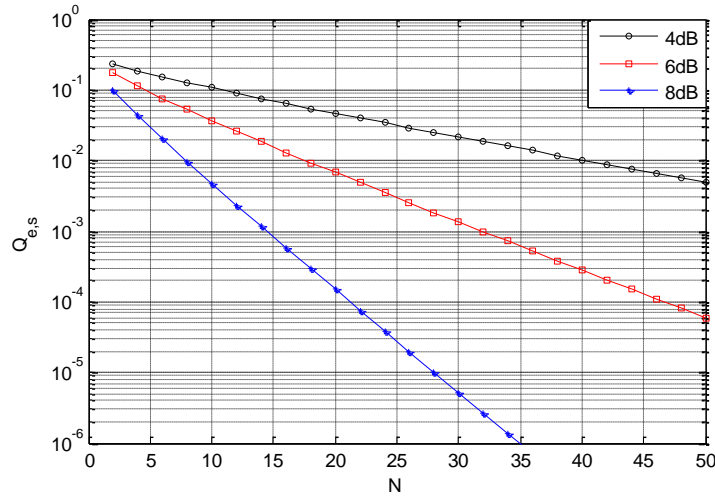


Fig. 4. Average sensing error probability versus number of cooperated CR users in cognitive network I with SNR=4, 6, 8 dB; optimal soft information fusion strategy applied; $p(H_0^1) = 0.7$.

6.1 Hard Information Fusion

With hard information fusion strategy, n in Half-voting rule is $\lceil N/2 \rceil$ consistently regardless

of the values of p_f and p_d [9]. In Fig. 5, it has been shown that the final probability of false alarm in MSE rule is noticeable smaller than that in single-user scheme and multiuser sensing with Half-voting rule. Especially when the individual probability of false alarm is very large, for example, $p_f = 0.8$, the final probability of false alarm is 0.8 in single-user scheme and is close to 1 in Half-voting rule. However, in MSE rule, the final probability of false alarm is only 0.1. This indicates that the performance of the final probability of false alarm can be greatly improved with a big value of the individual probability of false alarm in MSE rule.

In Fig. 6, it has been shown that MSE rule also improves the final probability of detection. Especially when the individual probability of detection is small, the performance of detection can be greatly improved compared with the single-user scheme and Half-voting rule. Another merit we derived in MSE rule is to quicken the asymptotic rate of convergence, so MSE rule greatly improves the performance of detection probability.

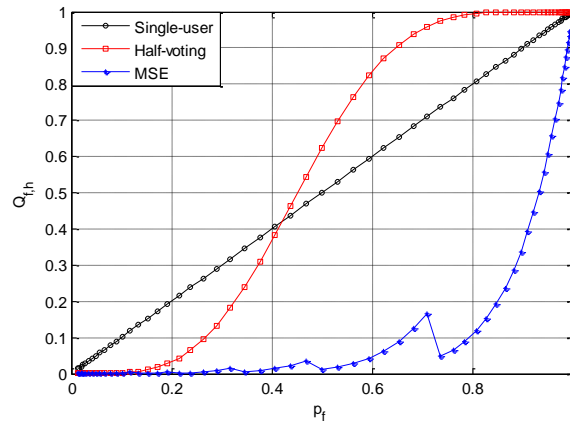


Fig. 5. $Q_{f,h}$ as functions of p_f under AWGN environment for different decision rules; $N = 10$; $SNR = 10dB$.

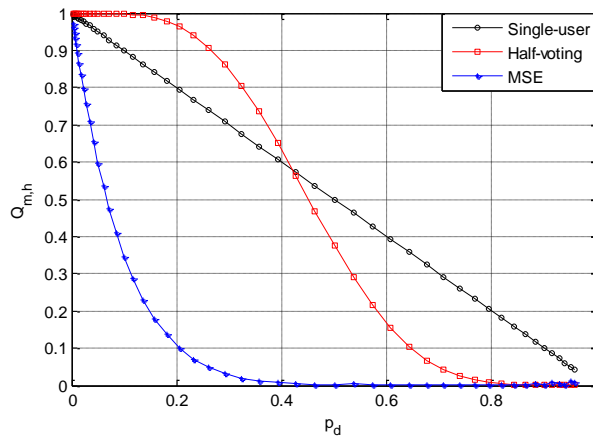


Fig. 6. $Q_{m,h}$ as functions of p_d under AWGN environment for different decision rules; $N = 10$; $SNR = 10dB$.

As indicated in **Fig. 5** and **Fig. 6**, $Q_{f,h}$ in Half-voting rule is larger than that in single-user scheme when $p_f > 0.42$ and the performance of detection probability is also worse when $p_d < 0.42$. Hence, Half-voting rule cannot show better performance for different objectives. Contrastingly, MSE rule appropriately improves both the final false alarm probability and detection probability simultaneously. Moreover, it obviously improves the related sensing outcomes to achieve different optimization objectives. In addition, it has been shown that there exists some sawtooths in **Fig. 5** and **Fig. 6**. The reason is that in MSE rule, the final decision threshold is an integer $\lceil \rho \rceil$ and ρ is adaptive to the local threshold of the energy detector.

