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# A Comparative Study on High School Students' Mathematical Modeling Cognitive Features[1]

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Comparative studies on mathematical modeling cognition feature were carried out between 15 excellent high school third-grade science students (excellent students for short) and 15 normal ones (normal students for short) in China by utilizing protocol analysis and expert-novice comparison methods and our conclusions have been drawn as below.
1. In the style, span and method of mathematical modeling problem representation, both excellent and normal students adopted symbolic and methodological representation style. However, excellent students use mechanical representation style more often. Excellent students tend to utilize multiple-representation while normal students tend to utilize simplicity representation. Excellent students incline to make use of circular representation while normal students incline to make use of one-way representation.
2. In mathematical modeling strategy use, excellent students tend to tend to use equilibrium assumption strategy while normal students tend to use accurate assumption strategy. Excellent students tend to use sample analog construction strategy while normal students tend to use real-time generation construction strategy. Excellent students tend to use im-

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mediate self-monitoring strategy while normal students tend to use review-monitoring strategy. Excellent students tend to use theoretical deduction and intuitive judgment testing strategy while normal students tend to use data testing strategy. Excellent students tend to use assumption adjustment and modeling adjustment strategy while normal students tend to use model solving adjustment strategy.
3. In the thinking, result and efficiency of mathematical modeling, excellent students give brief oral presentations of mathematical modeling, express themselves more logically, analyze problems deeply and thoroughly, have multiple, quick and flexible thinking and the utilization of mathematical modeling method is shown by inspiring inquiry, more correct results and high thinking efficiency while normal students give complicated protocol material, express themselves illogically, analyze problems superficially and obscurely, have simple, slow and rigid thinking and the utilization of mathematical modeling method is shown by blind inquiry, more fixed and inaccurate thinking and low thinking efficiency.

*Keywords:* mathematical modeling, cognitive feature, protocol analysis, expert-novice research
*MESC Classification*: M15, C35
*MSC2010 Classification*: 97M10, 97C30

## 1. INTRODUCTION

Based on mighty education function of mathematical modeling, many countries have put mathematical modeling into high school mathematics curriculums as a crucial component in recent decades. Mathematical modeling has been introduced into high school mathematics curriculums according to the Curriculum Standard of Mathematics in Senior High school (experiment) enacted by the Ministry of Education of the People's Republic of China in 2003. Some progress has been made in the execution of high school mathematical modeling curriculums and teaching effects are unsatisfactory (Li, 2007; Li & Yu, 2008; Li, 2009). One of primary causes is that it is hard to provide necessary cognitive psychological instructions for mathematical modeling curriculum planning and teaching because of the absence of researches on cognitive law (Xu, 2007; 2008). To address these problems, we carried out comparative studies on high school students' mathematical modeling cognitive features at different levels through the utilization of protocol analysis and expert-novice comparison to identify different students' mathematical modeling cognitive features and difference, to provide psychological basis for high school mathematical modeling curriculum planning and teaching execution and to increase the mathematical modeling teaching effects.

# 2. METHOD

## 2.1. Subjects

Four high schools were selected in Jiangsu and Guangdong province in China, including two high-level schools and two normal schools. Total 218 students were chosen in a third grade class in each school. Mathematical modeling Testing (Group) tests were performed (these tests were used to investigate factors which affected high school students' mathematical modeling academic achievement) (*cf.* Li, Cai & Wang, 2010). 15 students with the best scores were chosen as excellent student subjects (these subjects taking more mathematical modeling training were treated as expert subjects in our research) and 15 students with sorted test scores from 73%–27% were randomly chosen as normal student subjects (these subjects taking less mathematical modeling training were treated as novice subjects in our research).

## 2.2. Instruments

The test instruments consisted of 6 mathematical modeling problems, derived from the screening, adjustment and improvement of 8 preliminary constructed mathematical modeling problems advised by mathematical modeling experts and course teachers in the light of features of requirements of protocol test time and style (see the appendix). The first two problems were for practices and the latter four were for tests. Among the latter four problems, the third was simple, the forth was moderate and the fifth and sixth were complicate. For the sixth problem, subjects were not required for complete solving, but required to speak out of the thinking. These results of the sixth problem were not counted and only used for subjects' background reference materials of the thinking feature analyses. The instruction of test instruments was "please read problems aloud and say whatever you think during problem-solving. Do not say why you do like this, but read out your thinking aloud. I will be recording your saying to know how you solve problems."

