NEW ORBITAL PARAMETERS AND RADIAL VELOCITY CURVE ANALYSIS OF SPECTROSCOPIC BINARY STARS

Kamal Ghaderi, Ali Pirkhedri, Touba Rostami, Salem Khodamoradi, and Hedayat Fatahi
Department of Science and Engineering, Islamic Azad University, Marivan Branch, Marivan, Iran
E-mail : K.Ghaderi.60@gmail.com

ABSTRACT

We use a Probabilistic Neural Network (PNN) technique to derive the orbital parameters of spectroscopic binary stars. Using measured radial velocity data of five double-lined spectroscopic binary systems (i.e., EQ Tau, V376 And, V776 Cas, V2377 Oph and EE Cet), we find the corresponding orbital and spectroscopic elements. Our numerical results are in good agreement with those obtained by other groups via more traditional methods.

*Key words :* Stars, binaries, eclipsing — Stars, binaries, spectroscopic

1. INTRODUCTION

Determining the orbital elements of binary stars allows one to obtain fundamental information, such as the mass and radius of stars; their knowledge plays an important role in the evolution of stellar objects. Through the analysis of both the light and radial velocity (hereafter V_R) curves deduced from photometric and spectroscopic observations, respectively, we can infer the complete orbital parameters. There are different methods to determine the orbit of a spectroscopic binary from its V_R curve. Lehmann-Filhés (1894) introduced a geometrical technique to determine the orbital elements from the geometrical properties of the V_R curve, especially its maxima and minima. The method of Lehmann-Filhés (1984) has been very useful and little, if any, longer than other methods, providing a planimeter is used. In order to improve the final solution of the orbit, their technique also provides the differential corrections to the preliminary elements, using the form of the equations obtained by the method of least squares. Russell (1902) suggested an analytical technique to develop the observed V_R into a trigonometric series (Fourier series), and the elements are found by comparing this series with the corresponding analytical expression for the velocity. In this method the time consumed is considerably longer than that of Lehmann-Filhés (1984), and when the eccentricity is much greater than 0.4, it becomes laborious. King (1908) proposed a graphical method which enables one to use the entire velocity curve in finding the preliminary elements, and is applicable to orbits of any eccentricity. This method depends on the velocities at equal time intervals as read from a freehand curve drawn to represent the observations. Schlesinger (1910) presented the least square method, which differ-

entially corrects the preliminary orbital elements using the equations of condition. Schlesinger's method is suitable for all the orbits except those with very small eccentricities. It has a greater accuracy and it also enables one to vary all the unknown parameters simultaneously, instead of one or two at the time. Russell (1914) developed a quick method analogous to the graphical technique of King (1908), equally applicable to orbits with all eccentricities. It has the advantage that a single diagram replaces numerous protractors which must be constructed for each separate value of the eccentricity. Within king's method, the graphical processes which require precise drawings of lines and measurements of angles will consume more time than the quick strategy of Russell (1914). Sterne (1941) described two forms of least square solutions, both of which allow for the differential corrections to be set of preliminary elements obtained by any direct method like those of Russell (1902, 1914). The first form is a modification of the Schlesinger's method (1910), in which the date of periastron passage is replaced by the date at which the mean longitude is zero. The first form is suitable for all the orbits except those with very small eccentricities. The second form is particularly suitable for orbits having very small eccentricities. Singh (1984) introduced an approach similar to the method of Russell (1902) for expanding the V_R as a trigonometric series. This technique gives a standard set of velocity curves for a set of eccentricity, and the longitude of periastron values that can be used for the determination of preliminary elements preparatory to a least square correction. Karami and Teimoorinia (2007) introduced a new non-linear least squares velocity curve analysis technique for spectroscopic binary stars. Their method can be applied to orbits of all eccentricities and inclination angles, and the required time is considerably less than the one obtained with method of Lehmann-Filhés (1984). They showed the validity of their new