6.3 Performance Comparisons

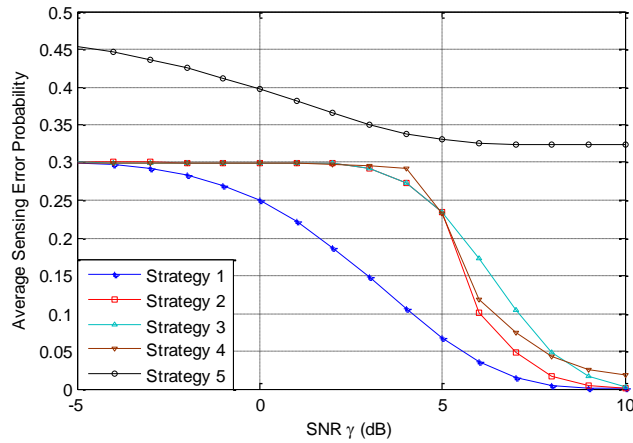


Fig. 7. Average sensing error probabilities under five strategies in cognitive network I; $\lambda = 21$ for strategy 4 and strategy 5; $N = 10$; $p(H_0^1) = 0.7$.

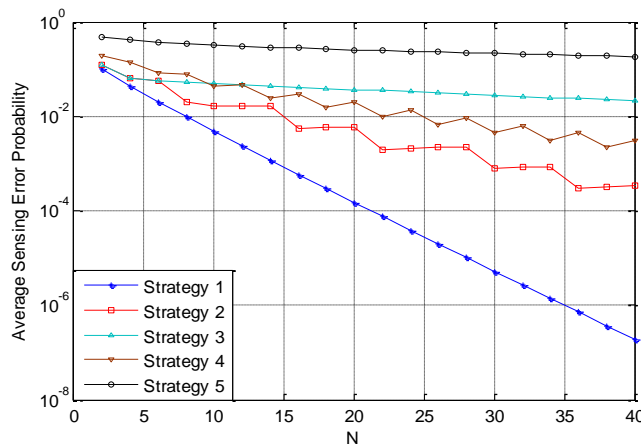


Fig. 8. Average sensing error probabilities under five strategies in cognitive network I; $\lambda = 21$ for strategy 4 and strategy 5; $SNR = 8dB$; $p(H_0^1) = 0.7$.

To illustrate the relative performances of various decision strategies, we show in **Fig. 7** and **Fig. 8** five strategies of multiuser sensing, i.e., *Strategy 1*: optimal soft information fusion;

Strategy 2: hard information fusion with locally optimal threshold and MSE rule; *Strategy 3*: hard information fusion with locally optimal threshold and Half-voting rule; *Strategy 4*: hard information fusion with locally fixed threshold ($\lambda = 21$) and MSE rule; *Strategy 5*: hard information fusion with locally fixed threshold ($\lambda = 21$) and Half-voting rule.

It has been shown in Fig. 7 that strategy 1 provides a theoretical bound on the performance of average sensing error probability in an ideal cooperative sensing setup. Under hard information fusion, the performance of our proposed strategy (strategy 2) is suboptimal among the five strategies since the local threshold of energy detector and the final decision threshold are both optimal. Strategy 3 and strategy 4 optimize only one level of decision making, namely, either the local threshold of energy detector or the final decision threshold. Thus, the average sensing error probability is larger than strategy 2. In strategy 5, since the local threshold and the final decision threshold are both fixed values, the sensing performance is the worst among five strategies.

The simulations in Fig. 8 match the analysis from Fig. 7. In addition, from Fig. 8, it has been shown that spectrum sensing performance of strategy 1 and strategy 2 can be greatly improved with an increase of the number of cooperative partners. In strategy 5, the average sensing error probability is almost unaltered when N is larger than 30. This is because both the local threshold and the final decision threshold in strategy 5 are fixed values regardless of the values of p_f and p_d and the related sensing performance is bad. In addition, it has been shown that there exists some sawtooths in strategy 2 and strategy 4. The reason is that MSE rule is employed and the final decision threshold is an integer $\lceil \rho \rceil$.

6.4 Sensing Performances of CR Users within the Cross-over Region

In practical cognitive radio networks, the soft information fusion strategy is not achievable. So we employed the hard information fusion strategy in the cross-over cognitive radio networks. FC1 and FC2 make final decision results to determine which channel to access for the CR users which are located in region C. These CR users can choose one licensed band to access either PU1 or PU2 is absent. As illustrated in Fig. 9 and Fig. 10, in region C, our proposed MSE rule also greatly improves the sensing performances for the SUs within the cross-over region.