## 2.3. Programs

### *2.3.1. Protocol practice*

The tester explained instructional requirements to the subject in advance and gave a protocol of the first problem as an example, then let the subject practice the protocol of the second problem. If the subject stopped for a long time in mathematics modeling, the tester could give him (her) an indication: "What are you thinking?" "Please speak out of whatever you think" to spur him (her) to voice his (her) thinking and give timely, continuous and complete protocol.

#### *2.3.2. Protocol test*

The formal tests started after subjects' observation, practice and adaptation of mathematical modeling protocol test and the time was unlimited. After the subject finished solving (or gives up solving), the tester recorded the used time, required the subject to recall and describe the thinking and procedure of mathematical modeling in time and ask questions and make discussions about his obscure and unclear expression necessarily.

#### *2.3.3. Protocol organization*

The tester organized each subject's recording of mathematical modeling protocol into the text after finishing tests, then checked and complemented preliminary the text along with the recording, pencil-and-paper test and questions along with discussions, and configure complete mathematical modeling protocol test and interview materials (text version).

#### *2.3.4. Protocol analysis*

Through expert-novice comparison, mathematical modeling protocol test and interview materials of each subject were analyzed from mathematical modeling problem representation, strategy utilization, thinking features, solving results and solving efficiency. We also classified these according to excellent subjects and normal subjects, and compared mathematical modeling cognitive features and difference between these two subjects.

## 3. RESULTS

### 3.1. The comparison of mathematical modeling problem representation features between excellent students and normal students

We analyzed and compared subjects' mathematical modeling protocol test and interview materials from mathematical modeling problem representation mode, span and method between excellent students and normal students. Results were shown in Table 1.

According to the analyses of subjects' oral presentation materials, we thought that representation modes of subjects' mathematical modeling problems consisted of symbolic representation, methodical representation and mechanical presentation. Symbolic representation of mathematical modeling means the comprehension and resolving of mathematical modeling problems from wording languages, figures and images and mathematical symbols, and the acquisition and comprehension of descriptive knowledge of mathematical modeling problems.

**Table 1.** The comparison of mathematical modeling problem representation features between excellent students and normal students

| Group | Problem No. | Mode | | | Span | | Method | |
|---|---|---|---|---|---|---|---|---|
| | | SR | MR | MeR | SiR | MuR | OR | CR |
| EG | 45 | 45 | 45 | 32 | 8 | 37 | 10 | 35 |
| NG | 45 | 45 | 45 | 11 | 34 | 11 | 36 | 9 |
| *Z* | | – | – | 4.316 | −4.887 | 4.863 | −4.871 | 4.879 |
| *p* | | | | 0.005 | 0.001 | 0.001 | 0.001 | 0.001 |

Abbreviation: EG, Excellent Group; NG, Normal group; SR, Symbolic Representation; MR, Methodical Representation; MeR, Mechanical Representation; SiR, Simplicity Representation; MuR, Multiple Representation; OR, One-way Representation; CR, Circular Representation.

Methodical representation of mathematical modeling is the comprehension and resolving of mathematical modeling problems from strategies and methods, and the acquisition and comprehension of programmable knowledge of mathematical modeling problems. Mechanical representation of mathematical modeling is the comprehension and resolving of mathematical modeling problems from problematic structures and required theories, and the activation of knowledge and structures of theories and categories on mathematical modeling problems. Symbolic representation is the most superficial resolving; mechanical representation is the deepest one and methodical representation is the resolving amid those two. It was shown in table 1 that there was significant difference in representation modes of mathematical modeling problems between excellent students and normal students. Although both of them used symbolic representation and methodical representation modes, excellent students had more problem numbers to use mechanical representation modes than normal students.