Corresponding Author: K. Ghaderi
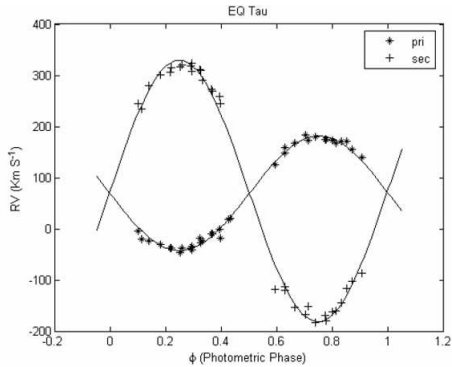
Fig. 1.— Radial velocities of the primary and secondary components of EQ Tau, plotted against the photometric phase. The observational data have been measured by Rucinski et al. (2001).


Fig. 2.— Same as Fig. 1, but for V376 And.

Fig. 3.— Same as Fig. 2, but for V776 Cas.

method using a wide range of different types of binaries - see Karami, Mohebi (2007a, b, 2009) and Karami et al. (2008).

So far, a large number of practical methods have been proposed. However, it is not useful to discuss their relative advantages: the method which gives the best result in one case may be unsuitable in another. For instance, for near circular orbits and small eccentricity, the V_R curve approaches the simple sine curve and graphical methods are not ideal. In that case, analytical methods become useful - see Curtis (1908), Plummer (1908) and Sterne (1941). Lucy and Sweeney (1971) suggested that, because of observational errors, most of the spectroscopic binaries with eccentricities close to zero should be assigned circular orbits. Their argument is based purely statistical considerations, regardless of the orbital period, and has been subsequently challenged by other authors. For instance, Skuljan et al. (2004) emphasize that modern observations do support the idea of Lucy and Sweeney (1971), at least for the shortest-period binaries (e.g., with period less than about 10 days for late-type dwarfs). However, the situation for longer periods is not clear, and the single-lined spectroscopic binary star ζ TrA is one such examples. They point out that the orbit of ζ TrA was proved to be a definite ellipse, although with an extremely small eccentricity equal to 0.01442. There is always the possibility that a third low mass unseen component could have perturbed the orbit of ζ TrA to non-circularity. For the evolution of low mass members of close binary systems see Han et al. (2000), Yakut and Eggleton (2005).

Artificial Neural Networks (ANNs) have become a popular tool in almost every field of science. In recent years, ANNs have been widely used in astronomy for applications such as star/galaxy discrimination, morphological classification of galaxies, and spectral classification of stars (see Bazarghan et al. 2008, and references therein). Following Bazarghan et al. (2008), we employ Probabilistic Neural Networks (PNNs), which
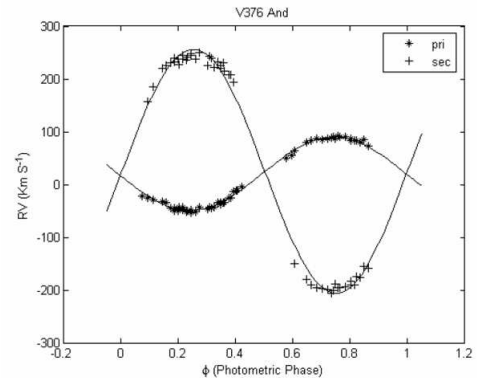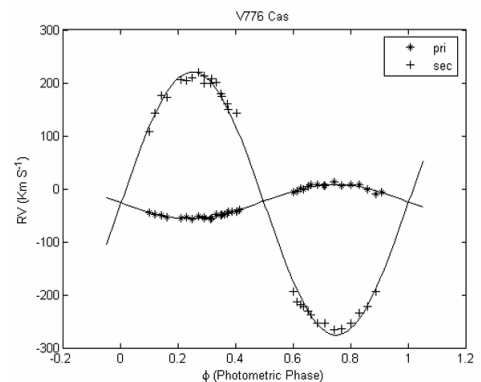
have been investigated in detail by Bazarghan et al. (2008).