We show in Fig. 9 and Fig. 10 four schemes in the cross-over cognitive radio networks, i.e., *Scheme 1*: cognitive network I: MSE rule and cognitive network II: MSE rule; *Scheme 2*: cognitive network I: MSE rule and cognitive network II: Half-voting rule; *Scheme 3*: cognitive network I: Half-voting rule and cognitive network II: MSE rule; *Scheme 4*: cognitive network I: Half-voting rule and cognitive network II: Half-voting rule.

As indicated in Fig. 9, since MSE rule is employed in both the two cognitive networks, scheme 1 has the best sensing performance, the final probability of false alarm is noticeable smaller than that in other schemes. Since scheme 2 and scheme 3 choose MSE rule in one cognitive network and Half-voting rule in another cognitive network, the final probability of false alarm is a little larger than that in scheme 1. Contrastingly, scheme 4 has the worst false alarm probability performance since Half-voting rule is employed in both the two cognitive networks. For example, when $\lambda = 16$, $Q_f^c \approx 0.02$ in scheme 1; $Q_f^c \approx 0.27$ in scheme 2; $Q_f^c \approx 0.15$ in scheme 3; $Q_f^c \approx 0.92$ in scheme 4.

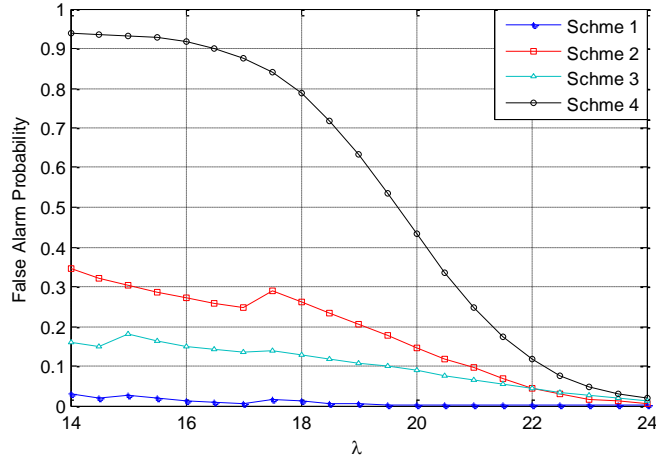


Fig. 9. False alarm probability versus threshold of energy detector in region C; hard information fusion strategy applied; $SNR = 10dB$; $N = 10$; $K = 20$; $p(H_0^I) = 0.7$; $p(H_0^{II}) = 0.8$.

In **Fig. 10**, scheme 1 also has the best detection performance among the four schemes. In low λ region, 14 ~ 22, as shown in **Fig. 9**, the final probability of false alarm with scheme 2 is larger than scheme 3. However, in high λ region, 34 ~ 48, as shown in **Fig. 10**, the final probability of missed detection with scheme 2 is lower than scheme 3. This is because both scheme 2 and scheme 3 choose MSE rule in one cognitive network and Half-voting rule in another cognitive network. Consequently, our proposed MSE rule optimizes the sensing performance for each CR user which is randomly distributed in the cross-over cognitive radio networks.

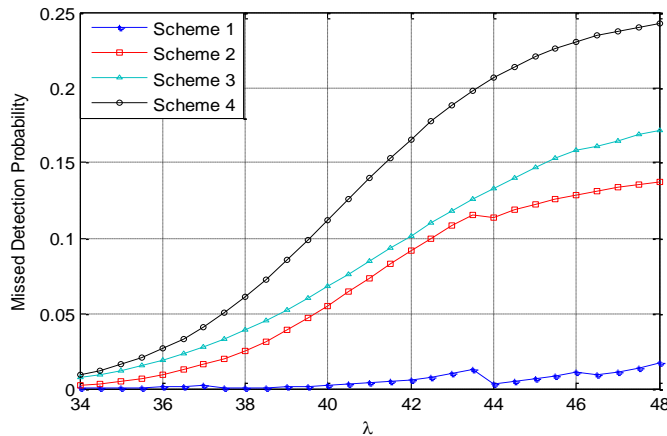


Fig. 10. Missed detection probability versus threshold of energy detector in region C; hard information fusion strategy applied; $SNR = 10dB$; $N = 10$; $K = 20$; $p(H_0^I) = 0.7$; $p(H_0^{II}) = 0.8$.

7. Conclusion

We consider the optimal strategy of cooperative spectrum sensing with soft information fusion

and hard information fusion in multiple cross-over cognitive radio networks. It has been found that the optimal local threshold of the energy detector and the optimal decision threshold are related to SNR of the CR users. The optimal soft information fusion provides a theoretical bound on error probability performance in an ideal cooperative sensing setup. With hard information fusion strategy, MSE rule appropriately improves both the final false alarm probability and detection probability simultaneously. By simulations, our proposed strategy optimizes the sensing performance for each CR user which is randomly distributed in the multiple cross-over cognitive radio networks.

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