According to the analyses of subjects' protocol test and interview materials, we considered that representation span of subjects' mathematical modeling problems could be depicted by multiple representation and Simplicity representation. The multiple representations is that a modeler uses multiple representation modes, such as reading, data repressing, diagrammatizing, symbolization, to apperceive and comprehend the information, the connotation and structures of mathematic modeling problems. The Simplicity representation means that a modeler only uses one of representation modes, such as reading, data repressing, diagrammatizing and symbolization, to apperceive and comprehend mathematic modeling problems. It was shown in table 1 that the significant difference of representation span of mathematical modeling problems occurred between excellent students and normal students. Excellent students used multiple representation modes more often while normal students were inclined to use simplicity representation modes.

According to the analyses of subjects' protocol materials, we considered that representation methods included circular representation methods and one-way representation methods. Circular representation methods are that a tester uses problematic representation over again and repeatedly in mathematic modeling. One-way representation methods are that a tester does not repeat any more after using problematic representation in the stage of problem comprehension. Table 1 showed that the significant difference of representation methods of mathematical modeling problems appeared between excellent students and normal students. Excellent students tended to use circular representation methods while normal students were inclined to use one-way representation methods.

### 3.2. The comparison of strategic features of mathematical modeling problem between excellent students and normal students

Based on subjects' protocol test and interview materials, analyses and statistic comparisons on strategic types used in mathematical modeling between excellent students and normal students have been taken. Results were shown in Table 2.

From Table 2, we can conclude that excellent students and normal ones differ greatly in the application of strategies of Mathematical Modeling Assumption, Mathematical Model Construction, Mathematical Modeling Self-monitoring, Mathematical Modeling Validation and Mathematical Modeling Adjustment Strategy.

Mathematical Modeling Assumption Strategy refers to the strategy used to assume the perspicuity and idealization of the situations, conditions and objectives of real problems in the process of mathematical modeling. Through the analysis of subjects' protocol materials, we can categorize the Mathematical Modeling Assumption Strategy that subjects use into Feasible Assumption Strategy, Accurate Assumption Strategy, and Equilibrium Assumption Strategy. Feasible Assumption Strategy refers to the approximate and simplified assumption strategy that the modeler uses in order to pursue the feasibility of the mathematical modeling problems. The adoption of this strategy tends to make much too simple assumptions so that there may be some inaccuracy between the real problem and the model. Accurate Assumption Strategy refers to the accurate and practical assumption strategy that the modeler uses in order to achieve the accuracy of the mathematical modeling problems. The adoption of this strategy tends to make much too complicated assumptions so that the process and results of the modeling may be too complex or even difficult to be realized. Equilibrium Assumption Strategy refers to the moderate and balanced assumption strategy that the modeler uses after the consideration of both the feasibility and accuracy of the mathematical modeling problems.

The adoption of this strategy helps not only to the smooth development of the mathematical modeling, but also to the better solution to the real problems. Table 2 indicates that there are few differentials in the adoption of Feasible Assumption Strategy between excellent and normal students, but excellent ones tend to choose Equilibrium Assumption Strategy, while normal ones Accurate Assumption Strategy.

**Table 2.** The comparison of strategic features of mathematical modeling problem between excellent students and normal students

| Group | MMAS | | | MMCS | | |
|---|---|---|---|---|---|---|
| | FS | AS | ES | PRS | SAS | IGS |
| EG | 9 | 6 | 30 | 9 | 24 | 12 |
| NG | 6 | 27 | 12 | 6 | 6 | 33 |
| $\chi^2$ | | 11.817 | | | 11.431 | |
| $p$ | | 0.001 | | | 0.001 | |

| Group | MMSMS | | | MMVS | | |
|---|---|---|---|---|---|---|
| | IMS | SEMS | RMS | DVS | TDS | IJS |
| EG | 27 | 6 | 12 | 8 | 18 | 19 |
| NG | 3 | 12 | 30 | 28 | 9 | 8 |
| $\chi^2$ | | 18.428 | | | 14.465 | |
| $p$ | | 0.001 | | | 0.001 | |

| Group | MMAJS | | |
|---|---|---|---|
| | AAS | MMEAS | MSAS |
| EG | 18 | 18 | 9 |
| NG | 9 | 9 | 7 |
| $\chi^2$ | | 13.859 | |
| $p$ | | 0.001 | |