PNNs are a new tool to derive the orbital parameters of spectroscopic binary stars. In this method, the time consumed is considerably inferior to that of Lehmann-Filhés, and even less than the non-linear regression method proposed by Karami & Teimoorinia (2007). In the present paper, we use a PNN technique to find the optimum match to the four parameters of the V_R curves of the five double-lined spectroscopic binary systems: EQ Tau, V376 And, V776 Cas, V2377 Oph, and EE Cet. Our aim is to show the validity of our new method for a wide range of binaries of different types.

In particular, EQ Tau is a close binary system. The spectral type is G2, with a period of P=0.341348 days (Rucinski et al. 2001). V376 And is a contact binary of W UMa-type with spectral type A4V. The relatively long period of 0.799 days is consistent with the spectral type (Rucinski et al. 2001). V776 Cas is a contact A-type system with a very small mass ratio due to a low orbital inclination. The spectral type is F2V and the orbital period is 0.440413 days (Rucinski et al. 2001). V2377 Oph is a fairly uncomplicated W-type contact binary. The spectral type is G0/1V and the orbital period is 0.425401 days (Lu et al. 2001). EE Cet is a
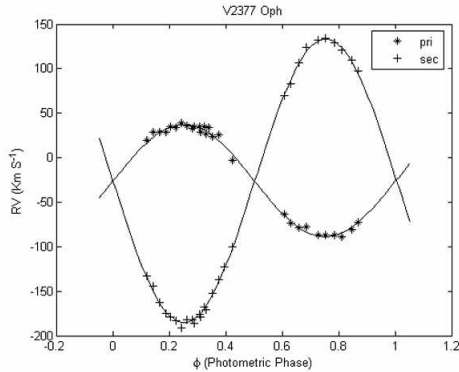
Fig. 4.— Radial velocities of the primary and secondary components of V2377 Oph, plotted against the photometric phase. The observational data have been measured by Lu et al. (2001).
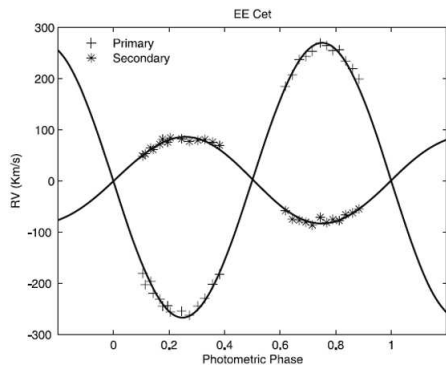
Fig. 5.— Radial velocities of the primary and secondary components of EE cet, plotted against the photometric phase. The observational data have been measured by Rucinski et al. (2002a,b).

contact double-lined spectroscopic binary with a period of P = 0.339917 days (Rucinski et al. 2002a, b).

This paper is organized as follows. In Section 2, we introduce a Probabilistic Neural Network (PNN) method to estimate the four parameters of the V_R curve. Section 3 contains the numerical results. Conclusions are given in Section 4.

2. PNN PARAMETER ESTIMATION

Following Smart (1990), the V_R of a star in a binary system is defined as

$$V_R = \gamma + K[\cos(\theta + \omega) + e \cos \omega], \qquad (1)$$

where γ is the V_R of the center of mass of the system with respect to the Sun. Also K is the amplitude of the V_R of the star with respect to the center of mass of the binary. Furthermore θ, ω and e are respectively the angular polar coordinate (true anomaly), longitude of periastron and eccentricity.

Here we apply the PNN method to estimate the four orbital parameters γ, K, e and ω of the V_R curve in Eq. (1). In this work, for the identification of the observational V_R curves, the input vector is the fitted V_R curve of a star. The PNN is first trained to classify the V_R curves corresponding to all the possible combinations of γ, K, e and ω. One can synthetically generate V_R curves given by Eq. (1), for each combination of the parameters:

- $-100 \leq \gamma \leq 100$ in steps of 1;
- $1 \leq K \leq 300$ in steps of 1;
- $0 \leq e \leq 1$ in steps of 0.001;
- $0 \leq \omega \leq 360°$ in steps of 5°.