Abbreviation: EG, Excellent Group; NG, Normal Group;
MMAS, Mathematical Modeling Assumption Strategy;
MMCS, Mathematics Model Construction Strategy; FS Feasible Strategy;
AS, Accurate Strategy; ES, Equilibrium Strategy; PRS, Pattern Recognition Strategy;
SAS, Sample Analog Strategy; IGS, Immediate Generation Strategy;
MMSES, Mathematical Modeling Self-monitoring Strategy;
MMVS, Mathematical Modeling Validation Strategy; IMS, Immediate Monitoring Strategy;
SEMS, Selective Monitoring Strategy; RMS, Review Monitoring Strategy;
DVS, Data Validation Strategy; TDS, Theoretical Deduction Strategy;
IJS, Intuitive Judgment Strategy; MMAJS Mathematical Modeling Adjustment Strategy;
AAS, Assumptive Adjustment Strategy; MMEAS, Modeling Methodical Adjustment Strategy;
MSAS, Model Solving Adjustment Strategy.

Mathematics Model Construction Strategy refers to the strategy used for the modeling of real problems in the process of mathematical modeling. Through the analysis of subjects' protocol materials, we can categorize the Mathematics Model Construction Strategy that subjects use into Pattern Recognition Strategy, Sample Analog Strategy, and Immediate Generation Strategy. Pattern Recognition Strategy refers to the strategy of matching current problems with the already-existing schemata of problems in mind in order to recognize the mode of current problems and directly adopt the model or method of this problem in the process of modelers' mathematical modeling. Sample Analog Strategy refers to the strategy of constructing models of current problems similar with the samples in the aspects of patterns, methods and models. That is, in the process of mathematical modeling, the modeler recalls the problems which he or she once solved successfully and which is analogous with the current problem, analyses the similarities and differences between previous problems and current problems, and accommodate the thoughts, methods and results of previous problems to apply them to current problems. Immediate Generation Strategy refers to the strategy of doing thinking activities freely according to current problems in modelers' mathematical modeling. Table 2 indicates that there are few differentials in the adoption of Pattern Recognition Strategy between excellent and normal students, but excellent ones tend to choose Sample Analog Strategy, while normal ones Immediate Generation Strategy.

Mathematical Modeling Self-monitoring Strategy refers to the strategy of planning, examining, evaluating, getting feedbacks from, accommodating and monitoring cognitive activities of mathematic modeling. Through the analysis of subjects' protocol materials, we can categorize the Mathematical Modeling Self-monitoring Strategy that subjects use into Immediate Monitoring Strategy, Selective Monitoring Strategy and Review Monitoring Strategy. Immediate Monitoring Strategy refers to the modeler's strategy of continuously monitoring every step and result of the mathematical modeling. Selective Monitoring Strategy refers to the modeler's strategy of selectively monitoring some of the steps and results of the mathematical modeling. Review Monitoring Strategy refers to the modeler's strategy of monitoring all the steps and results of the mathematical modeling after finishing it roughly. Table 2 indicates that excellent students tend to Immediate Monitoring Strategy, while normal ones Selective Monitoring Strategy and Review Monitoring Strategy.

Mathematical Modeling Validation Strategy refers to the strategy of the modeler's validation of the mathematical modeling and the correctness and rationality of the results. Through the analysis of subjects' protocol materials, we can categorize Mathematical Modeling Validation Strategy that subjects use into Data Validation Strategy, Theoretical Deduction Strategy, and Intuitive Judgment Strategy. Data Validation Strategy refers to the strategy of validating the mathematical modeling and the correctness and rationality of the results by checking the data involved in or related to the mathematical modeling. Theoretical Deduction Strategy refers to the strategy of validating the mathematical modeling and the correctness and rationality of the results by checking the existing theoretical deductive steps of the mathematical modeling. Intuitive Judgment Strategy refers to the strategy of validating the mathematical modeling and the correctness and rationality of the results through the modeler's intuition and perception. Table 2 indicates that excellent students tend to use Theoretical Deduction Strategy and Intuitive Judgment Strategy, while normal ones Data Validation Strategy.

Mathematical Modeling Adjustment Strategy refers to the strategy of the modeler's adjustment of some steps of the mathematical modeling when facing some obstacles or mistakes. Through the analysis of subjects' protocol materials, we can categorize Mathematical Modeling Adjustment Strategy that subjects use into Assumptive Adjustment Strategy, Modeling Methodical Adjustment Strategy and Model Solving Adjustment Strategy. Assumptive Adjustment Strategy refers to the strategy of adjusting the previous mathematical modeling assumption when facing obstacles in validation. Modeling Methodical Adjustment Strategy refers to the strategy of adjusting the previous mathematical modeling methods when facing obstacles in validation. Solving Adjustment Strategy refers to the strategy of adjusting the mathematical modeling solution and process when affirming or finding that there is some inaccuracy between the modeling results and the real problems. Table 2 indicates that excellent students tend to use Assumptive Adjustment Strategy and Modeling Methodical Adjustment Strategy, while normal ones Model Solving Adjustment Strategy.

### 3.3 The comparison of thinking patterns, results and efficiency of mathematical modeling between excellent students and normal students

Based on subjects' protocol test and interview materials, analyses and statistic comparisons on thinking patterns features, results and time-consuming of mathematical modeling problem between excellent students and normal students have been taken. Results were shown in Table 3.

From Table 3, we can conclude that excellent students and normal ones differ greatly in thinking patterns, results and efficiency of mathematical modeling. The number of thinking patterns conversion of mathematical modeling problem of excellent students is more than normal ones. It shows that excellent students are more flexible than normal students during the course of mathematical modeling problem; The significant difference of thinking patterns set of mathematical modeling problems appeared between excellent students and normal students, the number of thinking patterns set of normal students is obvious more than excellent ones, it means that thinking patterns set has a major influence on normal students during the course of mathematical modeling problem; The significant difference of the results of mathematical modeling appeared between excellent students and normal ones, the number of reasonable results gained by excellent students is more than normal ones ,while the unreasonable results gained by normal students is obvious more than excellent ones.

**Table 3.** The comparison of thinking patterns, results and efficiency of mathematical modeling between excellent students and normal students

| Group | Problem No. | Thinking patterns | | | Results | | |
|---|---|---|---|---|---|---|---|
| | | TPC | TPS | TPE | T | F | A |
| EG | 45 | 28 | 6 | 7 | 27 | 12 | 6 |
| NG | 45 | 8 | 30 | 35 | 9 | 31 | 5 |
| *Z* | | 3.667 | −4.938 | −5.282 | | | |
| $\chi^2$ | | | | | | 11.920 | |
| *p* | | 0.001 | 0.001 | 0.001 | | 0.001 | |

| Group | time-consuming of mathematical modeling problem (Min) | | |
|---|---|---|---|
| | Easy | Medium | Complex |
| EG | 11.8 | 18.4 | 27.5 |
| NG | 16.6 | 26.7 | 16.3 |
| *Z* | −3.411 | −4.271 | 4.852 |
| *p* | 0.001 | 0.001 | 0.001 |

Abbreviation: EG, Excellent Group; NG, Normal Group; TPC, Thinking Patterns Conversion; TPS, Thinking Patterns Set; TPE, Thinking Patterns Errors; T, True; F, False; A, abandon .

The significant difference of efficiency during mathematical modeling problem also appeared between excellent students and normal students, the time consumed for average numbers of easy and moderate mathematical modeling problems by excellent students is obvious less than normal ones, it showed that efficiency of mathematical modeling problem of excellent students is better than normal ones.