The training procedure provides a large set of k pattern groups, where k denotes the number of different V_R classes, one class for each combination of γ, K, e and ω. Since the large number of different V_R classes causes some computational limitations, one can first start with larger step-sizes. Note that, according to Petrie (1960), one can guess γ, K and e from the V_R curve. This enables one to restrict the range of parameters around their initial guesses. When the preliminary orbit is derived, one can use smaller step-sizes to obtain the final orbit. The PNN has four layers: input, pattern, summation, and output, respectively (see Fig. 5 in Bazarghan et al. 2008). When an input vector is present, the pattern layer computes distances from the input vector to the training input vectors and produces a vector whose elements indicate how close the input is to a training input. The summation layer adds these contributions for each class of inputs, to produce a vector of probabilities. Finally, a competitive transfer function on the output layer picks the maximum of these probabilities, and returns one for the current class, and zero for the others (Specht 1988, 1990). Thus, the PNN classifies the input vector into a specific k class labeled by the four parameters γ, K, e and ω, since that class has the maximum probability of occurrence.

3. NUMERICAL RESULTS

In this section we use the PNN method to derive the orbital elements for five different double-lined spectroscopic systems (i.e., EQ Tau, V376 And, V776 Cas, V2377 Oph and EE Cet). With measured V_R data for the two components of these systems (EQ Tau, V376 And, V776 Cas are data from Rucinski et al. 2001, V2377 Oph is from Lu et al. 2001, while EE Cet is taken from Rucinski et al. 2002a, b), we plot in figs. 1-5 the velocity curves as a function of their phases.

The orbital parameters obtained from the PNN for EQ Tau, V376 And, V776 Cas, V2377 Oph and EE Cet are reported in Tables 1, 3, 5, 7 and 9, respectively. The various tables show that results are in good agreement with those obtained by Rucinski et al. (2001) for EQ

Table 1.
Orbital parameters of EQ Tau

	This Paper	Rucinski et al. (2001)
$\gamma\left[\mathrm{kms}^{-1}\right]$	72 ± 1	$71.95(1.22)$
$K_p\left[\mathrm{kms}^{-1}\right]$	112 ± 1	$112.41(1.43)$
$K_s\left[\mathrm{kms}^{-1}\right]$	255 ± 1	$254.38(2.42)$
e	0.002 ± 0.001	—
$\omega[°]$	290 ± 5	—

Table 2.
Combined spectroscopic elements of EQ Tau

Parameter	This Paper	Rucinski et al. (2001)
$m_p \sin^3 i / M_\odot$	1.2147 ± 0.0180	—
$m_s \sin^3 i / M_\odot$	0.5335 ± 0.0106	—
$(m_p + m_s)\sin^3 i / M_\odot$	1.7482 ± 0.0286	$1.749(55)$
$a_p \sin i / R_\odot$	0.7553 ± 0.0067	—
$a_s \sin i / R_\odot$	1.7197 ± 0.0067	—
$(a_p + a_s)\sin i / R_\odot$	2.4751 ± 0.0135	—
m_s/m_p	0.4392 ± 0.0056	$0.442(7)$

Table 3.
Orbital parameters of V376 And

	This Paper	Rucinski et al. (2001)
$\gamma\left[\mathrm{kms}^{-1}\right]$	23 ± 1	$22.83(0.89)$
$K_p\left[\mathrm{kms}^{-1}\right]$	70 ± 1	$70.00(0.67)$
$K_s\left[\mathrm{kms}^{-1}\right]$	230 ± 1	$229.67(1.78)$
e	0.004 ± 0.001	—
$\omega[°]$	275 ± 5	—