## 4. DISCUSSION

### 4.1. On the differences in representation styles of mathematical modeling problems

This research finds that there are differences in representation styles of mathematical modeling problems between excellent students and normal ones. Although both excellent students and normal ones adopt Symbolic Representation and Methodical Representation, excellent ones use Mechanical Representation more. Moreover, excellent students are able to use Symbolic Representation, Methodical Representation and Mechanical Representation flexibly. Firstly, they use Symbolic Representation to perceive and understand the basic information of the problems; then they use Mechanical Representation to grasp the key points of the problems; thirdly they use Methodical Representation to search and select ways of thinking about and solving the mathematical modeling. On the other hand, normal students usually adopt Methodical Representation immediately after using Symbolic Representation, but they seldom adopt Mechanical Representation. Even though adopting Mechanical Representation, normal students' Mechanical analysis is somewhat unclear and general, so that a way of thinking about mathematical modeling cannot be formed. Actually, Mechanical Representation is the process of integrating the consisting elements of mathematical modeling problems and putting them into certain theoretical categories and frames. It is the key to solving mathematical modeling problems, and the key guideline of distinguishing subjects' levels of mathematical modeling representation. Excellent students are able to search and select certain information and conditions of modeling problems effectively, identify the patterns of problems correctly and quickly, and find rational routes of mathematical modeling in the guidance of general, abstract and transferred Mechanical Representation. However, normal students cannot be guided by Mechanical Representation, so their representation is trivial and disordered, without reaching correct mathematical modeling thought. Current researches (Xin Ziqiang 2004; Yu, 2005; Zhong, Chen & Zhang, 2009; Eysenck & Keane, 2005) finds that different styles of problem representation have different effects on the process and results of the solution to the problems, and that styles of problem representation are a key factor affecting the efficiency of problem solving.

### 4.2. On the differences in the adoption of mathematical modeling strategies

This research finds that there are obvious differences in the adoption of Mathematical Modeling Assumption, Mathematics Model Construction, Mathematical Modeling Self-monitoring, Mathematical Modeling Adjustment Strategy, and so on between excellent and normal students. Previous researches indicate that the degree of richness and the levels of schema affect the quality of representation, which affects the quality of strategy selection. Schema, mediated by representation, affects the search, selection, production and implement of strategies. The higher the level of schema is, the higher the level of strategy is (Li, Ren &Yu, 2001; Xin, 2004). Having rich cognitive schema, excellent students implement various representation of multi-level, form a system of problem representation covering Symbolic Representation, Mechanical Representation and Methodical Representation, and provide a presupposition and basis for activation, search, production and application of mathematical modeling strategies. The excellent subjects' problem schemas are richer, so they have large numbers of problem styles to be used for reference and analogy. Therefore, when constructing mathematical modeling, they firstly adopt Pattern Recognition Strategy. If there is no given model, they then use Sample Analog Strategy to search patterns of problems similar or related to the problems in the schema, analyze the problem structure, constructed model and adopted methods and set them for reference, thus gain the thought pattern and method of modeling. On the other hand, though adopting Pattern Recognition Strategy as well, normal students' problem schemas are relatively poor. So it is not easy for them to manage to search the applicable problem patterns, nor can they find sample problems similar or related to current problems. When failing to adopt Pattern Recognition Strategy, they use Immediate Generation Strategy, at which time they always search blindly in the schema and can seldom use problem patterns and methods for reference. As a result, it is difficult for them to find a right thinking pattern and method of the mathematical modeling. When facing obstacles, the excellent subjects usually find whether there is any problem in their assumption at first, and try to change their assumption or add new assumption. They may also maintain the previous assumption, but analyze and adjust the modeling method. On the other hand, when normal students have some difficulties, they usually examine and adjust the method of getting solutions, but ignore their assumption and method of modeling. Actually, the adoption of Assumptive Adjustment Strategy and Modeling Methodical Adjustment Strategy can effectively improve the probability of successful mathematical modeling.

### 4.3. On the differences in thinking patterns, results, and efficiency of mathematical modeling

This research finds that there are significant differences in thinking patterns, results and efficiency of mathematical modeling. These differences may be the effects of such factors as subjects' cognitive structures of mathematics and science, representation levels, information processing styles, and levels of mathematical modeling strategy adoption.