Table 4.
Combined spectroscopic elements of V376 And

Parameter	This Paper	Rucinski et al. (2001)
$m_p \sin^3 i / M_\odot$	1.7135 ± 0.0303	—
$m_s \sin^3 i / M_\odot$	0.5215 ± 0.0144	—
$(m_p + m_s)\sin^3 i / M_\odot$	2.2351 ± 0.0447	$2.232(55)$
$a_p \sin i / R_\odot$	1.1050 ± 0.0158	—
$a_s \sin i / R_\odot$	3.6307 ± 0.0158	—
$(a_p + a_s)\sin i / R_\odot$	4.7358 ± 0.0316	—
m_s/m_p	0.3043 ± 0.0057	$0.305(5)$

Table 5.
Orbital parameters of V776 Cas

	This Paper	Rucinski et al. (2001)
$\gamma\left[\mathrm{kms}^{-1}\right]$	-25 ± 1	$-24.71(1.11)$
$K_p\left[\mathrm{kms}^{-1}\right]$	32 ± 1	$31.97(0.64)$
$K_s\left[\mathrm{kms}^{-1}\right]$	245 ± 1	$245.31(1.83)$
e	0.001 ± 0.001	—
$\omega[°]$	285 ± 5	—

Tau, V376 And and V776 Cas, Lu et al. (2001) for V2377 Oph, and Rucinski et al. (2002a, b) for EE Cet.

Note that the Gaussian errors of the orbital parameters in Tables 1, 3, 5, 7 and 9 are the same selected steps for generating V_R curves, i.e., $\Delta\gamma = 1, \Delta K = 1, \Delta e = 0.001$ and $\Delta\omega = 5$. These are close to the observational errors reported in the literature. Regarding the estimated errors, following Specht (1990) the error of the decision boundaries depends on the accuracy with which the underlying Probability Density Functions (PDFs) are estimated. Parzen (1962) proved that the expected error gets smaller, as the estimate is based on a large data set. This definition of consistency is particularly important since the true distribution will be approached smoothly. Specht (1990) showed that a very large value of the smoothing parameter would cause the estimated errors to be Gaussian regardless of the true underlying distribution and the misclassification rate is stable and does not change dramatically with small changes of the smoothing parameter. The combined spectroscopic elements, including $m_p \sin^3 i$, $m_s \sin^3 i$, $(m_p + m_s)\sin^3 i$, $(a_p + a_s)\sin i$ and m_s/m_p, are calculated by substituting the estimated parameters K, e and ω into Eqs. (3), (15) and (16) in Karami & Teimoorinia (2007). The results obtained for the five systems are reported in Tables 2, 4, 6, 8 and 10 they show that our results are in good agreement with those obtained by Rucinski et al. (2001) for EQ Tau, V376 And and V776 Cas, Lu et al. (2001) for V2377 Oph and Rucinski et al. (2002a, b) for EE Cet, respectively. The errors of the combined spectroscopic elements in tables 2, 4, 6, 8 and 10 are obtained by the help of orbital parameters errors - see again Eqs. (3), (15) and (16) in Karami and Teimoorinia (2007).

4. CONCLUSIONS

We applied a Probabilistic Neural Network (PPN) technique to derive the orbital elements of spectroscopic binary stars. PNNs are used in both regression (including parameter estimation) and classification problems. However, one can discretize a continuous regression problem to such a degree that it can be represented as a classification problem (Specht 1988, 1990), as we did in this work.

Using the measured V_R from data of EQ Tau, V376 And, V776 Cas, V2377 Oph and EE Cet obtained by Rucinski et al. (2001), Lu et al. (2001) and Rucinski et al. (2002a, b), respectively, we find the orbital elements of these systems by the PNN technique. Our numerical results show that the orbital and spectroscopic parameters are in good agreement with those obtained by other groups using more traditional methods.