In excellent students' cognitive structure of mathematics and science, there are rich cognitive schemata, which enable the excellent students to make assumptions beyond the given information. That is to say, once the cognitive schema is activated, it can lead the modeler to search the space of mathematical modeling problems in a certain mode, look for the related features of the mathematical modeling problems, and construct the procedures and methods solving mathematical modeling problems. In excellent students' cognitive schemata, there are a lot of procedural knowledge related to certain areas, so it is easy to develop the procedures and methods of solving mathematical modeling problems. The knowledge in normal students' cognitive schemata is seldom organized, with its unit coupled with each other loosely and consisting of relatively little amount of declarative and procedural knowledge related to certain areas. So they can only develop simple representation superficially similar to mathematical modeling problems, and find it difficult to find out effective procedures and methods to solve mathematical modeling problems. Some researchers (Sweller, 1990; Gilhooly, 1988, pp. 75–90) think that the ultimate reason why experts and novices differ is that experts have the problem-solving schemata which novices do not have.

The levels of subjects' representation of mathematical modeling problems determine their selection and application of mathematical modeling strategies, which further influence the rationality, effectiveness and correctness of their mathematical modeling thinking patterns, which finally affect the process and results of finding solution to mathematical modeling problems. The knowledge in excellent students' schemata is well organized; the units of knowledge, including the declarative knowledge of related areas, are highly related. So in the problem solving process, excellent students can form complicated representation similar to the structure of problems, which provides an important basis for the selection and application of appropriate strategies.

There are differences in information processing modes between excellent students and normal ones. When dealing with related information, excellent subjects are able to retrieve the stored information from the long-term memory system, process it and find solution to it. Previous researches (Sternberg, 1995; Liang, 1997) indicate that the modes of focusing on and encoding problems are different between experts and novices. Novices perceive problems from the surface structure, while experts solve problems according to rules and principles. From the analysis of protocol materials, we know that the protocol materials of excellent subjects is comparatively simple with several smooth problem-solving steps, which indicates that they can quickly retrieve related information to process from their long-term memory system and make fast answers when dealing with new information. The time excellent students spend in solving problems is much less than that spent by normal ones. This phenomenon may have something to do with the "automation" of some procedures in the process of solving mathematical modeling problems, which makes many intermediate procedures in the problem-solving process not appear in short-term memory. On the other hand, when solving mathematical modeling problems, normal students always pay conscious attention to the relation of each conditions, especially the intermediate procedures, so they prolong the time for mathematical modeling problem solving, and reduce the efficiency.

Subjects' selection and application of mathematical modeling strategies directly determine the effectiveness of adopted mathematical modeling strategies, which further influences the rationality and effectiveness of mathematical modeling thinking patterns, which finally affect the results and efficiency of mathematical modeling.

The fact that the protocol material of excellent students is short and concise may result in the impossibility of presenting many intermediate procedures appeared in the short-term memory. When normal students solve mathematical modeling problems, they always pay attention to the relations of each conditions and operation consciously, As a result of which, they extend the time they use to solve problems and increase the steps. The reason why excellent students spend more time solving complicated mathematical modeling problems than normal ones is that the normal ones give up exploring the problems midway.

### 4.4. On the reliability and validity of the protocol testing materials of mathematical modeling presentation

In order to increase the objectivity of the analysis of protocol materials mathematical modeling , we discriminate, classify and analyze subjects' protocol materials of testing and interviewing materials twice (with a three months' interval between them) in the aspect of problem representation, strategy adoption, thinking patterns, modeling results and problem-solving efficiency. The result shows that the consistency coefficient between the two analyses is 0.902, which indicates that the reliability of the testing materials is high.

To insure the application of the methods of protocol analysis and expert-novice comparison to deeply analyze the cognitive features of excellent and normal students' mathematical modeling, and the possible differentials, the researchers make predictions and interviews for many times when working out testing materials of mathematical modeling presentation. The testing problems are adjusted for several times, and mathematical modeling testing materials of different levels of difficulty are prepared, aiming at sufficiently presenting the thinking patterns and possible differentials of students at different mathematical modeling levels when they face mathematical modeling problems of different levels of difficulty. The analysis of the testing results does show that there are characteristics and differentials in problem representation, strategy adoption, thinking patterns, modeling results and problem-solving efficiency between excellent and normal students when they construct mathematical modeling. This indicates that the testing materials can effectively measure the features and differentials of mathematical modeling between excellent students and normal ones, and that the validity of the testing problems of mathematical modeling is satisfactory.

# 5. CONCLUSION

## 5.1. The features and differentials in mathematical modeling problem representation between excellent students and normal ones

There are significant differentials in representation styles, span and methods of mathematical modeling problems between excellent students and normal students. Although both of them use symbolic representation and methodical representation styles, excellent students have more problem numbers to use mechanical representation styles than normal students; excellent students use multiple-representation styles more often while normal students are inclined to use simplicity representation styles; excellent students tend to use circular representation methods while normal students are inclined to use one-way representation methods.

## 5.2. The features and differentials in mathematical modeling strategy adoption between excellent students and normal ones

There are significant differentials in mathematical modeling strategy adoption between excellent students and normal students. Excellent students tend to choose equilibrium assumption strategy, while normal ones accurate assumption strategy; excellent students tend to choose sample analog construction strategy, while normal ones Immediate generation construction strategy; excellent students tend to use immediate self-monitoring strategy, while normal ones selective monitoring strategy and review-monitoring strategy; excellent students tend to use theoretical deduction testing strategy and intuitive judgment testing strategy, while normal ones data testing strategy; excellent students tend to use assumption adjustment strategy and modeling method adjustment strategy, while normal ones model solving adjustment strategy.

## 5.3. The features and differentials in thinking patterns, results and efficiency of mathematical modeling between excellent students and normal ones

There are significant differentials in thinking patterns, results and efficiency of mathematical modeling between excellent students and normal ones. On the thinking patterns of mathematical modeling, the protocol material of excellent students' mathematical modeling is comparatively concise; the logicality of expression is good; the analysis of problems is deep and clear; the thinking pattern is diverse, quick and agile; the mathematical modeling method adopted is heuristic inquiry. The protocol material of normal students' mathematical modeling is comparatively complicated; the logicality of expression is unsatisfactory; the analysis of problems is superficial and obscure; the thinking pattern is single, slow and stubborn; the mathematical modeling method adopted is blind inquiry. On the results of mathematical modeling problem solving, there are more excellent students getting correct (rational) results of mathematical modeling than normal students, who get more incorrect (irrational) results. On the efficiency of mathematical modeling problem solving, the time spent in solving mathematical modeling problems correctly (rationally) is significantly different, with less time spent by excellent students.

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# APPENDIX

**Testing and interview problems**

1. There are two types of fish, one is 10cm length labeled 3 yuans (RMB), and the other is 13cm length labeled 4 yuans (RMB), which type is more cost-effective if you buy?
2. A small lizard is 15cm length, 15kg weight, how its weight become when it grows up to 20cm?
3. The maximum amount of culture is a fixed number M in a certain fishery, however, to leave adequate free room, the actual amount is not up to the maximum one because of guaranteeing the room of growing. If you want to get the maximum amount of fish increment per year, how many fish you should cultivate in actually? (Tips: according to long-term statistical analysis, the amount of fish increment per year is direct proportion to the product between actual amount and amount of adequate free room)
4. One kind of ice-cream packaged by plastic has two specifications, one is 60g and the other is 150g, the 60g ice-cream sell at 1.5 yuans (RMB) which cost is 1 cent per gram, profit is 25%. If the profit of 150g ice-cream is the same to 60g one, how much is the 150g ice-cream? Which one is more cost-effective if you buy?
5. One solution is V1ml with W g solute, if we add organic solvent extraction of solute V2 ml, how much solute is in the solution after extracting N times? Can we extract all the solute? (Tips: the course of extraction meets extraction distribution law; the solute is a constant in the proportion between two solutions.)
6. Pineapple is rich in April every year. Usually, salesman will help you to peel. It is an interesting course because of screw thread in pineapple after peeling. Why people use this way to peel pineapple? This way is just for cutting off seed and peel or avoiding loss pulp? Please demonstrate you idea from mathematics.