Our technique is applicable to orbits of all eccentricities and inclination angles. In our method, the time spent to recover the various parameters is considerably inferior to that of Lehmann-Filhés. It is also more accurate as the orbital elements are deduced from all the

Table 6.
Combined spectroscopic elements of V776 Cas

Parameter	This Paper	Rucinski et al. (2001)
$m_p \sin^3 i/M_\odot$	0.8578 ± 0.0159	—
$m_s \sin^3 i/M_\odot$	0.1120 ± 0.0051	—
$(m_p + m_s) \sin^3 i/M_\odot$	0.9698 ± 0.0210	0.975(26)
$a_p \sin i/R_\odot$	0.2784 ± 0.0087	—
$a_s \sin i/R_\odot$	2.1318 ± 0.0087	—
$(a_p + a_s) \sin i/R_\odot$	2.4103 ± 0.0174	—
m_s/m_p	0.1306 ± 0.0046	0.130(4)

Table 10.
Combined spectroscopic elements of EE Cet

Parameter	This Paper	Rucinski et al. (2002a,b)
$m_p \sin^3 i/M_\odot$	0.3645 ± 0.0085	—
$m_s \sin^3 i/M_\odot$	1.1585 ± 0.0175	—
$(m_p + m_s) \sin^3 i/M_\odot$	1.5229 ± 0.0260	1.706(41)
$a_p \sin i/R_\odot$	1.7931 ± 0.0067	—
$a_s \sin i/R_\odot$	0.5641 ± 0.0067	—
$(a_p + a_s) \sin i/R_\odot$	2.3572 ± 0.0134	—
m_p/m_s	0.3146 ± 0.0049	0.315(5)

Table 7.
Orbital parameters of V2377 Oph

	This Paper	Lu et al. (2001)
$\gamma \left[\mathrm{kms}^{-1}\right]$	-25 ± 1	$-25.79(3.80)$
$K_p \left[\mathrm{kms}^{-1}\right]$	63 ± 1	62.99(0.62)
$K_s \left[\mathrm{kms}^{-1}\right]$	160 ± 1	159.64(0.70)
e	0.004 ± 0.001	—
$\omega[^\circ]$	45 ± 5	—

Table 8.
Combined spectroscopic elements of V2377 Oph

Parameter	This Paper	Lu et al. (2001)
$m_p \sin^3 i/M_\odot$	0.3507 ± 0.0085	—
$m_s \sin^3 i/M_\odot$	0.1381 ± 0.0047	—
$(m_p + m_s) \sin^3 i/M_\odot$	0.4888 ± 0.0132	0.487(9)
$a_p \sin i/R_\odot$	0.5295 ± 0.0084	—
$a_s \sin i/R_\odot$	1.3447 ± 0.0084	—
$(a_p + a_s) \sin i/R_\odot$	1.8742 ± 0.0168	—
m_s/m_p	0.3937 ± 0.0087	0.395(12)

Table 9.
Orbital parameters of EE Cet

	This Paper	Rucinski et al. (2002a,b)
$\gamma \left[\mathrm{kms}^{-1}\right]$	2 ± 1	1.6(0.93)
$K_p \left[\mathrm{kms}^{-1}\right]$	267 ± 1	266.92(1.54)
$K_s \left[\mathrm{kms}^{-1}\right]$	84 ± 1	84.05(1.24)
e	0.001 ± 0.001	—
$\omega[^\circ]$	305 ± 5	—

complete because certain phases could not have been observed, or when a star is surrounded by two dark companions with commensurable periods. In the latter situation, the resultant velocity curve may have several unequal maxima, causing the geometrical methods to fail.

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points of the velocity curve instead of just four as in the method of Lehmann-Filhés. The present method enables one to vary all the unknown parameters γ, K, e and ω simultaneously instead of one or two of them at the time. It is possible to make adjustments in the elements before the final result is obtained. There are some cases for which the geometrical methods are inapplicable, and in these situations the present technique may be useful. For example, when observations are in-

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